The Design of Macroprudential Stress Tests

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We study the design of stress tests that provide information about aggregate and idiosyncratic risk in banks' portfolios and impose contingent capital requirements. In the optimal static test, an adverse scenario fails all weak and some strong banks, limiting the stigma of failure. Sequential tests outperform static tests. Under natural conditions, the optimal sequential test consists of a precautionary recapitalization followed by a scenario that fails only weak banks, similar to TARP in 2008 followed by SCAP in 2009. Our results also shed light on the Federal Reserve's decision to test the banks twice in 2020 during the Covid-19 pandemic. **Keywords:** stress tests, capital requirements, systemic risk, macro-prudential regulation, mechanism design, dynamic mechanisms, Bayesian persuasion. (*JEL* G01, G21, G38, D83)

1. Introduction

Over the last decade, stress tests of large financial institutions have become an essential forward-looking tool for bank regulators. Stress testing is used to understand, evaluate, and address the risks posed to the financial system.

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There are two key aspects of stress tests. The regulator can manage risk at the individual bank level, i.e., the micro-prudential aspect, by making capital requirements conditional on the individual bank's stress test result. Moreover, the stress test aggregates dispersed information across individual bank portfolios and, by doing so, uncovers systemic risks and their implications on the stability of the financial system as a whole, i.e., the macro-prudential aspect. In other words, micro-prudential aspects of stress tests are about discovering and responding to idiosyncratic risks (that can spill over to the rest of the system). The macro-prudential aspects are about aggregate risks such as discovering a large correlation in individual banks' risk exposures.

The macro-prudential stress test design faces the challenge of informing market participants of underlying systemic risks without causing instability. To achieve this goal, the optimal (static) stress test must be partially informative if conducted on its own to avoid exposing the entire system to a capital shortfall. In this paper, we argue that such a need for opacity evaporates if the regulator can require banks to raise capital before conducting a stress test. We show that a round of precautionary recapitalization before the stress test not only dominates the expected welfare under a static stress test, but, under some conditions, also achieves the global optimum in a class of dynamic mechanisms, which we term "sequential stress tests", in which the regulator can communicate information and set capital requirements over multiple rounds.

This paper contains three main results. First, we show that the optimal (static) stress test relies on a partially informative adverse scenario that is failed by all weak banks and some strong ones too. Such partial informativeness reduces the stigma of failing the stress test and facilitates recapitalization. However, failing strong banks puts excessive limits on their risk-taking and leads to asset misallocation – the test's partial transparency is accordingly costly.

Second, the regulator can generate a net welfare improvement over the optimal stress test by recapitalizing the banks before the stress test, with the benefits being highest during times of high uncertainty. By implementing stringent capital requirements first, the regulator stabilizes banks and is then able to carry out a more informative stress test: stronger balance sheets limit the consequences of failing the stress test and eliminate the need to fail strong banks to support weak ones. All strong banks pass the stress test following a precautionary recapitalization and are able to take advantage of the slack in the capital constraints by increasing the amount of socially desirable risky investment. Consequently, by recapitalizing the banks and conducting a more informative stress test, the regulator increases the expected amount of socially desirable risky investment after the test is conducted and, thus, improves welfare. The beneficial substitution of an imperfectly informative stress test with precautionary capital requirements followed by a fully informative stress test makes for a powerful argument in favor of stress test transparency while maintaining bank stability. Precautionary recapitalization also improves risk reallocation across banks in good times while reducing fire sales in bad times, which in turn supports higher asset prices ex-ante.

Finally, we consider a broad class of dynamic mechanisms referred to as sequential stress tests. This set of policies includes precautionary recapitalization as a special case. Surprisingly, precautionary recapitalization followed by a single, fully informative, stress test not only dominates a static stress test but converges to the payoff of the optimal sequential stress test as trading frictions decline or the bank's balance sheet weakens. This is possible as, in the absence of trading frictions, the regulator can match the expected outcome of any sequential stress test by adjusting the magnitude of precautionary recapitalization, all while eliminating any non-fundamental, i.e., policy-induced, uncertainty in the bank's balance sheet. If trading frictions are sizeable, the optimal sequential stress test improves upon precautionary recapitalization by implementing an initial round of testing to determine whether the latter is warranted. However if the bank's balance sheet is weak, then this first stage test is not very informative, reducing the benefit of the sequential test relative to precautionary recapitalization. The characterization of the optimal sequential stress test serves both as a theoretical benchmark for evaluating the efficiency of simpler policies, such as static tests, but also offers insight into the Federal Reserve's decision to stress test the banks twice in 2020 to better assess the impact of the COVID-19 pandemic on the banking system.

We model the financial system as a collection of banks and a competitive market of investors. Banks own safe and risky assets and are partially financed by debt obligations. The riskiness of the banks' assets affects their market price and constitutes the focus of the stress test. The regulator designs the stress test to maximize the expected welfare of all the agents in the financial system net of distress costs that a bank's failure imposes on the broader economy. The (static) stress test consists of first acquiring and disclosing information about the risk of the banks' assets and then setting capital requirements contingent on the shared outcome. We show that the regulator can influence the informativeness of this exercise by publicly choosing a particular stress scenario, mild or adverse, and evaluating the performance of banks' balance sheets under it. The regulator then uses her supervisory authority to specify enforceable capital requirements, which we identify with risk-weighted capital adequacy ratios (CAR), contingent on the outcome of the stress test. To parsimoniously capture both capital regulation and interbank trade, in our main analysis, we assume that the bank improves its risk-weighted capital ratio only by selling risky assets to the market, but our main results hold if the bank can also raise capital via common equity.[1]

The static stress test trades off informed asset allocation with the possibility of socially costly distress stemming from imperfect risk-sharing. On the one hand, a more informative stress test gives the regulator the flexibility to fine-tune capital requirements freeing up the strong banks to make profitable investments while simultaneously imposing a strict capital requirement for weak banks. On the other hand, negative information released by the stress test increases perceived asset risk and, as a result, depresses market prices, making it harder for weak banks to recapitalize. As we show in Section 3, the optimal static stress test trades off these considerations by relying on an adverse scenario, passing only a fraction of the strong banks, and setting strict capital requirements for all those who fail the test. Notably, the stress test's adversity is higher when the financial system is riskier as a whole, setting a higher bar for strong banks to pass. The supervisory authority to set capital requirements is valuable whenever the banks' shareholders, or managers, do not internalize social distress costs to a sufficient extent.

A round of precautionary recapitalization prior to the stress test is beneficial when the financial system's risk is high. Precautionary recapitalization enhances risk-sharing and allows for a more informative stress test to follow. That, in turn, improves asset allocation without increasing distress costs. The partial irreversibility of the bank's portfolio dynamics due to the disclosure of stress test results is key to the optimality of such multi-step interventions. If the bank first raises capital via asset sales but then passes the stress test, it can reacquire its risky asset. Such round-trip transaction has a lasting effect on the bank's balance sheet as the bank suffers a capital loss: it reacquires the asset at a higher price if it passes the test and is, thus, disclosed as having good quality assets. The strong bank's capital loss is offset by the weak bank's capital gain; the latter ends up selling its assets at a premium during precautionary recapitalization, generating effective risk-sharing between weak and strong banks. However, the strict benefit comes from the regulator's ability to conduct a more informative stress test that fails fewer strong banks and specifies capital requirements that are efficient ex-post. The degree to which the bank's trading actions are irreversible is endogenous to the informativeness of the stress test (a less informative stress test creates smaller variation in asset prices, making it easier to

[1] See Section B.4 of Online Appendix for the case when the bank improves its capital ratio via equity issuance.

reacquire the asset) and the degree of precautionary recapitalization, both of which are optimally chosen by the regulator.

Precautionary recapitalization serves two additional benefits. First, as we show in Section 5, it increases the interbank market liquidity because the subsequent stress test fails fewer strong banks. The strong banks that pass the test can buy more risky assets from the banks that fail the test and reduce the need for sales to outside investors, which improves allocation. Second, as we show in Section 6, the increase in the banks' overall liquidity reduces the risks and severity of fire sales, which supports asset prices and reduces the amount of capital banks need to raise ex-ante. The optimal sequential stress test can do even better by implementing perfect risk-sharing across banks and resorting to outside investors only when aggregate risk is sufficiently high.

The beneficial impact of precautionary recapitalization before the stress test underscores the positive effect that equity infusions via the Troubled Asset Relief Program (TARP) had on the efficacy and transparency of the subsequent Supervisory Capital Assessment Program (SCAP) implemented by the U.S. bank regulators during the 2007-2009 financial crisis. The banks' recapitalization conducted via TARP significantly reduced their riskiness and could have been an important factor for why the stress tests administered by the U.S. bank regulators were significantly more informative and compelling than their E.U. counterpart, the Committee of European Banking Supervisors (CEBS). Precautionary recapitalization similar to TARP has also been considered more recently by E.U. bank regulators during the height of the Covid pandemic.[2]

Sequential stress tests also gained regulatory relevance during the Covid pandemic when the Federal Reserve chose to conduct two Dodd-Frank Act Stress Tests (DFAST) during 2020 in light of the material uncertainty associated with the effect of the pandemic on bank balance sheets. The outcome of the first stress test was disclosed in June of 2020 (see Federal Reserve Board [2020a]), with the Federal Reserve suspending share repurchases and limiting dividend growth to increase the banks' capital buffer. The Federal Reserve then tested the banks again in September of 2020 (see Federal Reserve Board [2020b]). Such a two-step approach taken by the Federal Reserve bears strong economic resemblance to the optimal sequential stress test we derive in Proposition 3.

[2] See Section B.1 of the Online Appendix for additional details.

1.1 Related Literature

The canonical trade-off between the allocative efficiency of transparency and the risk-sharing benefits of opacity plays a central role in our analysis and is known as the Hirshleifer [1971] effect. The implications of the Hirshleifer [1971] effect on the design of optimal stress tests were first studied by Goldstein and Leitner [2018].[3] We contribute to the literature by analyzing a joint design of stress tests and capital requirements. In contrast to Goldstein and Leitner [2018] we show that the optimal static stress test fails some strong banks (as opposed to passing some weak banks) when the average quality of banks is sufficiently high due to the complementarity between disclosing of positive information and relaxing capital requirements. Moreover, we are the first to consider sequential tests and point out the benefits of precautionary recapitalization prior to a stress test. Surprisingly, this finding circles back to the argument of Hirshleifer [1971] that trade followed by information disclosure and subsequent reallocation may be beneficial in the context of consumption risk-sharing. The regulator's role in requiring recapitalization is important as bank shareholders may not follow socially optimal actions if there is a wedge between social and private benefits of recapitalization, as highlighted by Williams [2017] in the context of static tests.

Ong and Pazarbasioglu [2014] point out the efficacy and transparency of SCAP in the U.S. in 2008, relative to CEBS in the E.U. in 2009 and 2010. Faria-e Castro, Martinez, and Philippon [2015] show that the optimal stress test may have been more informative in the U.S. due to the greater fiscal capacity available to its regulators at the time. In our model, a lower bailout cost, proxied by a lower social distress cost, also results in an incrementally more informative optimal static test. Such a stress test could, however, require ex-post government bailouts. In contrast, precautionary recapitalization leads to a more informative test and reduces the need for government funds, acting as a form of market insurance, advocated for by Kashyap, Rajan, Stein, et al. [2008] to manage systemic risk. We show in Section 4.4 that precautionary recapitalization can be a more efficient way to increase stress test transparency in times of high uncertainty and, moreover, the regulator would benefit from combining both approaches. Our results suggest that the efficacy and transparency of SCAP can, at least in part, be attributed to the U.S. regulators' successful efforts in recapitalizing the banks ex-ante via TARP which, as documented by Veronesi and Zingales [2010], was responsible for significantly reducing bank CDS spreads.[4]

[3] Monnet and Quintin [2017] study a similar trade-off in the context of securitization.

[4] In the model, banks improve their capital ratios via asset sales, rather than equity issuances. This is motivated by the leverage ratchet effect of Admati et al. [2018] and

Such implementation of the optimal mechanism via two distinct, but sequential, regulatory interventions highlights the importance of the joint effects of contemporaneous bank regulations, as emphasized by Greenwood et al. [2017].

We start by assuming that the regulator can commit to an arbitrary information disclosure policy, as in Kamenica and Gentzkow [2011] and show that the optimal policy can be implemented by subjecting banks to (public) adverse stress scenarios. Similar to Parlatore and Philippon [2018] greater scenario adversity reduces the informativeness of the outcome. In light of the portfolio choice problem solved by the regulator when setting capital requirements, our model provides a novel setting for analyzing the costs and benefits of disclosing information to an active market, complementing Goldstein and Yang [2017] who consider a CARA-normal model. The optimal information disclosure in our model is asymmetric and skewed towards disclosing good, i.e., low-risk states, as the bank's portfolio is further from distress and the regulator can relax capital requirements by a greater margin to increase welfare.

Our results on the optimality of sequential testing stem from partial irreversibility of the impact of trade on banks' portfolios in the presence of sequential information disclosures. While disclosure of a positive piece of information after a negative one could have a completely offsetting effect on the price, interim trade encodes the *path* of prices into the bank's portfolio. In this respect, our paper is related to Grenadier et al. [2016], Orlov et al. [2020], and Malenko and Tsoy [2019] who exploit irreversibility of time in the context of stopping games.

A large literature explores the effects of regulatory disclosures in a crisis. Bouvard et al. [2015] show that, in the face of a looming bank run, the regulator can prevent an inefficient run on the solvent banks from occurring by probabilistically disclosing bad banks. Shapiro and Skeie [2015] show the regulator could provide markets with information about bank balance sheets via costly bailout actions. Gick and Pausch [2013] show the benefits of credible but partial information disclosure, while Inostroza and Pavan [2018] emphasize the coordination role of regulatory communication if bank creditors are dispersed. Importantly, these papers focus on the optimal way to manage a crisis, whereas our model offers guidance on how the regulator can avoid it in a precautionary way via informative stress tests and capital requirements. Inostroza [2020] shows that, if the bank's short-

the added benefit of incorporating interbank trade in the model. Veronesi and Zingales [2010] point to the qualitative similarities between asset sales and equity infusions in the context of the Troubled Asset Relief Program, but acknowledge that equity infusions achieve greater efficiency than asset sales. We show in Appendix B.4 that our static and dynamic results are robust to equity issuance. See Philippon and Schnabl [2013] for an in-depth analysis of optimal recapitalizations outside the context of stress tests.

and long-term creditors are segmented, the bank can raise more funds from long-term creditors if the regulator subsequently conducts a stress test that minimizes the run of short-term creditors in response to liquidity shocks. Our results are qualitatively different as they speak to the optimality of sequential information disclosure and balance sheet adjustments, even if all parties are long-lived, and there is only one source of uncertainty, which is optimally resolved through the regulator's sequential disclosures. Alvarez and Barlevy [2015] show that, in the event of severe contagion, the banks' private incentives to disclose the quality of their balance sheet may be insufficient, and the regulator can contribute by revealing systemically important institutions. Huang [2020] considers a network model of banks and shows that the optimal stress test induces a correlation structure across stress test outcomes that benefits systemically important banks, allowing them to raise the necessary capital while undermining recapitalization of peripheral banks. We contribute to this literature by showing that precautionary recapitalization can be even more valuable in the presence of interbank trade since it increases the subsequent stress test's informativeness and, consequently, improves the liquidity of the interbank market.

2. Baseline Model

The model has three time periods indexed by $t \in \{0,1,2\}$. There is a numeraire good, referred to as cash, and two types of tradable assets. The first asset is safe, modeled as a riskless bond with a unit face value maturing at $t=2$. To economize notation, we, without loss, normalize all parties' discount rate to 0. The second asset is risky and pays an uncertain amount $X \sim U[\theta,1]$ at $t=2$. The random variable $\theta \in \{0,1\}$ captures the quality of the risky asset. The risky asset is payoff equivalent to the safe asset if $\theta=1$, but not if $\theta=0$.[5] Risky asset quality θ is realized at $t=1$, ahead of the actual cash flow realization X at $t=2$, but not directly observed by market participants. There is a common prior $\mathrm{P}_0(\theta=1)=\pi_0$. The economy is comprised of the financial sector, which, for now, we identify with a single bank (or a continuum of identical banks) and a competitive market of risk-neutral investors. The bank is the natural holder of the risky asset and, whereas the risky asset pays X if the bank holds it at $t=2$, it only pays δX if outside investors hold it at $t=2$, where $\delta<1$.[6]

The bank's starting portfolio at $t=1$ is given by $b>0$ units of the safe asset, equivalent to cash, and $a>0$ units of the risky asset. The

[5] We assume there is no cash flow risk in state $\theta=1$ to make the contrast between $\theta=0$ and $\theta=1$ stark. Our results hold if, conditional on $\theta=1$, the cash flow X is also risky.

[6] See DeMarzo and Duffie [1999] and Duffie et al. [2005] for a similar discount specification.

bank can trade the risky asset in the market in period $t=1$, before the risky cash flow X is realized. The market price of the risky asset at $t=1$ is equal to $\delta\mathrm{E}[X]$, where the expectation is taken with respect to all information available to the market about θ at the time of the trade in period $t=1$. The bank also has a liability in the amount d maturing in period $t=2$. We assume it is deterministic and refer to it as debt repayment, but the analysis can be extended for a stochastic realization of d stemming from the settlement of other financial contracts, such as options or credit default swaps. If the bank repays its period $t=2$ liability d, the remaining cash flows are distributed to the bank's shareholders. If the bank is unable to pay d in the second period, then it enters distress and imposes a social cost. To account for the multiple channels through which the bank's distress is socially costly, we model it in reduced form via an increasing convex function $c(\cdot)$ of the bank's cash shortfall at $t=2$, i.e., of how much additional cash the bank would need to repay its liability d when it comes due.[7] If the bank enters period $t=2$ with portfolio $(\hat{b},\hat{a})$, the realized social welfare is

$$\textit{Social Welfare} = \underbrace{b+\hat{a}\cdot X+(a-\hat{a})\cdot\delta X}_{(i)} - \underbrace{c\big(\max\{d-\hat{b}-\hat{a}\cdot X,0\}\big)}_{(ii)}, \qquad (1)$$

where (i) is the joint welfare of the bank's shareholders and creditors, as well as investors in the capital market and (ii) is the realized social cost of distress. The liability d nets out in (i) since it is a transfer from the bank's shareholders to creditors, but not in (ii) where it affects the potential severity of the bank's cash shortfall. The bank's rebalancing from its starting portfolio (b,a) to portfolio $(\hat{b},\hat{a})$ in period $t=1$ affects the allocative efficiency of the risky asset in (i) and the distress cost in (ii).

Capital Adequacy Ratio. The regulator's objective is to maximize the expected value of social welfare in (1), as the bank's shareholders do not internalize the social costs of distress. Traditionally, this goal has been achieved using various forms of capital and liquidity requirements, aimed at managing the bank's solvency risk, by way of limiting the banks' risk-taking.[8] Our model features no maturity mismatch, and,

[7] Bank failure may lead to financial contagion (Allen and Gale [2000]), fire sales and liquidity spirals (Caballero and Simsek [2013], Brunnermeier and Pedersen [2009]), and ultimately a contraction of credit for the real economy (Ivashina and Scharfstein [2010]). Alternatively, the regulator may be required to bailout the bank to avoid its failure (Faria-e Castro et al. [2015]). We assume $c(\cdot)$ is convex to capture the increasing marginal cost of either real effects of distress or bailout funds in the severity of the bank's bankruptcy.

[8] In the U.S., the Federal Reserve enforces minimal liquidity and capital ratios for SIFIs stipulated by Basel III and Dodd-Frank Act. While the minimum common equity tier 1 ratio is fixed at 4.5%, the Federal Reserve requires the banks to hold additional Stress Capital Buffer, which is determined from the stress test results. For details see `https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200810a.htm`.

consequently, both the capital and liquidity regulation can be identified with the bank's Capital Adequacy Ratio (CAR), defined in (2) as the ratio of the marked-to-market value of the bank's assets net of liabilities to the risk-weighted value of its assets. The bank's initial CAR can be written as

$$R_0 \stackrel{def}{=} \frac{b+a\cdot\delta\mathrm{E}[X]-d}{0\cdot b\cdot 1+1\cdot a\cdot\delta\mathrm{E}[X]} \qquad \textit{(Capital Adequacy Ratio)} \quad (2)$$

Whenever the market value of the bank's assets is above its liabilities, i.e., $b+a\cdot\delta\mathrm{E}[X]\geq d$ a reduction in the risky asset holdings improves the bank's capital ratio: retention of $A<a$ units of the risky asset by selling $a-A$ of them to the market increases its safe asset holdings to $B=b+(a-A)\cdot\delta\mathrm{E}[X]$ and does not affect the numerator in (2), but reduces the value of risk-weighted assets in the denominator of (2). Inversely, a bank can attain a prescribed CAR equal to R, by rebalancing its portfolio to

$$A \stackrel{def}{=} \frac{b+a\cdot\delta\mathrm{E}[X]-d}{R\cdot\delta\mathrm{E}[X]}, \qquad B \stackrel{def}{=} d-\frac{1-R}{R}\cdot\big(b+a\cdot\delta\mathrm{E}[X]-d\big). \quad (3)$$

We refer to capital ratio R as more *strict* than capital ratio R_0 if the implied risky asset holding is lower under R than under R_0, i.e., $A<a$.

Static Stress Test. In the midst of the financial crisis of 2007-2009, the uncertainty about the U.S. financial system's health led the Federal Reserve to supplement traditional regulation with a stress test combined with capital requirements contingent on its result. The test provided the broad market with information about the riskiness of the banks' portfolios by evaluating their performance under a hypothetical macro-economic scenario.[9] The choice of this scenario provided the regulator with a degree of flexibility about what kind and how much information the stress test outcome conveyed, which, in the context of our model, we identify with a signal S about θ. The regulator then used its supervisory authority to tighten the capital requirements for banks which performed poorly in the stress test. Such contingent capital

[9] Flannery, Hirtle, and Kovner [2017], Petrella and Resti [2013], Ong and Pazarbasioglu [2014], and Peristiani, Morgan, and Savino [2010] provide empirical evidence that stress test disclosures in the U.S. via SCAP in 2009 and in the E.U. via CEBS in 2010 contained new information manifested through abnormal returns and trading volume. We show explicitly in Section 6 that the stress test is warranted as the regulator has a comparative advantage of identifying bank risks by virtue of being able to observe the cross-section of banks' portfolios.

requirements are naturally captured by a target capital ratio $R(s)$ which depends on the outcome $S=s$.[10]

Definition 1. A (static) stress test $\mathcal{S}=\{S,R(\cdot)\}$ is a signal S, correlated with θ, and a capital adequacy ratio $R(s)$ contingent on the stress test outcome $S=s$.

- A stress test is pass/fail if it induces a binary outcome $S\in\{pass, fail\}$ and is followed by a contingent capital ratio that is more strict if the bank fails the test.

A (static) stress test $\mathcal{S}^*=\{S^*,R^*(\cdot)\}$ is *optimal* if it maximizes the expected social welfare

$$\mathcal{S}^*\in \operatorname*{argmax}_{\mathcal{S}} \mathrm{E}\Big[b+A^*(S)\cdot X+(a-A^*(S))\cdot\delta X-c\big(\max\{d-B^*(S)-A^*(S)\cdot X,0\}\big)\Big]. \tag{4}$$

The regulator designs and commits to a stress test ex-ante, i.e., at $t=0$, before the riskiness θ is realized, capturing the fact that she has no private information about θ relative to the market when committing to a test.[11] Figure 1 summarizes the timing of events in the model.

Motivated by adverse scenarios used by U.S. and E.U. bank regulators, we define a particular class of pass/fail stress tests which *always* fail the bank if $\theta=0$ and *probabilistically* fail the bank if $\theta=1$. An adverse stress scenario can fail the bank even if it has good assets, and, thus, reduces the overall informativeness of the stress test by generating false negative outcomes. It turns out that such pass/fail tests feature prominently in the characterization of the optimal stress test.

Definition 2. A stress test implements an adverse scenario if it is a pass/fail test with zero false positive and some false negative outcomes, i.e.,

$$\mathrm{P}(S=pass|\theta=0)=0 \quad \textit{and} \quad \mathrm{P}(S=fail|\theta=1)>0.$$

The regulator's joint design of the informativeness of the stress test signal S and associated capital requirements $R(S)$ leads to an economic trade-off. On the one hand, a highly informative test allows the regulator

[10] State contingent capital requirements depend on S and not θ to ensure they do not convey any information to the market above and beyond the stress test outcome. This is without loss of generality since S is a sufficient statistic for θ. We implicitly assume that the regulator cannot make private recommendations to the bank in a way that would allow the bank to sell some assets before the market learns about the recommendation. Such recommendations are suboptimal in the context of sequential stress tests defined below.

[11] It does not matter whether the bank knows (but cannot credibly disclose) θ due to the regulator's supervisory authority to enforce capital requirements.

$t=0$: –Regulator designs stress test $\mathcal{S}=\{S,R(\cdot)\}$.

$t=1$: –Risky asset quality θ is realized;
–Regulator conducts stress test $\mathcal{S}$ and discloses signal S;
–Bank buys or sells assets to meet capital requirements $R(S)$.

$t=2$: –Risky asset payoff $X\sim U[\theta,1]$ is realized;
–Bank settles liability d and pays dividends if not in distress;
–If bank is unable to repay d the economy suffers distress cost.

Figure 1
Timing of the stress test and cash flows.

to impose strict capital requirements only if they are truly needed, i.e., if $\theta=0$. On the other hand, revealing that $\theta=0$ leads to a drop in the market price of the risky asset and reduces the market value of the bank's portfolio in this state making its distress unavoidable.

Sequential Stress Test. During the 2007-2009 financial crisis, bank regulators implemented several programs aimed at stabilizing the financial system. One of the most successful such programs was the combination of the Troubled Asset Relief Program (TARP),[12] that recapitalized largest U.S. banks, followed by the Supervisory Capital Assessment Program (SCAP), that provided a highly informative insight into the banks' riskiness and required some banks to raise additional capital. During the market turmoil caused by the COVID-19 pandemic the Federal Reserve expanded its toolkit even further by conducting two stress tests sequentially to better assess the impact of the pandemic on the banking system. To account for such *precautionary* recapitalization prior to a stress test and to consider joint design of multiple related stress tests, we extend Definition 1 to a (very) broad class of mechanisms that allow the regulator to recapitalize the bank over multiple steps and share additional information about θ with the market participants at each of these steps.[13]

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[12] A similar program was active in the U.K. in the summer of 2008, see Veronesi and Zingales [2010] for details.

[13] By the revelation principle, the optimal sequential stress test dominates a mechanism in which the regulator privately communicates stress test outcomes to the bank.

Definition 3. An N-step sequential stress test $\mathcal{S} = \{S_n, R_n(\cdot)\}_{n=1}^N$ is a sequence of signals $\{S_n\}_{n=1}^N$ and contingent capital ratios $\{R_n(\cdot)\}_{n=1}^N$. At each step $n \in \{1, \dots, N\}$, the regulator first discloses the stress test outcome $S_n = s_n$ and the bank is required to rebalance its portfolio to meet the new capital requirement $R_n(s_1, \dots, s_n)$.

For expositional convenience, we assume the sequential stress test is completed within period $t = 1$, similar to Figure 1, and we abstract from the delay costs associated with the multiple steps.[14] A sequential stress test is *optimal* if the bank's resulting portfolio, consisting of B_N units of the safe asset and A_N units of the risky asset, maximizes the expected social welfare in (4).

Notably, TARP did not reveal any information about the banks' assets and simply reduced the riskiness of bank's portfolios. We can formalize this regulatory intervention as an uninformative signal S_1 with capital requirement R_1 that improves the ratio of safe to risky assets. SCAP then contained a much more informative outcome $S_2 \in \{pass, fail\}$ and associated capital requirements $R_2(S_2)$. The sequential implementation of these two policies is naturally embedded in the framework of sequential stress tests.

Definition 4. Precautionary recapitalization followed by a stress test is a two-step sequential stress test $\mathcal{S} = \{S_i, R_i(\cdot)\}_{n=1}^2$ such that S_1 is uninformative and capital ratio R_1 is stricter than R_0.

Surprisingly, such precautionary recapitalization followed by the stress test may achieve the maximum expected welfare among *all* sequential stress tests, as we show in Section 4.1.

2.1 Discussion, Interpretation, and Implementation

We assume that the only information market participants learn about θ comes from the regulatory stress test. This assumption is founded on the idea that the financial regulator is able to evaluate systemic risks by collecting proprietary information about bank portfolios. First, the regulator has access to a cross-section of bank portfolios and identifies the sources of systemic risk in a forward-looking way based on the commonality of their exposures – we show this explicitly in Section 6. Second, the regulator may be better informed about the state of the macroeconomy, leading her to better evaluate the systematic exposure of the banks' portfolios. Finally, even if the banks were better informed

[14] This is an acceptable approximation as the optimal sequential stress test often has only two steps, and the banks make a significant improvement to their capital ratios at the first step, limiting the carry-over of risk across periods.

about their assets than the regulator, this information may not be reflected in the risky asset's price due to adverse selection. The stress test alleviates this problem both via the credible public disclosure, i.e., certification, of risky asset quality, which reduces the information asymmetry, *and* the publicly observable capital requirements, which alleviate the adverse selection problem by disallowing the bank to strategically retain high-quality assets.

Stress Test Scenarios. The stress test's informativeness can be naturally mapped to the adversity of the stress test scenarios used by the Federal Reserve and the European Central Bank. To do so in the context of our stylized model, we relate the payoff of the risky asset X to the already introduced asset quality θ realized at $t=1$, but also an exogenous shock Z realized at $t=2$. The risky asset of quality θ has an unobservable *stochastic* distress threshold z_θ such that, whenever shock Z is less than z_θ, the risky asset pays full value 1 and whenever shock Z exceeds the distress threshold z_θ the asset loses its value and generates a cash flow in [0,1]. Formally,

$$X(Z,\theta)=1\cdot\mathbb{1}\{Z\leq z_\theta\}+U[0,1]\cdot\mathbb{1}\{Z>z_\theta\}. \tag{5}$$

We assume the high quality asset $\theta=1$ has a distress threshold $z_1 \overset{P\text{-}a.s.}{>} 1$, while the low quality asset $\theta=0$ has a distress threshold $z_0 \overset{P\text{-}a.s.}{<} -1$. The shock Z is drawn at $t=2$ from a distribution with support on $[-1,1]$[15]. The regulator does not observe the shock Z prior to its realization, but can evaluate payoff $X(z,\theta)$ in (5) under a hypothetical realization $Z=z$ at $t=1$, before the actual shock Z is realized at $t=1$. Very mild $(z\ll-1)$ or very severe $(z\gg1)$ stress scenarios reveal little information since the bank either passes or fails such scenarios regardless of the underlying θ, i.e., regardless of whether a bank has high or low-quality assets. However, tests of intermediate severity allow the regulator to distinguish between high and low quality assets. The severity of the stress scenario z determines not only the informativeness of the stress test outcome but also the type of errors that the stress test produces. For example, mild stress scenarios with $z<-1$ pass some of the banks even if $\theta=0$, thus generating partially informative signals with type 1 errors (false positives). Such scenarios uncover that the asset is of low quality only if its distress threshold is low, i.e., $z_0<z$. If the low quality asset's threshold z_0 exceeds z, however, then the mild scenario

[15] The distribution of shocks satisfies $\mathrm{P}(z_0<Z<z_1)=1$, i.e., the low quality asset always pays $U[0,1]$ and the high quality asset always pays 1. As a result, stress scenarios with $z<-1$ and with $z>1$ are outside of the support of future shocks Z. Condition $\mathrm{P}(z^0<Z)=1$ is inconsequential and can be relaxed within our model. Condition $\mathrm{P}(Z<z^1)=1$ is necessary due to assumption that risky asset quality can take, at most, two values $\theta\in\{0,1\}$ in our model. This condition can be relaxed in the version of this model where the asset quality θ is continuous.

cannot distinguish it from the high quality asset. On the other end of the spectrum, extremely severe stress scenarios with $z > 1$ generate partially informative tests with type 2 errors (false negatives). They reveal the high quality asset whenever $z_1 > z$, but high quality assets with $z_1 < z$ perform poorly in the test similar to the low quality assets. It is natural to refer to *a scenario as adverse if* $z \geq 1$, leading the weak bank to always fail it. Increasing the severity of the scenario from $z = 1$ to $+\infty$ increases the stress test's failure rate but does so by failing more $\theta = 1$ banks. As a result, failing an adverse scenario does not impose as much stigma as failing a mild scenario. The intuition for such scenario specification is consistent with the Gaussian framework of Parlatore and Philippon [2018] in which scenario adversity reduces the test's informativeness.
Regulatory Capital Requirements. We assume that banks raise capital by selling assets to the market, rather than diluting existing shareholders, consistent with the leverage ratchet effect of Admati, DeMarzo, Hellwig, and Pfleiderer [2018]. We show in Section B.4 of the Online Appendix that our main results are unchanged if we focus on recapitalizations via secondary equity issuance. Asset sales change the denominator while equity issuance affects the numerator of the capital ratio in (2). We focus on the former in the main text as, in the presence of multiple banks in the economy, it allows us to model an active interbank market and study the implications it has on optimal stress test design in Sections 5 and 6. Despite both capital and liquidity requirements being equivalent in our baseline setting, the introduction of interbank trade allows banks to share liquidity, paving way for superior liquidity management to be another channel through which precautionary recapitalization, and sequential tests more generally, improve upon static policies.

We do not focus on the private incentives of the banks in the portfolio choice problem. If the bank's shareholders do not internalize all of the social distress costs, their private cost of distress is always weakly lower than the social one. This assumption leads the bank's shareholders to prefer more risk relative to the regulator's optimum and, thus, makes the capital adequacy requirement bind.

3. Static Stress Tests

The stress test outcome conveys an informative signal S about the riskiness of the bank's asset, which maps to a posterior belief

$$\pi(s) \stackrel{def}{=} \mathrm{P}(\theta = 1 | S = s)$$

shared by all market participants. For example, under an adverse scenario test, the posterior belief is $\pi(pass) = 1$ and $\pi(fail) < \pi_0$. Posterior belief $\pi(S)$ affects the price $p(S)$ of the risky asset via

$$p(S) \stackrel{def}{=} \delta \cdot \mathrm{E}[X|S] = \delta \cdot \Big[\overbrace{\mathrm{E}[X|\theta=1]}^{=1} \cdot \overbrace{\mathrm{P}(\theta=1|S)}^{=\pi(S)} + \overbrace{\mathrm{E}[X|\theta=0]}^{=1/2} \cdot \overbrace{\mathrm{P}(\theta=0|S)}^{=1-\pi(S)}\Big]$$
$$= \delta \cdot \frac{1+\pi(S)}{2}.$$

Belief $\pi(S)$ captures the knowledge about state θ and is important in two ways. First, it helps the regulator determine asset allocation to mitigate distress costs – an allocation effect. Second, it affects the value of the bank's initial portfolio through price $p(S)$ – a wealth effect. To illustrate these two distinct channels, it is useful to consider the case when all of the bank's starting wealth is in the safe asset, i.e., the bank has no risky asset. Denote the bank's (constant) starting wealth by w and define by $V(\pi,w)$ to be the regulator's expected value given belief π about θ and the bank's starting wealth w:

$$V(\pi,w) \stackrel{def}{=} \max_{\hat{b},\hat{a}} \mathrm{E}_\pi \Big[w + \hat{a}(1-\delta) \cdot X - c\Big(\max\Big\{d - \hat{b} - \hat{a} \cdot X, 0\Big\}\Big)\Big] \qquad (6)$$

subject to a no borrowing $\hat{b} \geq 0$, a no short sales $\hat{a} \geq 0$, and a budget $\hat{b} + \hat{a} \cdot \delta \frac{1+\pi}{2} \leq w$ constraints. If the bank's starting wealth is independent of the market price of the risky asset then social welfare increases with full information disclosure since it allows the regulator to impose strict capital requirements only if $\theta = 0$.

Lemma 1 (Benefit of Information, Allocation Effect). For any convex cost function $c(\cdot)$ and any starting wealth w, the regulator's value function $V(\pi,w)$ is strictly increasing in π. Moreover, full disclosure of θ dominates non-disclosure for any prior π, as captured by

$$V(\pi,w) \leq \pi \cdot V(1,w) + (1-\pi) \cdot V(0,w).$$

The cost of disclosing information about θ stems form the bank's exposure to the price of the risky asset. The prospect of distress costs makes the regulator averse to fluctuations in the bank's wealth.

Lemma 2 (Cost of Information, Wealth Effect). For any convex cost function $c(\cdot)$ and any belief π, the regulator's value function $V(\pi,w)$ is weakly concave in starting wealth w.

The bank's starting position in the risky assets leads the market value of its portfolio to fluctuate in response to new information about θ, captured by the identity $w = b + a \cdot \delta(1+\pi)/2$. A more informative stress test generates a mean-preserving spread in π and increases the riskiness of the bank's starting portfolio. This exposes the regulator to greater distress costs.

3.1 Optimal Static Stress Test

The regulator designs the stress test to balance the cost of transparency, entering through the wealth effect in Lemma 2, with the benefit of transparency, entering through the allocation effect in Lemma 1. Given belief π about θ, the expected social surplus is given by

$$V(\pi) \stackrel{def}{=} V\left(\pi, b + a \cdot \delta \frac{1+\pi}{2}\right).$$

Define by π_{DF} to be the lowest belief about θ such that the bank can avoid distress with certainty at $t=2$ were it to sell all of its risky asset to the market at $t=1$:

$$\pi_{DF} \stackrel{def}{=} \min\left\{\pi \geq 0: \quad b + a \cdot \delta \frac{1+\pi}{2} = d\right\}.$$

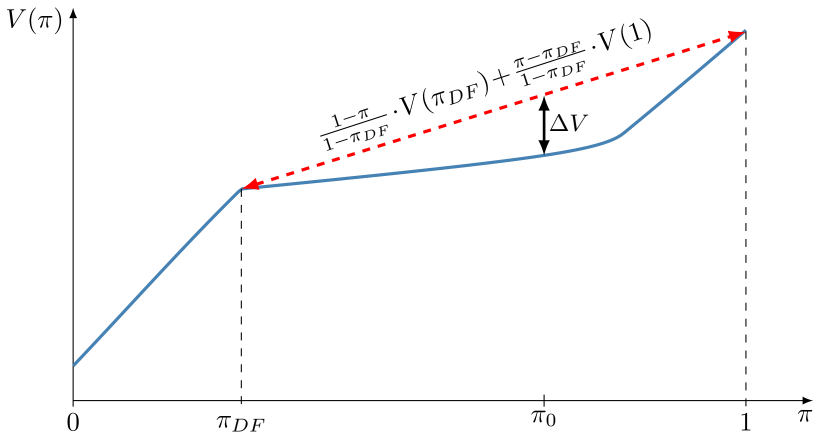


Figure 2
Expected value $V(\pi)$ (solid blue) line as a function of belief π. Concavification of $V(\pi)$ (dashed red) over $[\pi_{DF}, 1]$ line is the expected social value of an adverse pass-fail signal S with posteriors in $\{\pi_{DF}, 1\}$. The difference ΔV captures the incremental social value of such a signal. Parameters: $b=0.25$, $a=1$, $d=0.75$, $\pi_0=0.7$, $\delta=0.8$, $c(x)=4 \cdot \max(x,0)$.

Figure 2 illustrates that $V(\pi)$ is convex for $\pi \geq \pi_{DF}$, i.e., when the market value of the bank's assets exceeds its liabilities in period $t=1$. In this region the allocation effect, captured by Lemma 1 dominates the wealth effect as, by setting efficient capital requirements, the regulator can avoid bank distress and the associated costs. This implies that for $\pi_0 \geq \pi_{DF}$, the regulator can improve upon the no-information value $V(\pi)$ by disclosing some information about θ. Figure 2 illustrates the expected benefit the regulator obtains from an adverse pass-fail signal S with the failing posterior of π_{DF}: the marginal gain from relaxing the capital constraint and allowing the bank to hold the efficient amount of the risky asset, conditional on passing the stress test, exceeds the marginal welfare loss from a larger asset sale during the bank's recapitalization, conditional on failing the stress test. Proposition 1 shows the optimality of adverse pass-fail stress test structure that discloses *at least* the information between π_{DF} and 1, regardless of the cost function $c(\cdot)$ as long as $\pi_0 \geq \pi_{DF}$.[16]

Proposition 1 (Optimal Static Stress Test). Suppose $\pi_0 \geq \pi_{DF}$. The optimal static stress test is an adverse pass/fail test characterized by a failing belief $\pi^* \leq \pi_{DF}$ such that

$$\mathrm{P}(\theta = 1 | S = \mathit{fail}) = \pi^*.$$

The optimal capital adequacy ratio $R(\mathit{fail})$ if the bank fails the stress test implements a portfolio that maximizes expected welfare (6) subject to the bank's balance sheet being evaluated at the failing posterior belief π^*, i.e., $w = b + a \cdot \delta(1+\pi^*)/2$.

Proposition (1) shows that, if $\pi_0 \geq \pi_{DF}$, then the optimal stress test can be implemented via a pass/fail message. With probability $\frac{\pi_0 - \pi^*}{1-\pi^*}$ the stress test passes the bank, revealing that the risky asset is of high quality, and allows the bank to not only hold its initial position but even to expand its risky holdings – a rebalancing shareholders are happy to do given the positive expected return. With probability $\frac{1-\pi_0}{1-\pi^*}$, the optimal stress test fails the bank and requires it to recapitalize. The optimal stress test features false-negative errors: it always fails the bank if $\theta = 0$ and, in addition, it fails the bank if $\theta = 1$ with conditional probability $\frac{1-\pi_0}{1-\pi^*} \cdot \frac{\pi^*}{\pi_0}$. False-negative errors introduce cross-state subsidization by inefficiently restricting the bank's portfolio if $\theta = 1$ while simultaneously ensuring that, even if $\theta = 0$, the price of the risky asset is sufficiently high that recapitalization can, at least partially, mitigate the distress costs. An immediate corollary of Proposition 1 is:

[16] Proposition 1 characterizes the optimal stress test when $\pi_0 \geq \pi_{DF}$. If $\pi_0 < \pi_{DF}$ distress is unavoidable and the optimal stress test depends on the shape of the cost function $c(\cdot)$. For example, if $c(\cdot)$ is sufficiently convex, then the optimal stress test reveals no information for $\pi_0 < \pi_{DF}$.

Corollary 1 (No Solvency Constraints). Suppose the bank can avoid distress even when the risky asset price is $p = \delta \cdot \mathrm{E}[X|\theta = 0] = \delta/2$, i.e., $\pi_{DF} = 0$. Then, the optimal stress test fully reveals θ.

If $\pi_{DF} = 0$, then the value of the bank's portfolio is high enough to avoid distress even if state $\theta = 0$ were to be revealed. In this case the allocation effect makes cross-state risk-sharing suboptimal and the regulator chooses a fully transparent stress test that features no false negatives.

3.2 Optimality of a Default-Free Stress Test

The interaction of optimal information disclosure, i.e., scenario choice, and design of capital requirements can lead to an overall reduction in bank distress risk relative to solely managing information or setting capital requirements.

Lemma 3 (Risk-free Stress Test). Suppose $\pi_0 \geq \pi_{DF} \in (0,1)$. The optimal static stress test protects the bank from distress with certainty, i.e., $\mathrm{P}(\theta = 1|S = \mathit{fail}) = \pi_{DF}$, if and only if the marginal cost of distress is sufficiently high, i.e., $c'(0) \geq \bar{c}$, where $\bar{c}$ is increasing in liability d and decreasing in discount δ.[17]

Lemma 3 highlights that safe recapitalization is optimal when the marginal distress cost is sufficiently large *and* the bank is not overly leveraged. Safe recapitalization, however, might be socially *sub-optimal* even if it is feasible, i.e., $\pi_0 > \pi_{DF}$, as it requires too much misallocation. The optimal stress test allows the bank to enter distress with positive probability either when the welfare gains from holding the asset by the bank are higher, as captured by a lower δ, or when avoiding distress requires too much asset sales, as captured by a higher d. Surprisingly, when $c'(0) \geq \bar{c}$ the optimal stress test is safer than both a fully opaque ($S = \emptyset$) and a fully informative ($S = \theta$) stress tests that may result in bank distress when the asset quality is low ($\theta = 0$), highlighting a complementarity between information/scenario choice and capital requirements.

4. Optimal Sequential Stress Test

The optimal static stress test characterized by Proposition 1 is imperfectly informative about θ so as to limit the loss of value of the bank's portfolio in the event of unfavorable news. It comes at the cost

[17] A closed form upper bound for $\bar{c}$ is $\frac{1-\delta}{3} \cdot \frac{a}{b+a\delta-d} \cdot \frac{a}{d-b} \cdot \left(\frac{d-b}{a} + \frac{b/\delta+a}{b+a\delta-d}\right)$.

of having a significant rate of false-negative results in the stress test's pooling outcome. Importantly, the scope to set capital requirements only once coupled with the desire to make them contingent on the stress test outcome implies that the regulator must first share information about θ, and only then specify capital requirements. Such a "one-shot" view of capital regulation can be unnecessarily restrictive as demonstrated by the Federal Reserve Bank's actions during the financial crisis of 2007-2009 and the COVID-19 pandemic. During the great financial crisis the Federal Reserve (jointly with U.S. Treasury) carried out a combination of a precautionary recapitalization of the major U.S. financial institutions via TARP followed by a highly informative SCAP stress test. While not formally referred to as capital regulation, TARP reduced the banks' riskiness by making them sell preferred equity to the Federal Reserve at fair market prices.[18] By stabilizing the banks first, the Federal Reserve ensured that they could handle the stress test outcome even if it were unfavorable. A similar two stage approach was carried out by the Federal Reserve during the market turmoil caused by the onset of the COVID-19 pandemic. Despite encouraging results of the annual Dodd-Frank Act Stress Test (DFAST), published in July of 2020, the Federal Reserve restricted bank's capital distributions in a precautionary manner prior to conducting an additional (DFAST) stress test to better assess the impact of the COVID-19 pandemic on the banking system.

This section analyzes such multi-stage regulation under the framework of sequential stress tests and demonstrates how sequential capital adjustments can resolve the trade-off between risk-sharing and allocative efficiency better than static policies. In the absence of trading frictions, we show that precautionary recapitalization, followed by a highly informative stress test, constitutes an optimal sequential test. We also characterize the optimal sequential stress test under trading frictions and show that its expected payoff converges to the outcome of precautionary recapitalization if the bank's balance sheet is weak, or the trading frictions are low.

[18] Participation in TARP was de facto mandatory even for banks that were later found to be adequately capitalized by SCAP, e.g., NYT:"JPMorgan never needed the money, but was asked to take it – and complied for the sake of the weaker banks". As such, TARP could be viewed as a form of bank capital regulation. The Federal Reserve can provide loans to member banks through the "discount window" under the conditions specified in Section 10B of the Federal Reserve Act, which require these loans to be "be collateralized to the satisfaction of the lending Reserve Bank." Loans to individuals, partnerships, and corporations, such as primary dealers and insurance companies, are provided only under exigent circumstances and can be collateralized by sound valued privately issued collateral under Section 13.3 of the Federal Reserve Act or by government-issued securities under Section 13.13 of the Federal Reserve Act. In all cases, the collateral must "ensure protection for the taxpayer."

4.1 Precautionary Recapitalization as an Optimal Sequential Stress Test

Definition 4 introduced *precautionary recapitalization* as a full or partial sale of the risky asset by the bank before any information about θ is revealed. A large precautionary sale can significantly reduce the bank's riskiness if $\theta = 0$ but also lead to a large asset misallocation if $\theta = 1$. The regulator can, however, partially address this inefficiency by fully disclosing θ once the asset is sold and relaxing the capital requirements in the event of $\theta = 1$, i.e., allowing the bank to buy back some of the asset at the new price reflecting that $\theta = 1$. In what follows, we show that precautionary recapitalization dominates any, potentially quite complex, sequential stress test by reducing non-fundamental, i.e., policy-induced, risks in the bank's portfolio.

A sequential stress test $\mathcal{S} = \{(S_n, R_n)_{n=1}^N\}$ specifies a (stochastic) posterior belief π_n about θ at the end of each step n, given by $\pi_n \overset{def}{=} \mathrm{E}[\theta | S_1, \ldots, S_n]$. Since any sequential test can be augmented by a final step that fully reveals θ without imposing additional capital requirements, it is without loss to impose that $\pi_N = \theta$. The sequence of signals and capital requirements also specifies the bank's portfolio (B_n, A_n) at the end of each step n determined inductively for $n \geq 1$ via (3) with $(B_0, A_0) = (b, a)$. It is convenient to denote the corresponding (stochastic) market value of portfolio (B_n, A_n) at step n by $W_n \overset{def}{=} B_n + A_n \cdot \delta(1 + \pi_n)/2$. The expected payoff at the end of step N of the sequential stress test $\mathcal{S}$ is given by $V(\pi_N, W_N)$, formally introduced by (6), which denotes the expected value of optimal asset allocation given belief π_N and wealth W_N. The variation in the bank's terminal wealth W_N reflects both fundamental uncertainty about θ, as well as policy-induced, i.e., non-fundamental, uncertainty associated with the probabilistic nature of the disclosure of θ by the sequential test. The regulator would prefer to avoid the policy-induced uncertainty implied by the test since it introduces additional variability in the bank's balance sheet. Formally, given that $\pi_N = \theta$, the expected welfare from the sequential stress test $\mathcal{S}$ can be expressed and bounded by[19]

$$\mathrm{E}\left[W_0 + A_N \cdot X \cdot (1-\delta) - c\left(d - B_N - A_N \cdot X\right)\right] = \mathrm{E}\left[V\left(\theta, W_N\right)\right] \leq \mathrm{E}\left[V\left(\theta, \mathrm{E}_\theta[W_N]\right)\right] \quad (7)$$

where the inequality in (7) follows from Jensen's inequality due to concavity of $V(\pi, w)$ in w, established in Lemma 2. We proceed to show that the upper bound in (7) can be achieved by a single round of trading followed by a static stress test that fully discloses θ. The bank's portfolio

[19] For notational convenience, we write $\mathrm{E}_\theta[\cdot] \overset{def}{=} \mathrm{E}[\cdot | \theta]$.

in any sequential stress test is *self-financing* implying that the bank's expected wealth remains the same between the start and end of the stress test, i.e., $\mathrm{E}[W_N] = \pi_0 \cdot \mathrm{E}_1[W_N] + (1-\pi_0) \cdot \mathrm{E}_0[W_N] = W_0$. While all sequential stress tests preserve the bank's wealth in expectation, they differ in how much risk exposure to the fundamental uncertainty about θ the bank carries through its risky asset holdings. This exposure is summed up by the sensitivity of the bank's expected wealth W_N to the realization of θ, i.e., $\mathrm{E}_1[W_N] - \mathrm{E}_0[W_N] \geq 0$. Consider a two-step stress test in which the bank first adjusts its risky asset holdings to $\hat{A} \stackrel{def}{=} \frac{\mathrm{E}_1[W_N] - \mathrm{E}_0[W_N]}{\delta \cdot (\mathrm{E}_1[X] - \mathrm{E}_0[X])}$ and then the regulator conducts a fully informative static test. The difference between the bank's wealth in state $\theta = 1$ and $\theta = 0$ in this two-step test is $\hat{A} \cdot \delta\big(\mathrm{E}_1[X] - \mathrm{E}_0[X]\big) = \mathrm{E}_1[W_N] - \mathrm{E}_0[W_N]$, matching the bank's exposure to θ in the original sequential stress test. Moreover, because in this two-period test the only signal communicated by the regulator is full disclosure of θ, the bank's terminal wealth $\hat{W}_2$ is solely determined by the realization of θ, thus avoiding any non-fundamental, i.e., policy-induced, variability in the bank's portfolio, formally stated as $\hat{W}_2 = \mathrm{E}_\theta\left[\hat{W}_2\right] = \mathrm{E}_\theta[W_N]$. The expected value generated by such a two-step test thus attains the upper bound obtained in (7) for the expected welfare of the originally considered N-step test. Proposition 2 further shows that such a test constitutes precautionary recapitalization by first verifying that the asset holding $\hat{A}$ defined above is feasible for any sequential stress test and that the optimal A^* is lower than a, implying that the initial rebalancing is indeed a sale of the risky assets.

Proposition 2 (Precautionary Recapitalization). Suppose $b + a \geq d$. Then precautionary recapitalization followed by a fully informative stress test constitutes an optimal sequential stress test for any $\pi_0 \in (0,1)$. Moreover, the optimal precautionary recapitalization requires a complete sale of the risky asset prior to the stress test if, upon completing it, the bank is able to avoid distress if state $\theta = 1$ is disclosed. That is, for $\pi_0 \geq 2 \cdot \frac{d - b/\delta}{a} - 1$ the bank is first required to sell all of its risky assets, then a stress test fully reveals θ and the regulator sets optimal capital requirements for each $\theta \in \{0,1\}$.

An initial sale of risky assets prior to disclosure of θ is a profitable trade for the bank if state $\theta = 0$ is disclosed and the risky asset depreciates in value, but it is a losing trade if state $\theta = 1$ is disclosed and the risky asset appreciates in value. Hence, precautionary sale of the assets effectively transfers wealth from state $\theta = 1$ to state $\theta = 0$, while a precautionary purchase of the risky asset does the opposite. Consequently, the magnitude and direction of the initial capital adjustment depends on the difference in the marginal values

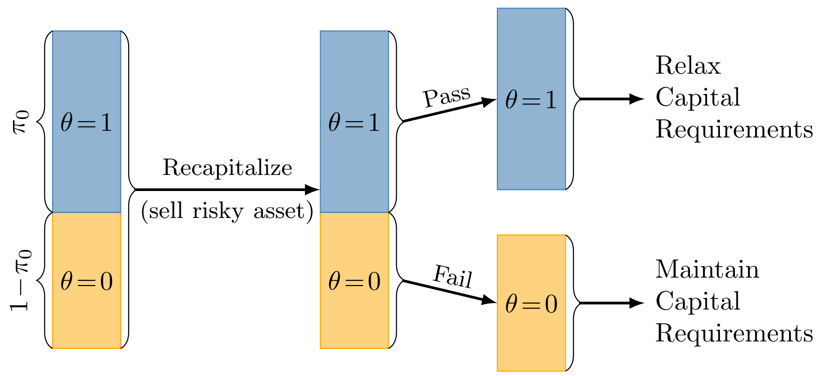


Figure 3
Timing and structure of optimal precautionary recapitalization.

of wealth between states $\theta=1$ and $\theta=0$. If the bank's initial portfolio is sufficiently good in state $\theta=1$ to avoid distress, i.e., $b+a \geq d$, then the marginal value of wealth is higher in state $\theta=0$ since distress in this state is still possible. This implies that such cross-subsidization is valuable and the *optimal* quantity of the risky asset retained A^* is less than a implying that the optimal sequential stress test takes a form of precautionary recapitalization followed by a fully informative stress test. If the market value of the bank's assets is so high that it can avoid distress in state $\theta=1$ even if it sells all of its assets, a condition given by $\pi_0 > 2\frac{d-b/\delta}{a}-1$, this same argument implies that full precautionary recapitalization $A^*=0$ is optimal and generates perfect risk-sharing across states, i.e., $W_2^* = \mathrm{E}_1[W_2^*] = \mathrm{E}_0[W_2^*] = W_0$. Importantly, however, if the bank's initial wealth is insufficient to avoid distress in state $\theta=1$, i.e., $\pi_0 < 2\frac{d-b/\delta}{a}-1$, then full precautionary recapitalization $A=0$ is generally excessive and the optimal magnitude depends on the convexity of the social distress cost $c(\cdot)$.[20]

The optimality of precautionary recapitalization in Proposition 2 stands in sharp contrast to the optimal static stress test in Proposition 1. The static stress test has to balance the benefits of risk-sharing with the costs of inefficient asset allocation. In contrast, precautionary recapitalization in Proposition 2 separates the problems of risk-sharing and allocation into two distinct steps, split by the irreversible disclosure of θ. The latter disclosure makes the bank's initial asset sale, effectively, irreversible, making it reacquire the asset in the second period at prices which reflect the true fundamentals about θ and, crucially, without risk-sharing concerns, permitting ex-post efficient allocation. Even

[20] For instance if the distress cost $c(\cdot)$ is linear, then full precautionary recapitalization is always suboptimal if it leads to distress in state $\theta=1$, i.e., if $\pi_0 \leq 2\cdot\frac{d-b/\delta}{a}-1$.

though the bank may end up with excess capital after precautionary recapitalization, the subsequent stress test is used to free up bank capital when risks are low by implementing a smaller rate of false negatives relative to the optimal static test. This increased likelihood of passing the test conditional on $\theta = 1$ leads to an overall expected increase in the amount of capital the bank can use for asset purchases/investment and, thus, improves welfare. This second step of the stress test fulfills its macro-prudential objective by uncovering sources of risks in the broader market, i.e., beyond the bank's balance sheet, in order to inform the market participants about the riskiness of new investments going forward.[21] The irreversibility of information sharing θ is similar to the irreversibility of time in Malenko and Tsoy [2019], but does not stem from agency conflicts. Instead, the optimality of sequential trade stems from the interaction between balance sheet adjustments and information disclosure and relies on the market price reflecting all information about θ disclosed by the regulator.

Precautionary recapitalizations are not just a theoretical construct and have been used during the financial crisis of 2007-2008 to stabilize the banks. The Troubled Asset Relief Program[22] (TARP) was implemented by the Federal Reserve and Treasury in the fall of 2008, not long before the implementation of the Supervisory Capital Assessment Program (SCAP), the inaugural stress test, in early 2009. Having stabilized the banks, the regulator was then able to conduct a highly informative test without triggering unwanted distress. This argument highlights another, subtle benefit of recapitalization prior to the stress test – it removes the need for the regulator to commit to a partially informative signal S, implemented as a fine-tuned adverse scenario, in the optimal static test. Instead, the policy involves full disclosure of θ once the bank is sufficiently well-capitalized.

Despite the benefits of precautionary recapitalization outlined earlier, the requirement for all banks to sell all of the risky assets for any level of belief and then allowing them to reacquire it in state $\theta = 1$ seems unrealistically burdensome. As such, it is best viewed as a theoretical benchmark highlighting the interaction between sequential balance sheet adjustments and information disclosure. Interestingly, the optimality of such extreme asset turnover holds despite the discount δ suffered by the bank when selling assets to the market, highlighting that it is not

[21] In Section 6 we show how the regulator can uncover the magnitude of systematic risk by evaluating and comparing the cross-section of bank's portfolios.

[22] As discussed in Veronesi and Zingales [2010], TARP was initially aimed at purchasing assets from the bank's balance sheets, but then pivoted to buying the banks' preferred equity. Our results hold if we allowed the banks to adjust their Capital Adequacy Ratios via equity issuance. Yet, the analysis of multiple banks in Sections 5 and 6 incorporates cross-bank trade and, focusing on the former, allows for a more self-contained analysis.

the initial underpricing that is important but, rather, the possibility of reacquiring these assets at that same discount δ in the future. In the next section, we aim to capture a more nuanced notion of transaction costs and study their implications on the optimal test.

4.2 Optimal Sequential Stress Test under Trading Frictions

Trading frictions may limit the bank's ability to reacquire the asset from the market, thus introducing costs to precautionary recapitalizations. Such market imperfections can be naturally captured in the model by endowing investors with bargaining power, which we model as different discounts δ_S and δ_B, such that $\delta_S \leq \delta \leq \delta_B \leq 1$, at which they are willing to buy and sell the risky asset respectively. Investors are willing to pay $\delta_S \mathrm{E}[X]$ for the risky asset, below their autarky valuation $\delta \mathrm{E}[X]$, to generate a positive expected return $(\delta - \delta_S)/\delta_S > 0$ for purchasing the asset. Similarly, investors are willing to sell the risky asset for $\delta_B \mathrm{E}[X]$, which is above their autarky payoff of $\delta \mathrm{E}[X]$, to generate a positive expected return $(\delta_B - \delta)/\delta > 0$ for selling the asset. From the perspective of the regulator, the bargaining power of investors does not itself make trading the asset inefficient since it is a zero-sum transaction between the banks and investors. The positive discount wedge $\delta_B > \delta_S$, however, drains the bank's balance sheet if it engages in round-trip transactions of first selling (at discount δ_S) and then buying (at discount δ_B) the asset.[23]

The advantage of a sequential stress test is best understood by considering the effect of a small precautionary recapitalization in which the bank sells $\varepsilon \ll a$ units of the risky asset prior to full disclosure of θ. This is costly for the bank if the state is $\theta = 1$ as it ends up selling the asset at $\delta_S(1+\pi_0)/2$, which is below the price δ_B of reacquiring it after $\theta = 1$ is disclosed. This cost incorporates both a capital loss, captured by the ε sale carried out when the belief $\pi_0 < 1$, as well as market illiquidity, captured by the difference in discounts $\delta_S \leq \delta_B$. Such transactions drain the safe asset from the bank, which is socially costly as a unit of the safe asset in state $\theta = 1$ can be used to purchase $1/\delta_B$ units of the risky asset, for a total welfare gain of $(1-\delta)/\delta_B$. An expected social cost of an ε sale of the risky asset before θ is disclosed is the product of the ex-ante probability that $\theta = 1$, the magnitude of the capital loss, and the

[23] We introduce transaction costs as a percentage mark-up for buying and selling the asset. These discounts can be micro-founded by modeling a competitive market of small investors who pay a search cost to transact in the market for the risky asset. The banks compete via posted prices to attract investors and, thus, compensate them for the search costs. Moreover, as the optimal stress test minimizes asset turnover to preserve bank balance sheet capacity, our results hold if we were to account for investors dead-weight search costs. Similar round-trip financing costs are present if the bank were to raise capital via equity issuance both due to secondary market imperfections, but also distortions associated with income taxes faced by investors.

marginal social value of the safe asset in that state

$$\pi_0 \quad \times \quad \underbrace{\varepsilon\cdot\left(\delta_B-\delta_S\frac{1+\pi_0}{2}\right)}_{\text{wealth loss if }\theta=1} \quad \times \quad \underbrace{\frac{1-\delta}{\delta_B}}_{\substack{\text{marginal value}\\ \text{of wealth if }\theta=1}} \quad . \qquad \textit{(presale cost)} \tag{8}$$

To illustrate the potential benefits of a presale, denote by A the optimal portfolio of the bank in state $\theta=0$ if the bank enters it with its starting portfolio of (b,a), obtained as the solution to[24]

$$A\overset{def}{=}\underset{\hat{a}}{\operatorname{argmax}}\,\mathrm{E}_0\left[\hat{a}\cdot(1-\delta)X-c\left(d-b-[a-\hat{a}]^+\cdot\frac{\delta_S}{2}+[\hat{a}-a]^+\cdot\frac{\delta_B}{2}-\hat{a}\cdot X\right)\right]. \tag{9}$$

The bank benefits most from precautionary recapitalization in state $\theta=0$ if the optimal portfolio A does not prescribe to reacquire the asset at the higher discount, i.e., if $A<a$. In this case, an ε asset presale leads to a net wealth gain of $\varepsilon\cdot\delta_S((1+\pi_0)/2-1/2)=\varepsilon\cdot\delta_S\pi_0/2$. The expected social benefit of the precautionary recapitalization is the product of the ex-ante probability of $\theta=0$, the capital gain obtained from the presale if $\theta=0$, and the marginal value of safe asset holdings in state $\theta=0$, obtained by differentiating (9) with respect to b and applying the Envelope theorem:

$$(1-\pi_0)\ \times\ \underbrace{\varepsilon\delta_S\frac{\pi_0}{2}}_{\substack{\text{wealth gain}\\ \text{if }\theta=0}}\times\underbrace{\mathrm{E}_0\left[c'\!\left(d-b-[a-A]^+\cdot\frac{\delta_S}{2}+[A-a]^+\cdot\frac{\delta_B}{2}-A\cdot X\right)\right]}_{\geq(1-\delta)/\delta_B,\quad \text{marginal social value of wealth if }\theta=0}. \qquad (\textit{presale benefit})$$

The marginal value of wealth in state $\theta=0$ is always weakly higher than $(1-\delta)/\delta_B$: the per-dollar social gain of purchasing more of the risky asset – and, moreover, strictly exceeds it whenever there is a positive probability of distress. Comparing the marginal benefit of a presale to its cost, we see that precautionary recapitalization is beneficial if the reduction in distress costs in state $\theta=0$ exceeds the expected misallocation in state $\theta=1$:

$$\underbrace{\mathrm{E}_0\left[c'\!\left(d-b-[a-A]^+\cdot\frac{\delta_S}{2}+[A-a]^+\cdot\frac{\delta_B}{2}-A\cdot X\right)\right]}_{\geq(1-\delta)/\delta_B}\geq\frac{1-\delta}{\delta_B}+\frac{1-\delta}{1-\pi_0}\cdot\left(\frac{2}{\delta_S}-\frac{2}{\delta_B}\right). \tag{10}$$

Inequality (10) is always satisfied if $\delta_S=\delta_B$ as, in the absence of trading frictions, the marginal value of wealth is weakly higher in state $\theta=0$ than in state $\theta=1$, and offers an alternative intuition for the optimality of

[24] The safe asset holdings can be imputed from the budget constraint $B=b+[a-A]^+\cdot\delta_S/2-[A-a]^+\cdot\delta_B/2$.

full precautionary recapitalization in Proposition 2. If, however, $\delta_S < \delta_B$, then trading frictions impose a cost on precautionary recapitalization, especially if state $\theta = 0$ is unlikely: if π_0 is close to 1, then (10) is never satisfied as the r.h.s. becomes exceedingly large. Precautionary recapitalization can, thus, be valuable only if the ex-ante riskiness $1-\pi_0$ is high relative to trading frictions.

Lemma 4 (Precautionary Recapitalization under Frictions). It is optimal to precautionary recapitalize the bank only if the probability of distress is sufficiently high, i.e., $\pi_0 \leq \bar{\pi}$, where $\bar{\pi} < 1$ if and only if $\delta_S < \delta_B$. Moreover, if the marginal cost of distress is sufficiently large, i.e., $c'(0) \geq \bar{c}$, then for $\pi_0 \in [\pi_{DF}, \bar{\pi}]$ the optimal precautionary recapitalization requires the bank to sell only the necessary fraction $\frac{d-b-a\cdot\delta_S/2}{\delta_S\cdot\pi_0/2}$ of its risky asset as to avoid distress in state $\theta = 0$.

In the presence of transaction costs, the magnitude of the optimal precautionary recapitalization reflects the expected riskiness of the bank. Once it is conducted, the bank is required to raise additional capital only if it fails the stress test, similar to the SCAP outcome in 2009 when ten out of nineteen banks failed the test and were required to raise additional capital. A sequential test provides an additional degree of adjustment to the regulator by letting her acquire and disclose information about θ prior to engaging in precautionary recapitalization characterized in Lemma 4. We show that, in contrast to Proposition 2, the regulator is better off by disclosing information about θ before requiring the bank to raise capital – the optimal stress test features both sequential communication and sequential recapitalization. First, the regulator acquires a partially informative signal about θ, similar to the adverse scenario of the static test in Proposition 1. Then, she imposes precautionary recapitalization *only* if the bank fails the first test and the posterior belief about asset riskiness is sufficiently high, thus avoiding recapitalization if the strong bank passes the first test. Finally, the regulator fully discloses θ and lets the bank reacquire some of its sold asset if it failed the first stress test but the state is $\theta = 1$. Similar to Proposition 2, the optimal sequential stress test can be implemented in two steps, as depicted in Figure 4.

Proposition 3 (Optimal Sequential Stress Test). Suppose $c'(0) \geq \hat{c}$, $\pi_0 \geq \pi_{DF}$, and $\delta_B \leq \delta \leq \delta_S$ with at least one of the inequalities being strict. Then the optimal sequential stress test involves two steps. The first test is a pass/fail adverse stress test with $P(\theta = 1 | S = \textit{fail}) = \pi^{**} \leq \pi_{DF}$. If the bank fails the first test, it undergoes recapitalization and, after that, a second, fully informative, stress test is conducted.

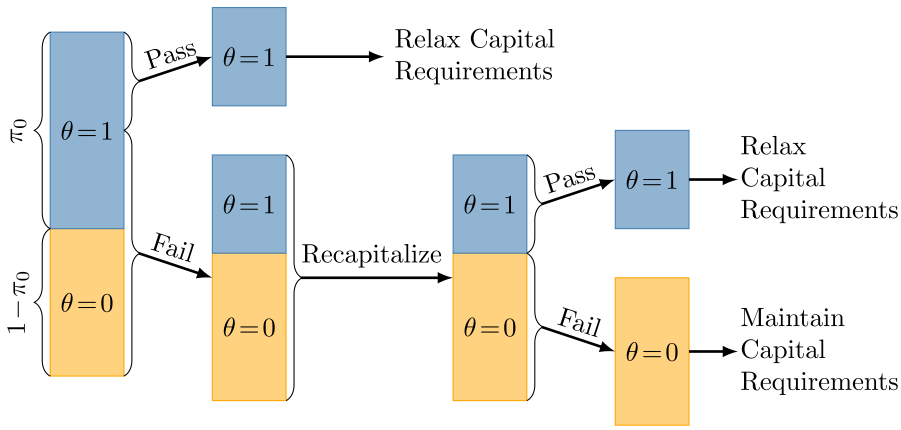


Figure 4
Optimal sequential stress test begins with an adverse pass/fail signal followed by optimal precautionary recapitalization in the event the bank fails it, followed by a fully informative test.

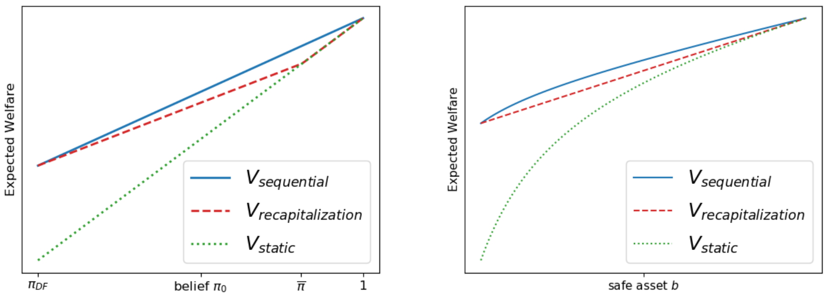
The first stress test probabilistically passes the bank if the state is $\theta=1$ and fails it otherwise. The optimal rate of false negatives ensures that the market value of the bank's portfolio upon failing the stress test is still sufficiently high to mitigate distress costs by selling the risky asset to the market.[25] If the bank passes the first test, then it is found adequately capitalized, and it can increase its asset holdings. Upon failing the first test, however, the bank is required to improve its capital ratio by selling some of its risky asset. Following such recapitalization the regulator then conducts a fully informative stress test. This second stress test aims not to improve the stability of the bank further but, instead, to undo the negative effects of the initial recapitalization in case of the first test producing a false negative outcome for the $\theta=1$ bank. If the second stress test reveals that $\theta=0$, then capital requirements continue to be strict to mitigate the risk of distress. However, if the regulator discloses that $\theta=1$ and the asset is not risky, the regulator allows the bank to purchase some of the asset back from the market.

[25] When $c'(0)$ is sufficiently large, then $\pi^{**}=\pi_{DF}$ and the bank is risk-free even if $\theta=0$. However, for intermediate values of marginal distress cost $c'(0)$ the optimal sequential test described by Proposition 3 entails $\pi^{**}<\pi_{DF}$, corresponding to the bank entering distress with a positive probability if $\theta=0$.

If the marginal cost of distress is sufficiently high, then the first test of the optimal sequential stress test relies on an adverse scenario, failing some of the $\theta = 1$ banks to preserve the value of the bank's balance sheet. The intuition is similar to Proposition 1, but the severity of the optimal scenario may differ. Once the bank recapitalizes at the first step of the test, the regulator implements a less adverse scenario for the failed banks. Given the second scenario's less adverse nature, the strong bank passes it with certainty, leading to the full revelation of the underlying state θ.

4.3 Comparison of Optimal Policies

Our analysis of optimal tests and precautionary recapitalization now allows us to evaluate the efficacy of simpler policies, such as the static test in Proposition 1 and precautionary recapitalization in Lemma 4, relative to the globally the optimal sequential stress test in Proposition 3.



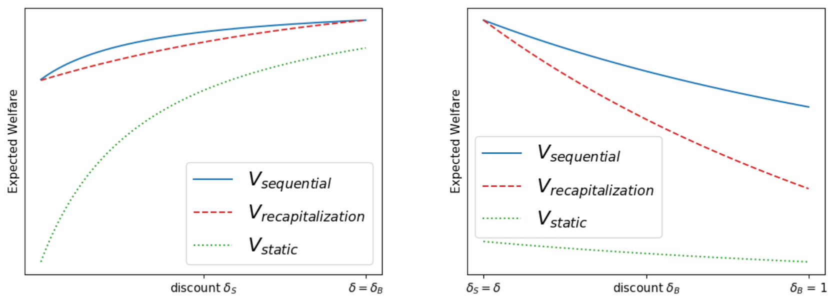
(a) Expected social welfare as a function of the initial belief π_0.

(b) Expected social welfare as a function of the bank's initial safe asset holdings b.

Figure 5
Expected social welfare under optimal static test, precautionary recapitalization, and sequential test as functions of beliefs $\pi \in [\pi_{DF}, 1]$. Parameters: $b = 0.1$, $a = 1$, $d = 0.65$, $\delta = 0.8$, $\delta_S = 0.7$, $\delta_B = 0.9$, $c(x) = 100 \cdot \max(0, x)$.

Figure 5 compares the three policies as a function of the market value of the bank's initial portfolio. Figure 5a shows that precautionary recapitalization does just as well as the sequential test when beliefs are low, i.e., when the price of the bank's risky asset and the starting capital adequacy ratio are low, and performs substantially better than the static test by avoiding asset misallocation in state $\theta = 1$. The sequential stress test generates the greatest gain over precautionary recapitalization in the vicinity of $\bar{\pi}$ since the first step test in Proposition 3 informs the regulator whether precautionary recapitalization is warranted and, thus, has the greatest allocative benefits. Figure 5b paints a similar picture when the variation in the market value of the bank's portfolio stems from the size of its initial safe asset holding b. Both Figures 5a and 5b show that if the bank's balance sheet is weak, then the simpler approach of precautionary recapitalization approximates the expected welfare that can be achieved in a broad class of sequential tests.

Figure 6 illustrates the social welfare achieved by the optimal policies as a function of the trading frictions. Figure 6a shows that as δ_S decreases, while keeping a constant discount $\delta_B = \delta$, precautionary recapitalization approximates the expected welfare of the sequential test and both perform substantively better than the static test. The intuition is very similar to Figures 5a and 5b since a lower δ_S depresses the value of the bank's assets.



(a) Expected social welfare as a function of δ_S, given $\delta_B = \delta = 0.8$.

(b) Expected social welfare as a function of δ_B, given $\delta_S = \delta = 0.7$.

Figure 6
Expected social welfare under optimal static test, recapitalization, and sequential test as functions of discounts δ_S and δ_B. Parameters: $\pi_0 = 0.8$, $b = 0.1$, $a = 1$, $d = 0.65$, $c(x) = 100 \cdot \max(0, x)$.

Figure 6b plots the expected welfare under the three policies as one increases the price at which the bank buys back the assets from the market, captured by increasing δ_B from δ to 1, while keeping $\delta_S = \delta$. A change in δ_B does not change the stress tests' informativeness if $c'(0)$ is sufficiently large so that all stress tests under consideration are default-free. However a greater δ_B makes it more costly for the bank to buy the risky asset from the market and, thus, proxies for liquidity frictions. The first stage of the optimal sequential and optimal static tests are equally affected by the increase in δ_B. However, the reduced ability to purchase risky assets back after a large recapitalization further reduces welfare under the sequential test. If, however, δ_B is low, then precautionary recapitalization again approximates the welfare under the optimal sequential test. Figures 6a and 6b illustrate the consequence of Proposition 2 that the expected welfare under precautionary recapitalization converges to the optimal sequential test if trading frictions are low.

4.4 Comparison of Precautionary Recapitalization and Bailouts

Precautionary recapitalization improves upon a static stress test by increasing the informativeness of the subsequent test and, consequently, improving asset allocation. Consider the effect of the marginal precautionary recapitalization, i.e., a sale of $\varepsilon \ll a$ assets, on the informativeness of the subsequent optimal stress test S_ε. For a sufficiently high marginal distress cost, the test S_ε is an adverse pass-fail with[26] $\pi_{DF}(\varepsilon) \approx \pi_{DF} - \frac{\varepsilon}{a} \cdot (\pi_0 - \pi_{DF})$, i.e., an ε presale of risky assets increases the stress test informativeness by $\frac{\varepsilon}{a} \cdot (\pi_0 - \pi_{DF})$. If the bank fails the stress test, the expected welfare is 0 as the bank sells all of its assets while avoiding distress. The cost of such precautionary recapitalization is incurred from the reduction of the bank's balance sheet capacity in state $\theta = 1$ if the bank passes the test. It is obtained from (8) by adjusting the probability of passing the test:

$$\textit{Presale Cost} \approx \underbrace{\frac{\pi_0 - \pi_{DF}}{1 - \pi_{DF}}}_{\textit{passing probability}} \times \underbrace{\varepsilon \cdot \left(\delta_B - \delta_S \frac{1 + \pi_0}{2} \right)}_{\textit{wealth loss if } \theta = 1} \times \underbrace{\frac{1 - \delta}{\delta_B}}_{\substack{\textit{marginal value} \\ \textit{of wealth if } \theta = 1}}. \quad (11)$$

As shown by Faria-e Castro et al. [2015], access to fiscal capacity allows the regulator to reduce the cost of distress, by providing bailout funds to failing banks, and increase the informativeness of the optimal static stress test. The fundamental distinction between bailouts and precautionary recapitalizations is that the latter is an *ex-ante* intervention while the former is an *ex-post* one. So even though both can be used to increase the stress test transparency, they induce economically different distortions.[27] Precautionary recapitalization

[26] In this section we use approximately equal sign $\approx$ to denote equality up to the second order terms, i.e, up to $o(\varepsilon)$.

[27] In Section B.5 of the Online Appendix we characterize the optimal static test in the presence of bailout costs as well as offer additional comparisons of precautionary recapitalizations and bailouts.

relies on risk-sharing across states and its cost is borne when the fundamental state is good, i.e., $\theta = 1$, and the bank passes the stress test. In contrast, bailouts are conducted only after a bank fails a stress test. If the regulator were to increase the stress test informativeness from π_{DF} to $\pi_{DF}(\varepsilon)$ without precautionary recapitalizing the banks first, the failing banks would be unable to raise enough capital to meet their liabilities. To avoid bank distress the regulator would have to cover the capital shortfall in period $t=2$: the magnitude of such bailout is $a \cdot \delta_S(\pi_{DF} - \pi_{DF}(\varepsilon))/2 \approx \varepsilon \cdot \delta_S(\pi_0 - \pi_{DF})/2$. The probability of needing these bailout funds is the probability that the bank fails the stress test equal to $(1-\pi_0)/(1-\pi_{DF}(\varepsilon))$. Denoting by c_B the marginal social cost of providing \$1 of bailout funds we see that the expected social cost of increasing stress test informativeness to $\pi_{DF}(\varepsilon)$ by relying on fiscal capacity is

$$\textit{Bailout Cost} \approx \underbrace{\frac{1-\pi_0}{1-\pi_{DF}}}_{\textit{failing probability}} \times \underbrace{\varepsilon \cdot \delta_S \frac{\pi_0 - \pi_{DF}}{2}}_{\textit{bailout magnitude}} \times \underbrace{c_B.}_{\substack{\textit{marginal cost} \\ \textit{of bailout funds}}} \qquad (12)$$

Comparing (11) and (12), precautionary recapitalization is a more effective tool to marginally increase the informativeness of the stress test whenever (11) is smaller than (12), which can be simplified as

$$c_B \geq \frac{1-\delta}{\delta_B} + \frac{1-\delta}{1-\pi_0} \cdot \left(\frac{2}{\delta_S} - \frac{2}{\delta_B} \right). \qquad (13)$$

Inequality (13) differs from inequality (10) only in the left hand side, highlighting the fact that bailouts are a substitute for distress costs at the margin. If trading frictions are small, i.e., $\delta_B = \delta_S$, inequality (13) simplifies to $c_B \geq (1-\delta)/\delta_B$. Such requirement on c_B must be satisfied, since the reverse inequality implies that the regulator would, counterfactually, use bailout funds to expand the bank's balance sheet even in the absence of distress. This reasoning shows that precautionary recapitalization dominates fiscal capacity if trading frictions are small.

If trading frictions are large, the optimal policy depends on the uncertainty π_0 about the bank's balance sheet. When the prior π_0 is sufficiently close to 1 the probability of the bank failing the stress test and requiring bailouts is close to 0, and as a result, the expected bailout cost is small as well. Precautionary recapitalization in contrast still incurs a positive cost (11) since, being an ex-ante policy, it requires recapitalization before the state is disclosed. Since the right hand side of (13) is increasing in π_0 there exists a $\hat{\pi}$, at which (13) is binding, such that bailouts are a cheaper tool to achieve transparency for $\pi_0 \geq \hat{\pi}$. For intermediate levels of beliefs $\pi_0 \in [\pi_{DF}, \hat{\pi}]$ precautionary recapitalization shines for two reasons: it has a low marginal cost and a large capacity. First, for $\pi_0 \leq \hat{\pi}$ condition (13) fails and precautionary recapitalization is a cheaper way to marginally increase the transparency of the optimal test. Second, if $\pi_0 > \pi_{DF}$, precautionary recapitalization leverages substantial risk-sharing capacity, as discussed in Section 4.1, and allows the regulator to conduct a fully transparent subsequent stress test without inducing distress. If the regulator's fiscal capacity is limited, however, it may constrain the transparency of the optimal stress test and result in lower expected welfare. Consequently, precautionary recapitalization is more efficient than the use of government funds both in terms of its implied marginal cost and its capacity when the uncertainty about θ is high.

For beliefs π_0 below π_{DF} and especially close to 0, the value of the bank's assets is low. Lack of risk-sharing opportunities limits the effective capacity of precautionary recapitalization. Consequently, precautionary recapitalization alone cannot avoid the bank's distress even though (13) is satisfied. Nevertheless, Proposition 2 shows that, if trading frictions are small, the regulator benefits from a partial precautionary recapitalization prior to conducting a fully informative stress test and, subsequently, bailing out the failing banks. The two approaches interact with each other as access to bailouts changes the effective shape of the social cost of distress faced by the regulator, while precautionary recapitalization acts to equalize the marginal values of wealth across states.

4.5 Regulatory Commitment to Stress Test Disclosures

The literature on stress test disclosure, surveyed by Goldstein and Leitner [2020], has long emphasized the importance of regulatory commitment to partial transparency.[28] Nevertheless, central bankers are at times hesitant in accepting the benefits of incomplete information acquisition and disclosure, guided by their desire to have a clear understanding of bank risks while, at the same time, being worried about how the market will interpret withholding of information. In this context, the fact that precautionary recapitalization followed by a fully informative stress test can approximate the globally optimal sequential stress test and, at the same time, alleviate the need for partial transparency presents the regulators with a useful policy tool. Moreover, post recapitalization, it is in the best interest of the regulator to both acquire as much information as possible, as well as report it truthfully – misreporting a $\theta = 0$ as a $\theta = 1$ state would lead the bank to acquire lots of risky assets at high prices, exposing the regulator to distress costs.

[28] There are multiple ways to justify such commitment power: stylized scenarios as in Section 2.1, public information disclosure as in Dye [1985], or regulator's long-run reputation as in Mathevet et al. [2019].

Precautionary recapitalization can thus be viewed as a substitute for the opacity required by the optimal static stress test.

5. Stress Tests and Interbank Liquidity

The analysis of Sections 3 and 4 considered the optimal stress tests in a model with a single bank. In this context we identified a fundamental economic trade-off between asset misallocation and the bank's safety and showed how sequential stress tests, such as precautionary recapitalization, achieve a better balance between these costs and benefits relative to static stress tests. In practice, however, regulators deal with a banking sector comprised of multiple banks holding heterogeneous portfolios. On the one hand, it makes the regulator's job harder as the optimal stress test differs bank by bank. On the other hand, the co-existence of multiple banks gives rise to an interbank market for risky assets allowing a given bank to reallocate its assets within the banking sector, which is, arguably, more efficient than selling these assets to outside investors.

Consider an extension of our model in which there is a unit mass of banks indexed by $j \in [0,1]$. Just like in Section 2, each bank has b units of the safe asset, a units of the risky asset X_j, and liability d maturing in period $t=2$. Banks differ in the quality of their risky asset X_j with a measure μ of banks, which we refer to as "strong", holding the high quality risky asset with cash flow $X_j=1$, while the remainder $1-\mu$ of banks, which we refer to as "weak", holding the low quality risky asset with cash flow $X_j=X\sim U[0,1]$.[29] The quality of the bank's risky asset is unobservable and is the subject of the regulatory stress test. To minimize new notation, we assume that the bank selling the risky asset in the interbank market has no bargaining power, as it has to satisfy regulatory constraints. We also assume that each unit of the risky asset pays X if it is held by one of the banks, independent of whether the retaining bank is the asset's originator or not, motivated by the banks' superior ability to monitor and manage the risky asset. Finally, for tractability (to manage the problem's increased dimensionality when working with multiple banks), we focus on default-free stress tests which ensure that no bank enters distress in period $t=2$.[30]

[29] When the total fraction of the strong banks μ is known the model features only idiosyncratic risk. In Section B.2 of the Online Appendix we consider an extension that has both aggregate and idiosyncratic uncertainty by allowing the fraction of strong banks μ_θ to depend on the aggregate state $\theta \in \{0,1\}$. State $\theta=1$ denotes low aggregate risk, captured by $\mu_1 \geq \mu_0$. The single-bank model of Section 2 corresponds to the case of pure aggregate risk given by parameters $1=\mu_1>\mu_0=0$ in which all banks are identical, and the portfolio of each one is pinned down by the realization of aggregate state θ.

[30] This simplifying assumption is motivated by the fact that the optimal static and sequential stress tests for a single bank derived in Sections 3 and 4 are default-free as long as $c'(0)$ is large enough.

The optimal static stress test passes the maximum number of strong banks subject to keeping asset prices for the failing banks sufficiently high so that they can recapitalize safely. In the presence of an interbank market, the passing banks can use their safe assets to purchase the risky assets of the failing banks, leading to an unambiguous improvement to social welfare over an economy without interbank trade. Bank liquidity, as measured by the banks' safe asset holdings b, provides a crucial medium for such trade in the interbank market. When the liquidity needed by the failing banks, however, exceeds that available to the passing banks, the optimal static stress test is unable to reallocate *all* of the failing banks' risky assets within the banking system and resorts to inefficient sales to outside investors. Precautionary recapitalization improves upon the static test whenever the latter resorts to such outside sales. The optimal sequential stress test generates further improvement by tapping into *all* available bank liquidity, which achieves maximum risk-sharing across banks.

Proposition 4 (Idiosyncratic Risk). Suppose that $\mu \geq \pi_{DF}$, and strong banks have sufficient balance-sheet capacity, i.e., $b + a \cdot \mu \geq d$. The **optimal default-free**

- **static stress test** is an adverse pass/fail test with $\mathrm{P}(X_j = 1 | S_j = \mathit{fail}) = \pi_{DF}$. Moreover, the banks avoid selling risky assets to outside investors if and only if b is sufficiently large, i.e.,

$$\frac{b + a \cdot \delta_S (1+\mu)/2 - d}{b + a \cdot \delta_S - d} \geq \frac{d - b}{d}. \tag{14}$$

- **precautionary recapitalization** is always valuable if the passing banks do not have sufficient liquidity to purchase the risky asset from the failing banks, i.e., if condition (14) is not met.
- **sequential stress test** reallocates all of the risky asset across the banks and allows the banking system to purchase more risky assets from the market.

Just like the optimal static stress test for a single bank in Proposition 1, the optimal test for a cross-section of banks relies on subjecting every bank to the same adverse scenario, failing each weak bank with certainty, and failing each strong bank with probability $f = f_{DF} = \frac{1-\mu}{\mu(1-\pi_{DF})}$. The rate of false negatives f is just high enough to allow the weak banks to recapitalize and avoid distress at $t = 2$. A total of $\mu \cdot (1 - f_{DF})$ strong banks pass the test and step in as natural buyers of the assets sold by the banks that failed the test. The strong passing banks can acquire at most $\mu \cdot (1 - f_{DF}) \cdot b$ worth of assets while the failing banks need to raise $\big(\mu \cdot f_{DF} + 1 - \mu\big) \cdot \big(d - b\big)$ to avoid distress, which gives rise to condition

(14). Whenever the passing banks lack the liquidity to purchase all of the assets sold by the banks that fail the test, the remaining assets are sold to outside investors imposing a welfare loss. While the resulting allocation differs from the optimal stress test for a single bank in Proposition 1 by virtue of interbank trade, the form of the optimal static stress test and prescribed capital requirements for each bank are identical.

The optimal static test does not fully take advantage of the cross-bank risk-sharing since failing some strong banks to support asset prices also takes up the interbank market's liquidity. Precautionary recapitalization prior to a stress test has two effects. First, it increases the safe assets held by each bank at the cost of selling some of the high quality risky assets to the outside market. A small presale at $t=0$ alone, however, does not improve upon the expected payoff of the optimal static test as, in the aggregate, it is a perfect substitute to an asset sale conducted after the static stress test, given that the banking sector would have to sell some of the risky asset to investors to begin with at $t=1$. Second, it increases the informativeness of the subsequent stress test (lower f), passing more strong banks and unlocking their liquidity to purchase the assets of the failing banks. Precautionary recapitalization, thus, improves welfare by exploiting the complementarity between a transparent stress test and interbank trade.[31]

A sequential stress test consisting of multiple steps of information disclosure and capital adjustments allows the regulator to make use of all liquidity available to the strong banks and reallocate the risky assets only within the financial system whenever strong banks have enough liquidity, formally stated as $b+a\cdot\mu\geq d$. For some parameters[32] the optimal sequential stress test can be implemented in just two steps. During the first step, the stress test fails strong banks with probability $f>f_{DF}$. The failing banks undergo partial recapitalization by selling a fraction of their risky assets to μf strong banks that pass the first stress test. The high false-negative rate $f>f_{DF}$ allows the regulator to then conduct a fully informative test during the second step. At this point, only weak banks fail the test and are required to further improve their capital ratios by selling more risky assets, which can be purchased by the strong banks which pass the second, fully informative test.

[31] See Section B.2 of the Online Appendix for a comparison of the optimal precautionary recapitalization policies with and without interbank trade.

[32] For example, if $a=1$, $d=0.5$, $\delta_S=0.5$, and $\mu=0.5$, then constraint (14) requires $b\geq 0.298$ while a two-step sequential stress test generates an efficient outcome for $b\geq 0.183$. See condition (A.80) in Lemma A.15 in Appendix A for a sufficient condition for the two-step sequential test that reveals the strong banks in two steps is optimal.

5.1 Observed Bank Heterogeneity

In practice, banks may also differ along publicly observable dimensions such as the safe asset holdings b, risky asset holdings a, or liabilities d. To explore the effects of observable heterogeneity, suppose, without loss,[33] that there are two groups of banks $j \in [0,1)$ and $j \in [1,2)$ that, in addition to unobservable riskiness of portfolios $X_j \in \{X,1\}$, also publicly differ in their holdings of safe assets. Specifically, we assume that $b_j = b_L$ for $j \in [0,1)$ and $b_j = b_H$ for $j \in [1,2)$ with $b_H > b_L$. As shown by Lemma 3 and Proposition 4, the optimal default-free test S^L for the $b_j = b_L$ bank is pass-fail and is less informative than the optimal test S^H, which is also pass-fail, for the $b_j = b_H$ bank, captured by

$$\mathrm{P}\big(\theta = 1 \,|\, S_j^L = fail, b_j = b_L\big) = \pi_{DF,L} \stackrel{def}{=} \frac{2}{\delta_S} \cdot \frac{d - b_L}{a} - 1$$

$$> \frac{2}{\delta_S} \cdot \frac{d - b_H}{a} - 1 \stackrel{def}{=} \pi_{DF,H} = \mathrm{P}\big(\theta = 1 \,|\, S_j^H = fail, b_j = b_H\big).$$

If the regulator could run *different* stress test scenarios for the two groups of banks, then she would subject poorly capitalized $b_j = b_L$ banks to the more adverse scenario, corresponding to signal S^L, to reduce the stigma of failing the test, while subject the better capitalized $b_j = b_H$ banks to a less adverse scenario, corresponding to signal S^H. Bank regulators, however, are often hesitant in allowing for heterogeneous stress scenarios as it may give the banks an incentive to conceal losses, as the poorly capitalized b_L shareholders would, generally, benefit from a less adverse scenario at the expense of social distress costs. When the stress test scenario has to be *the same* for the whole system we show that the stress test informativeness has to cater to the poorly capitalized $b_j = b_L$ banks in order to keep them safe, thus distorting allocations for the well capitalized b_H banks. Such an externality creates an additional benefit for sequential stress tests.

Corollary 2 (Observed Heterogeneity). Suppose the regulator must subject all banks to the same scenarios. Then, the **optimal static test** is given by S^L and implements the optimal static test for the $b_j = b_L$ banks. The well-capitalized $b_j = b_H$ banks are allowed to retain more risky assets upon failing this test than the $b_j = b_L$ banks, but fail the test more frequently than their individually optimal test. The **optimal sequential stress test** achieves greater welfare than the sum of the individually optimal stress tests by implementing the allocation of the aggregate bank, i.e., the bank that holds the aggregate

[33] We focus on heterogeneity only in safe asset holdings b to minimize new notation, but the argument applies if the public heterogeneity is in any of these, even combined, dimensions, including ex-ante priors of asset quality.

portfolio of the whole banking system, while subjecting both banks to the same stress scenarios at each step.

Since the optimal static stress test S^L for $b_j = b_L$ banks is less informative, it fails too many of the weak $X_j = X$ but well-capitalized $b_j = b_H$ banks. Such a high failing rate forces the failing $b_j = b_H$ banks to hold an excessive amount of capital at test's S_j^L failing belief $\pi_{DF,L} > \pi_{DF,H}$. Moreover, the increased number of such failing banks reduces their purchasing capacity in the interbank market, further reducing efficiency. The optimal sequential stress test utilizes the fact that S^H is strictly more informative than S^L. Running the latter scenario first, and subsequently refining the signal partition allows the regulator to apply the optimal bank-specific test for each bank all while subjecting banks to the same scenario at each step.

Proposition 4 and Corollary 2 show that static stress tests do not take full advantage of the balance sheet capacity of the entire banking system in the presence of bank heterogeneity. Precautionary recapitalization and sequential stress tests more generally improve upon static policies in two ways. First, they allow the regulator to better reallocate risky assets across banks through the interbank market thus reducing the misallocation costs of selling them to outside investors. Second, they allow the regulator to implement stress scenarios tailored to an individual bank's balance sheet, even when these scenarios differ across banks.

6. Correlation Risk

So far we have modeled the risk in the economy stemming from uncertainty θ about the level of the banks' risky asset cash flows. Stress test disclosure about θ, thus, directly, affected the prices of these assets. In this section, we show that similar economic forces arise endogenously, even without uncertainty about the level of expected cash flows, when asset riskiness stems from the degree of correlation across the banks' exposures.[34] Asset prices incorporate the information about how correlated bank portfolios are through expectations of future fire sales. Fire sales arise when an adverse shock forces a large fraction of banks to sell their assets simultaneously, generating excess supply in the market, and leading the price of these assets to fall below their fundamental value. Higher correlation across banks' portfolios, ceterus paribus, makes future fire sales more likely (a less adverse shock leads to a fire sale) and more severe (the same adverse shock leads more banks

[34] Benoit, Colliard, Hurlin, and Pérignon [2016] provide a literature review documenting that systemic risk often stems from the correlated exposures between bank portfolios.

to sell their assets). The prospect of fire sales in the future depresses asset prices today as market participants internalize that the value of these assets may fall below fundamentals should they need to liquidate them. A regulatory stress test that reveals to the market that banks hold highly correlated exposures also depresses asset prices today as the market learns that future fire sales are more likely. This, in turn, hinders the bank's ability to recapitalize, similar to earlier analysis.

Modeling correlation risk brings forward a general equilibrium feedback loop between capital requirements and asset prices. Stricter capital requirements reduce banks' exposure to adverse shocks for any given level of correlation risk and consequently reduce the likelihood and severity of future fire sales. Lower expectations of future fire sales, in turn, increase asset prices today and allow the banks to improve their capital ratios by preemptively selling some of the risky assets. A better capitalized banking system can support higher asset prices creating a virtuous cycle.

Modeling correlation risk as the source of aggregate uncertainty also provides a micro-foundation for the regulator's superior ability to forecast systemic risk. When the economic uncertainty stems from the correlation between individual banks' portfolios, the regulator has a natural advantage in assessing the quantity of aggregate risk over any outside observer or single bank by being able to monitor the cross-section of the banks' portfolios and evaluate their similarity.[35]

6.1 Embedding Correlation Risk into the Model

We extend our baseline model by an additional period $t=3$ to incorporate the role of bank liquidity in asset prices at $t=2$. There is a continuum of banks $i \in [0,1]$. Each bank has b units of the safe bond, $d>b$ units of debt both maturing in period $t=2$, and a units of the long-term tradable asset that pays 1 unit of the numeraire good in period $t=3$. We assume the total quantity of this long-term asset in the economy is $n \geq a$. Since we normalize the discount rate to 0 and all parties are risk-neutral, the fundamental value of this long-term asset is 1 in each period.[36] The market price of this asset, however, may fall below its fundamental value due to financial constraints of the banking system. We say that there is a fire sale if the market price for the long-term asset in period $t=2$ falls below its fundamental value. Each bank i also faces liquidity risk stemming from a non-tradable, i.e., perfectly illiquid, risky

[35] Duffie [2010] advocates for collection of exposures of the largest banks to the stress scenarios to evaluate how correlated their losses are in these scenarios to establish a forward-looking metric for systemic risk.

[36] It is possible to incorporate tradable idiosyncratic bank risk, in the spirit of Section 5, by replacing the safe cash flow in period $t=3$ with a risky cash flow.

cash flow Y_i that realizes in period $t=2$. Depending on the realization of $\theta \in \{0,1\}$, the fraction μ_θ of banks carry idiosyncratic risk, and fraction $1-\mu_\theta$ carry correlated risk, captured by

$$Y_i = \begin{cases} \xi_i & \text{with probability} \quad \mu_\theta, \\ Y & \text{with probability} \quad 1-\mu_\theta, \end{cases}$$

where random variables $\{Y,(\xi_i)_{i\in[0,1]}\}$ are independent and uniformly distributed over $[0,1]$ and $\mu_1 > \mu_0$. The aggregate risk of the financial system is captured by the fraction $1-\mu_\theta$ of banks that hold correlated risk. The level of systemic risk θ is realized at $t=1$, putting the regulator in a unique position to uncover θ from the cross-sectional similarity of the banks' portfolios *before* cash flows Y_i are realized at $t=2$. By the Exact Law of Large Numbers (Duffie and Sun [2012]), the regulator can forecast the correlation between the banks' portfolios by computing the average similarity of the banks' risky cash flows in period $t=2$ for any *hypothetical* outcome $\omega \in \Omega$, which can be identified with the regulator conducting a scenario test:[37]

$$\begin{aligned} \underbrace{\int_0^1 \int_0^1 \mathbb{1}\{Y_i(\omega) = Y_j(\omega)\}\, di\, dj}_{\substack{\textit{cross-sectional portfolio} \\ \textit{similarity for a hypothetical} \omega\in\Omega}} \overset{P-a.s.}{=} \mathrm{P}_\theta(Y_i = Y_j)^2 &= (1-\mu_\theta)^2 \\ &= \underbrace{\mathrm{corr}(Y_i, Y_j).}_{\substack{\textit{realized correlation} \\ \textit{between cash flows}}} \end{aligned} \tag{15}$$

The conceptual insight of (15) is that the regulator is well-positioned to forecast θ by exploiting her privileged private access to the cross-section of banks' portfolios.[38]

The timing of the model is as follows. At $t=0$ the regulator designs the stress test and implements it at $t=1$. At this time banks improve their risk-weighted capital ratios by selling some of their long-term asset at $t=1$ to short-term risk-neutral investors who consume in period $t=2$. If these investors own any long-term assets at $t=2$, they sell them to the banks in exchange for the numeraire good. This is also the time when the bank's cash flow shocks $\{Y_i\}_{i\in[0,1]}$ are realized and the banks have to satisfy their liabilities d. To do so banks that have excess liquidity buy the long-term tradable asset from banks that need to raise liquidity

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[37] Under the stylized structure of Y_i, the regulator can acquire all of the information about θ just by running a single scenario. While we assume the regulator can manage this acquired information with commitment, it's possible to modify the underlying probability space structure to map partially informative signals with stress test scenarios of different magnitudes of adversity, similar to Section 2.1.

[38] The banks, on the other hand, are likely to be in a better position to learn θ from the time-series of their own portfolio. We assume that this is already incorporated in the common prior π_0.

as well as investors who sell their remaining long-term asset back to the banks. If this available liquidity is insufficient, then the long-term asset may trade at fire sale prices, i.e., at prices lower than its fundamental value. Finally, at $t=3$, the long-term tradable asset pays out its face value 1 to its owners. For tractability, and similar to Section 5, we restrict attention to stress tests that maximize social welfare subject to keeping the banks default-free.

6.2 Fire Sales and Feedback Effect

Bank balance sheet risk stems from individual cash flow shocks Y_i but is amplified by the possibility of fire sales occurring when a large number of banks simultaneously become liquidity constrained, i.e., experience low realizations of Y_i, and have to sell their long-term asset to meet liabilities. Stricter capital requirements reduce the severity of such sales and, as a result, increase ex-ante asset prices. To illustrate the interaction between correlation risk and capital requirements, we first derive asset prices under a fully opaque static stress test, i.e., one in which $S=\emptyset$.

Denote by A to be the maximum quantity of the long-term asset that the regulator allows the bank to retain[39] and by $p_1(A;\pi_0)$ to be the endogenously determined equilibrium price of the long-term tradable asset at $t=1$ given retention quantity A and belief π_0. In period $t=2$, banks suffering negative shocks (low Y_i) sell their long-term asset to the market. The banks who receive positive shocks (high Y_i) pay off their debt d and buy the long-term asset from investors and other banks. Since investors consume only in period $t=2$, the price $p_2(A,\theta,Y)$ of the long-term asset in period $t=2$ in state θ given systematic shock Y is pinned down by the minimum between its face value of 1 and its cash-in-the-market price price $p_2^C(A,\theta,Y)$ given by

$$n\cdot p_2^C(A,\theta,Y)=b+(a-A)\cdot p_1(A;\pi)+A\cdot p_2^C(A,\theta,Y)-d+\overbrace{\int_0^1 Y_i di}^{\mu_\theta/2+(1-\mu_\theta)\cdot Y}. \quad (16)$$

The left hand side of (16) is the market price of the long-term asset in period $t=2$. The right hand side of (16) is the aggregate budget of the banking sector in period $t=2$. It is equal to the sum of the banks' initial endowment b, the proceeds from the sale of the risky asset in period $t=1$, the value of the remaining risky asset in period $t=2$, the realization of the aggregate liquidity shock, averaged across all banks, and net of the aggregate liability d. Whenever the long-term asset trades below its fundamental value in period $t=2$, the market clearing price

[39] Similar to Section 2 there is a one-to-one mapping between asset retention A and the bank's risk-weighted capital ratio R.

$p_2^C(A,\theta,Y)$ equalizes the banks' demand to the overall supply of the long-term asset and is given by (16).

Assumption 1. The quantity of systematic risk $1-\mu_\theta$ is sufficiently low if $\theta=1$ so that $p_2^C(a,1,Y)\geq 1$, and sufficiently high if $\theta=0$, so that $p_2^C(0,0,0)<1$, captured by $\mu_1>2n>\mu_0$.

Assumption 1 guarantees that when $\theta=1$ the long-term asset is trading at par in state $\theta=1$ for any shock Y. If, however, $\theta=0$, then the price of the long-term asset falls below fundamentals for a sufficiently severe aggregate shock, i.e., a sufficiently low realization of Y – the event we refer to as a fire sale. The magnitude of this price deviation from the fundamental value of 1 corresponds to the severity of the fire sale. The price of the risky asset at $t=1$ is given by the investors' resale expectation at $t=2$ as

$$p_1(A;\pi_0)=\delta\cdot\Big(\pi_0+(1-\pi_0)\cdot\mathrm{E}\Big[\min\Big\{p_2^C(A,0,Y),1\Big\}\Big]\Big). \tag{17}$$

The next lemma shows the feedback effect between the bank's exposure A, and the price $p_1(A,\pi)$.

Lemma 5 (Feedback Effect). The period $t=1$ price of the risky asset $p_1(A;\pi_0)$ is decreasing in the bank's exposure to risky assets A if and only if the sale discount $1-\delta$ at $t=1$ is sufficiently low relative to the probability of fire sales $\mathrm{P}\big(p_2^C(A;0,Y)<1\big)$ in period $t=2$.

The dependence of $p_1(A;\pi_0)$ on A is obtained from analyzing the fixed point condition (17). When the sale discount $1-\delta$ is low then selling some of the risky assets at $t=1$, i.e., reducing A, increases the banks' liquidity position at $t=2$. Hence for a fixed level of aggregate shock Y, a reduction in asset holdings A lowers the funding need in period $t=2$ of banks suffering such shock. Moreover, the banks having good idiosyncratic realizations of Y_i have more cash to buy risky assets in the market and support their price. This force makes the fire sale less severe and, on average, supports higher asset prices $p_2(A,0,Y)$. As a result, the price $p_1(A;\pi_0)$ goes up in period $t=1$ as well, which increases the initial sale proceeds $(a-A)\cdot p_1(A;\pi_0)$ for the bank and further improves its liquidity position. This, in turn, results in less severe fire sales and gives rise to the aforementioned virtuous cycle. The feedback effect from stricter capital requirements (lower A) to higher asset prices is this section's key contribution to the message of the paper.

6.3 Optimal Stress Test

Similar to Section 3, the optimal static stress test is an adverse pass-fail test that imposes strict capital requirements conditional on failing it. Importantly, fire sales may still occur in period $t=2$ under the optimal static stress test, but the associated capital requirements reduce their likelihood and severity and the banks are able to weather them safely. A precautionary recapitalization is strictly beneficial whenever fire sales in period $t=2$ are not completely eliminated by the static stress test. Moreover, the optimal precautionary recapitalization is followed by a fully informative stress test, similar to Proposition 2.

Proposition 5. A default-free stress test exists if and only if

$$\pi_0 \geq \pi_{DF}^c \stackrel{def}{=} \max\left\{1-2\cdot\frac{1-\mu_0}{\delta(1-\mu_0/2n)^2}\cdot\frac{b+a\delta-d}{a\cdot n},\quad 0\right\}.$$

The optimal **static test** is adverse pass-fail with $P(\theta=1|S=fail)=\pi_{DF}^c$ with the associated capital requirement $R(fail)=1$. Moreover, the optimal **precautionary recapitalization** is such that the subsequent stress test is fully informative about θ.

Two opposing forces pin down the optimal transparency of the stress test. On the one hand, in order to maintain the solvency of the banks in state $\theta=0$, the stress test needs to be sufficiently opaque, similar to Section 3. On the other hand, imposing strict capital requirements on the banks failing the stress test, improves the asset prices through the virtuous cycle discussed in Section 6.2 and reduces the need for opacity. This macro-prudential nature of the test, which accounts for the asset pricing implications of stricter capital requirements, reduces the stress test's optimal adversity and leads to a more transparent outcome relative to the absence of such a feedback loop.

In Section 4 we have shown that precautionary recapitalization is welfare improving as it increases ex-ante risk-sharing without the costly over-capitalization in state $\theta=1$. In the presence of correlation risk, precautionary recapitalization (and ex-ante risk-sharing) additionally supports the price of the risky asset by ameliorating fire sales. A better capitalized banking system endogenously reduces the riskiness of the $\theta=0$ state (by decreasing the likelihood and severity of fire sales) and, as a result, the necessary amount of capital the banks need to raise at $t=1$ after the stress test. Moreover, a higher price of the long-term asset in period $t=2$ also increases its price in period $t=1$, making it easier for the banks to raise capital. As a result, precautionary recapitalization is strictly welfare improving for all beliefs as long as there is a possibility of fire sales in period $t=2$. The general optimality of precautionary recapitalization whenever there is possibility of a fire sale resonates with our earlier result in Proposition 2 given that, for simplicity, we have abstracted from transaction costs in this setting.

We show that, in presence correlation risk and financially constrained asset markets, capital requirements decrease the riskiness of banks' assets through a reduction in the likelihood of fire sales. However, capital requirements alone might not be enough to ensure bank safety and the static stress test still relies on opacity to mitigate distress. Precautionary recapitalization increases transparency by further reducing the likelihood of fire sale, presenting another stabilizing aspect of this regulatory approach.

7. Conclusion

A stress test is a forward-looking tool used to identify emerging risks and prepare the financial system for their eventuality. Information policy and capital requirements work together in ensuring that banks are safe going forward at the lowest cost to the banking system and society. We show the optimal test relies on an adverse scenario that all weak but also some strong banks fail with the regulator imposing strict capital requirements on failing banks. Precautionary recapitalization prior to the stress test stabilizes the banks and allows the regulator to conduct a fully informative stress test, achieving both efficient risk-sharing and efficient asset allocation. Furthermore, precautionary recapitalization approximates the welfare of the optimal sequential stress test, i.e., a test consisting of multiple rounds of information disclosure and capital adjustments, either when the bank's balance sheet is relatively weak or trading frictions are relatively low. Our analysis of optimal precautionary recapitalizations provides novel insights into the beneficial interaction between TARP in 2008 and SCAP in 2009. Our characterization of the optimal sequential stress test sheds light on the Federal Reserve's approach of stress testing the banks twice in 2020 in response to the Covid pandemic.

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