Frequent Monitoring in Dynamic Contracts
Online Appendix

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A Online Appendix A: Main Text Proofs

A.1 Proof of Lemma 1 (truthful reporting)

The principal has long-term commitment, which implies that any allocation that can be implemented via a mechanism can be implemented via a direct mechanism in which the agent truthfully reports his private information, i.e., the revelation principle applies in this setting.

A.2 Proof of Lemma 4 (continuation value dynamics)

Agent's ex-ante expected value conditional on information available at time $t < \tau$ is given by

$$W_t \stackrel{def}{=} \mathrm{E}_a\left[e^{-r\tau}w_\tau - \int_0^\tau e^{-rs}a_s h\,ds \,\middle|\, \mathcal{F}_t^c\right]$$
$$= e^{-rt}\cdot \mathrm{E}_a\left[e^{-r(\tau-t)}w_\tau - \int_t^\tau e^{-r(s-t)}a_s h\,ds \,\middle|\, \mathcal{F}_t^c\right] - \int_0^t e^{-rs}h\,ds. \tag{A.1}$$

Process $W = \{W_t\}$ is a Levy martingale with respect to filtration $\mathbb{F}^c$ which is generated by two Poisson processes M and R. By Theorems T9 and T17 from Brémaud (1981) there exists a predictable process $(\phi_t, \psi_t)_{t\geq 0}$ with respect to the filtration $\mathbb{F}^c$ such that

$$dW_t = e^{-rt}\phi_t \cdot (\mu f_t\,dt - dM_t) + e^{-rt}\psi_t \cdot \Big((\lambda + \Delta(1-a_t))\,dt - dR_t\Big). \tag{A.2}$$

On the other hand, applying Ito's lemma to (A.1) obtain

$$dW_t = -re^{-rt}w_t\,dt + e^{-rt}dw_t - e^{-rt}a_t h \tag{A.3}$$

The result of the Lemma then follows from equating (A.2) and (A.3).

A.3 Proof of Proposition 1 (necessary and sufficient IC conditions)

Consider an alternative reporting and effort plan of the agent given by $(\hat{a}, \hat{R})$. The off-path reporting strategy of the agent can be identified with a predictable process $\hat{d} = (\hat{d}_t)_{t\geq 0}$ such that $d\hat{R}_t = \hat{d}_t\,d\hat{Y}_t$. Consider the difference $\hat{Y}_t - \hat{R}_t$ – if it is positive then the agent knows there was a bad project that the principal has invested in. According to Lemma 3, the agent's compensation at τ is 0 if $\hat{Y}_t - \hat{R}_t > 0$. Process $\hat{Y} - \hat{R}$ is a counting process with arrival intensity

$$\left(1-\hat{d}_t\right)\cdot\left(\lambda+\Delta(1-\hat{a}_t)\right)\cdot(1-f_t).$$

Define the continuation value of the agent conditional on information available at time $t<\tau$ from following $(\hat{a},\hat{d})$ until time t and then switching to the recommended strategy of (a,d):

$$\hat{W}_t \overset{def}{=} e^{-rt}\cdot \mathrm{E}_t\left[e^{-r(\tau-t)}C_\tau - \int_t^\tau e^{-r(s-t)}a_s h ds\right]\cdot \mathbb{1}\left\{\tau\geq t, \hat{Y}_t-\hat{R}_t=0\right\}+e^{-r\tau}\cdot C_\tau\cdot\mathbb{1}\{\tau\leq t\}-\int_0^{t\wedge\tau}\hat{a}_s h ds$$
$$=e^{-rt}\cdot w_t\cdot\mathbb{1}\left\{\tau\geq t,\hat{Y}_t-\hat{R}_t=0\right\}+e^{-r\tau}\cdot w_\tau\cdot\mathbb{1}\{\tau\leq t\}-\int_0^{t\wedge\tau}\hat{a}_s h ds.$$

Applying Ito's formula to process $\hat{W}$ obtain

$$e^{rt}d\hat{W}_t = -rw_t\,dt + dw_t - w_t\,d(\hat{Y}_t-\hat{R}_t) - \hat{a}_t h\,dt. \tag{A.4}$$

Using the representation for dw_t derived in (7), the drift of $e^{rt}\hat{W}_t$ in (A.4) can be expressed as

$$\begin{aligned}
\mathrm{E}_t\left[e^{rt}d\hat{W}_t\right] &= -rw_t dt + \mathrm{E}[dw_t] - w_t\mathrm{E}\left[d\hat{Y}_t - d\hat{R}_t\right] - \hat{a}_t h dt\\
&= -rw_t dt + rw_t dt + a_t h dt + \phi_t\left[\mu f_t - (\mu + (1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)f_t\right]dt\\
&\quad + \psi_t\left[\left(\lambda+\Delta(1-a_t)\right) - d_t\left(\lambda+\Delta(1-\hat{a}_t)\right)\right]dt - w_t(1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)(1-f_t)dt - \hat{a}_t h dt\\
&= -(\hat{a}_t - a_t)h dt - \phi_t(1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)f_t dt + \psi_t\left[(\lambda+\Delta)(1-d_t)+\Delta(d_t\hat{a}_t - a_t)\right]dt\\
&\quad - w_t(1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)(1-f_t)dt\\
&= -(\hat{a}_t - a_t)h dt - \phi_t(1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)f_t dt + \psi_t\left[\left(\lambda+\Delta(1-\hat{a}_t)\right)(1-d_t)+\Delta(\hat{a}_t - a_t)\right]dt\\
&\quad - w_t(1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)(1-f_t)dt\\
&= -(\hat{a}_t - a_t)h dt + \psi_t\Delta(\hat{a}_t - a_t)dt + (1-d_t)\left(\lambda+\Delta(1-\hat{a}_t)\right)\left(-\phi_t f_t + \psi_t - w_t(1-f_t)\right)dt.
\end{aligned}$$

If incentive constraints (8) and (9) are satisfied, then $\hat{W}$ is a supermartingale which establishes the sufficiency of the corresponding incentive compatibility conditions for the agent. If (8) or (9) are not satisfied with positive probability then we can construct a global deviation for whenever these conditions are violated as

$$\hat{d}_t = \begin{cases} 1 & \textit{if} \quad \psi_t \leq f_t\cdot\phi_t + (1-f_t)\cdot w_t \quad \textit{and} \quad R_t + \int_0^t dY_s dM_s = Y_t,\\ 0 & \textit{otherwise}.\end{cases}$$

and

$$\hat{a}_t = \begin{cases} 1 & \text{if} \quad \min\{\psi_t, f_t \cdot \phi_t + (1-f_t)\cdot w_t\} \geq h/\Delta \quad \text{and} \quad R_t + \int_0^t dY_s dM_s = Y_t, \\ 0 & \text{otherwise.} \end{cases}$$

Process $(\hat{a}, \hat{d})$ disagrees with (a, d) with positive probability if (8) and (9) are not met with a positive probability, leading to a strictly positive profit for the agent. Moreover, if the agent under-reports a bad project and the principal does not catch it, the agent reverts to a global deviation in which he does does not report and does not exert further effort, since his future compensation will be equal to 0 following Lemma 3.

A.4 Proof of Proposition 2 (Principal's value function)

To allow the proof to handle renegotiation-proof contracts, which are more meaningful when project termination has a positive value to the principal, I perform the proof under the additional assumption that the principal collects an outside option of K when he lets go of the agent.[1] In order for the existence and concavity results to also extend to the case of an impatient agent, I prove assume the agent discounts future consumption at rate $\rho \geq r$ which nests the main model. To slightly simplify the exposition, I prove the existence and concavity of the principal's value function under the agent exerting high effort until retirement, and the proof concludes with Lemma A.7 that proves it's global optimality under stated conditions.

A.4.1. Existence of a Solution to (11)

First, I establish existence of a solution under general conditions. Second, I provide sufficient conditions under which the solution is concave. In this second step I set $\lambda = 0$ as it reduces the analysis to a single delay term in (11). The concavity for a small λ holds by continuity since the solution is strictly concave for $\lambda = 0$. For notational convenience define

$$\delta \overset{def}{=} h/\Delta. \tag{A.5}$$

Lemma A.1. *For any given initial condition $v(\delta) = v_0$ there exists a maximal solution to* (11) *and* (12).

Proof. Given an initial condition $v(\delta)$ construct function $v_\varepsilon(w)$ from (11).

[1] In the main text, $K = 0$. The proof consists of two main parts.

(i) For $w \in [0,\delta]$ define function $v_\varepsilon(w)$ as

$$v_\varepsilon(w) = \Big(1 - \frac{w}{\delta}\Big) \cdot K + \frac{w}{\delta} \cdot v_0.$$

For every $x, x' \in \mathbb{R}$ and $w \in [0,\delta]$ define the auxiliary function $G_\varepsilon(w; x, x')$ as

$$G_\varepsilon\big(w;x,x'\big) \overset{def}{=} \max_{\phi,\psi,f}\Big\{\mu f(1-l) + \Big(\mu f\phi + \lambda\psi\Big)\cdot x' + \mu f\cdot\Big(v_\varepsilon(w-\phi) - x\Big) + \lambda\cdot\Big(v_\varepsilon(w-\psi) - x\Big)\Big\} \quad \text{(A.6)}$$

subject to (8) and (9) and the additional constraint that $\phi \geq \min(\varepsilon,\, w-\delta)$ and $\psi \geq \varepsilon$. Since $v_\varepsilon(w)$ is linear for $w \in [0,\delta]$, function $G_\varepsilon(w; x, x')$ is differentiable in all arguments for $w \in [0,\delta]$ and $G_\varepsilon(w; x, x') > 0$ is weakly increasing in x'.

(ii) For $w \geq \delta$ extend the definition of $G_\varepsilon(w; x, x')$ via (A.6) for every $x, x' \in \mathbb{R}$. As long as $v_\varepsilon(w)$ is continuous in w, function $G_\varepsilon(w; x, x')$ is also continuous in w. Define $v_\varepsilon(w)$ as the unique solution to the ordinary differential equation

$$(r+\gamma)\cdot v_\varepsilon(w) = \alpha - \lambda l - \mu + \gamma(K - w) + G_\varepsilon\big(w; v_\varepsilon(w), v_\varepsilon'(w)\big) + v_\varepsilon'(w)\cdot(\rho w + h) \qquad \text{(A.7)}$$

This solution is uniformly continuous, as the term multiplying $v_\varepsilon'(w)$ given by $(\rho w + h)$ is bounded away from 0 and $G(w; x, x')$ is weakly increasing in x'. The solution to ordinary differential equation (A.7) is, thus, uniquely specified given the specification of $v_\varepsilon(w)$ and $G_\varepsilon(w; x, x')$ for $w \in [0,\delta]$. Moreover, $v_\varepsilon(w)$ is continuous in w since $v_\varepsilon'(w)$ is bounded for every w, due to the boundedness of $\frac{\partial}{\partial x'}G(w; x, x')$ as a function of w.

For the constructed solution $v_\varepsilon(w)$ define the corresponding controls

$$\Big(\phi_\varepsilon(w), \psi_\varepsilon(w), f(w)\Big) = \arg\max_{\phi,\psi,f}\Big\{\mu f(1-l) + \Big(\mu\phi f + \lambda\psi\Big)\cdot v_\varepsilon'(w) + \mu f\cdot\Big(v_\varepsilon(w-\phi) - v_\varepsilon(w)\Big) + \lambda\cdot\Big(v_\varepsilon(w-\psi) - v_\varepsilon(w)\Big)\Big\}$$

subject to (8) and (9) and the additional constraint that $\phi \geq \min(\varepsilon,\, w-\delta)$ and $\psi \geq \varepsilon$.

We proceed to show that $v_\varepsilon(w)$ is increasing in ε. Suppose that $\varepsilon_1 > \varepsilon_2$ and consider the two corresponding solutions $v_{\varepsilon_1}(w)$ and $v_{\varepsilon_2}(w)$. We aim to show that $v_{\varepsilon_1}(w) \geq v_{\varepsilon_2}(w)$. Note that for $w \in [0, \delta+\varepsilon_2]$ the two functions coincide, i.e., $v_{\varepsilon_1}(w) = v_{\varepsilon_2}(w)$, since $\phi_{\varepsilon_1}(w) = \phi_{\varepsilon_2} = w - \delta$, implying that $\psi_{\varepsilon_1}(w) = \psi_{\varepsilon_2}(w)$. For $w \geq \delta + \varepsilon_2$ we have

$$(r+\gamma)v_{\varepsilon_2}(w) \geq \alpha - \lambda l - \mu\cdot\Big(1 - f_{\varepsilon_1}(w) + f_{\varepsilon_1}(w)l\Big) + \gamma(K-w) + v_{\varepsilon_2}'(w)\cdot\Big(\rho w + h + \mu f_{\varepsilon_1}(w)\phi_{\varepsilon_1}(w) + \lambda\psi_{\varepsilon_1}(w)\Big)$$

$$+\mu f_{\varepsilon_1}(w)\cdot\Big(v_{\varepsilon_2}\left(w-\phi_{\varepsilon_1}(w)\right)-v_{\varepsilon_2}(w)\Big)+\lambda\cdot\Big(v_{\varepsilon_2}\left(w-\psi_{\varepsilon_1}(w)\right)-v_{\varepsilon_2}(w)\Big)$$

$$\begin{aligned}(r+\gamma)v_{\varepsilon_1}(w)&=\alpha-\lambda l-\mu\cdot\Big(1-f_{\varepsilon_1}(w)+f_{\varepsilon_1}(w)l\Big)+\gamma(K-w)\\&+v'_{\varepsilon_1}(w)\cdot\Big(\rho w+h+\mu f_{\varepsilon_1}(w)\phi_{\varepsilon_1}(w)+\lambda\psi_{\varepsilon_1}(w)\Big)\\&+\mu f_{\varepsilon_1}(w)\cdot\Big(v_{\varepsilon_1}\left(w-\phi_{\varepsilon_1}(w)\right)-v_{\varepsilon_2}(w)\Big)+\lambda\cdot\Big(v_{\varepsilon_1}\left(w-\psi_{\varepsilon_1}(w)\right)-v_{\varepsilon_1}(w)\Big)\end{aligned}$$

Taking the difference between the two expressions above we have

$$\begin{aligned}(r+\gamma)\Big(v_{\varepsilon_2}(w)-v_{\varepsilon_1}(w)\Big)\geq&\Big(v'_{\varepsilon_2}(w)-v'_{\varepsilon_1}(w)\Big)\cdot\Big(\rho w+h+\mu f_{\varepsilon_1}(w)\phi_{\varepsilon_1}(w)+\lambda\psi_{\varepsilon_1}(w)\Big)\\&+\mu f_{\varepsilon_1}(w)\cdot\Big(v_{\varepsilon_2}\left(w-\phi_{\varepsilon_1}(w)\right)-v_{\varepsilon_2}(w)-v_{\varepsilon_1}\left(w-\phi_{\varepsilon_1}(w)\right)+v_{\varepsilon_1}(w)\Big)\\&+\lambda\cdot\Big(v_{\varepsilon_2}\left(w-\psi_{\varepsilon_1}(w)\right)-v_{\varepsilon_2}(w)-v_{\varepsilon_1}\left(w-\psi_{\varepsilon_1}(w)\right)+v_{\varepsilon_1}(w)\Big).\end{aligned}\tag{A.8}$$

We proceed to show that $v'_{\varepsilon_1}(w)\geq v'_{\varepsilon_2}(w)$ for all $w>\delta$. For $w\leq\delta+\varepsilon_2$ the optimal controls coincide implying that $v_{\varepsilon_1}(w)=v_{\varepsilon_2}(w)$. This implies that at $w=\delta+\varepsilon_2$ we have

$$0\geq v'_{\varepsilon_2}(\delta+\varepsilon_2)-v'_{\varepsilon_1}(\delta+\varepsilon_2)\qquad\Rightarrow\qquad v'_{\varepsilon_1}(\delta+\varepsilon_2)\geq v'_{\varepsilon_2}(\delta+\varepsilon_2).$$

Suppose there exists $\hat{w}>\delta+\varepsilon_2$ such that $v'_{\varepsilon_1}(\hat{w})=v'_{\varepsilon_2}(\hat{w})$. Then

$$\begin{aligned}0&=\Big(v'_{\varepsilon_2}(\hat{w})-v'_{\varepsilon_1}(\hat{w})\Big)\cdot\Big(\rho\hat{w}+h+\mu f_{\varepsilon_1}(\hat{w})\phi_{\varepsilon_1}(\hat{w})+\lambda\psi_{\varepsilon_1}(\hat{w})\Big)\\&\overset{(i)}{\leq}(r+\gamma)\cdot\Big(v_{\varepsilon_2}(\hat{w})-v_{\varepsilon_1}(\hat{w})\Big)+\mu f_{\varepsilon_1}(\hat{w})\cdot\Big(v_{\varepsilon_2}(\hat{w})-v_{\varepsilon_2}\left(\hat{w}-\phi_{\varepsilon_1}(\hat{w})\right)-v_{\varepsilon_1}(\hat{w})+v_{\varepsilon_1}\left(\hat{w}-\phi_{\varepsilon_1}(\hat{w})\right)\Big)\\&\qquad+\lambda\cdot\Big(v_{\varepsilon_2}(\hat{w})-v_{\varepsilon_2}\left(\hat{w}-\psi_{\varepsilon_1}(\hat{w})\right)-v_{\varepsilon_1}(\hat{w})+v_{\varepsilon_1}\left(\hat{w}-\psi_{\varepsilon_1}(\hat{w})\right)\Big)\\&=(r+\gamma)\cdot\Big(v_{\varepsilon_2}(\hat{w})-v_{\varepsilon_1}(\hat{w})\Big)+\mu f_{\varepsilon_1}(\hat{w})\cdot\int_{\hat{w}-\phi_{\varepsilon_1}(\hat{w})}^{\hat{w}}v'_{\varepsilon_2}(x)\,dx-\mu f_{\varepsilon_1}(\hat{w})\cdot\int_{\hat{w}-\phi_{\varepsilon_1}(\hat{w})}^{\hat{w}}v'_{\varepsilon_1}(x)\,dx\\&\qquad+\lambda\cdot\int_{\hat{w}-\psi_{\varepsilon_1}(\hat{w})}^{\hat{w}}v'_{\varepsilon_2}(x)\,dx-\lambda\cdot\int_{\hat{w}-\psi_{\varepsilon_1}(\hat{w})}^{\hat{w}}v'_{\varepsilon_1}(x)\,dx\\&=(r+\gamma)\cdot\Big(v_{\varepsilon_2}(\hat{w})-v_{\varepsilon_1}(\hat{w})\Big)+\mu f_{\varepsilon_1}(\hat{w})\cdot\int_{\hat{w}-\phi_{\varepsilon_1}(\hat{w})}^{\hat{w}}\Big(v'_{\varepsilon_2}(x)-v'_{\varepsilon_1}(x)\Big)\,dx\\&\qquad+\lambda\cdot\int_{\hat{w}-\psi_{\varepsilon_1}(\hat{w})}^{\hat{w}}\Big(v'_{\varepsilon_2}(x)-v'_{\varepsilon_1}(x)\Big)\,dx\overset{(ii)}{\leq}0.\end{aligned}$$

If either (i) or (ii) is strict, then the ranking of solutions is strict. This implies that both $v'_\varepsilon(w)$ and $v_\varepsilon(w)$ are decreasing in ε. For each w define $v(w)$ as the monotone decreasing limit

$$v(w)\overset{def}{=}\lim_{\varepsilon\to 0}v_\varepsilon(w).$$

The derivative of $v(w)$ with respect to w is

$$
\begin{aligned}
v'(w) &= \lim_{\epsilon\to 0}\left[\frac{v(w+\epsilon)-v(w-\epsilon)}{2\epsilon}\right] = \lim_{\epsilon\to 0}\left[\frac{\lim_{\varepsilon\to 0} v_\varepsilon(w+\epsilon)-\lim_{\varepsilon\to 0} v_\varepsilon(w-\epsilon)}{2\epsilon}\right] \\
&= \lim_{\epsilon\to 0}\lim_{\varepsilon\to 0}\left[\frac{v_\varepsilon(w+\epsilon)-v_\varepsilon(w-\epsilon)}{2\epsilon}\right] = \lim_{\epsilon\to 0}\lim_{\varepsilon\to 0}\left[\frac{1}{2\epsilon}\int_{w-\epsilon}^{w+\epsilon} v'_\varepsilon(x)\,dx\right] \\
&\overset{(i)}{=} \lim_{\epsilon\to 0}\left[\frac{1}{2\epsilon}\int_{w-\epsilon}^{w+\epsilon}\left(\lim_{\varepsilon\to 0} v'_\varepsilon(x)\right)dx\right] = \lim_{\varepsilon\to 0} v'_\varepsilon(w),
\end{aligned}
\tag{A.9}
$$

where (i) holds because $v'_\varepsilon(x)$ is monotonically decreasing in ε. The sequence of equalities (A.9) implies that the derivative of the limiting function $v(w)$ is equal to the limit of derivatives $v'_\varepsilon(w)$.

The final step is to show that the limiting function $v(w)$ satisfies (11) subject to (9) and (8). This follows from the following set of inequalities

$$
\begin{aligned}
(r+\gamma)\cdot v(w) &= \lim_{\varepsilon\to 0}\Big[(r+\gamma)\cdot v_\varepsilon(w)\Big] \\
&= \lim_{\varepsilon\to 0}\Big[\alpha-\lambda l-\mu+\gamma(K-w)+G_\varepsilon\big(w; v_\varepsilon(w), v'_\varepsilon(w)\big)+v'_\varepsilon(w)\cdot(\rho w+h)\Big] \\
&\overset{(i)}{=} \lim_{\varepsilon\to 0}\Big[\alpha-\lambda l-\mu+\gamma(K-w)+G_\varepsilon\big(w; v(w), v'(w)\big)+v'(w)\cdot(\rho w+h)\Big] \\
&\overset{(ii)}{=} \lim_{\varepsilon\to 0}\Big[\alpha-\lambda l-\mu+\gamma(K-w)+G_0\big(w; v(w), v'(w)\big)+v'(w)\cdot(\rho w+h)\Big] \\
&= \max_{(\psi,\phi,f)}\Big[\alpha-\lambda l-\mu+\gamma(K-w)+\mu f(1-l)+v'(w)\cdot(\rho w+h+\mu f\phi+\lambda\psi) \\
&\qquad\qquad +\mu f\cdot\big(v(w-\phi)-v(w)\big)+\lambda\cdot\big(v(w-\psi)-v(w)\big)\Big]
\end{aligned}
\tag{A.10}
$$

where (i) holds by continuity of $G_\varepsilon(w; x, x')$ in x and x', and (ii) holds because $G_\varepsilon(w; x, x')$ is decreasing in ε for each triplet (w, x, x'), as a lower ε relaxes the constraint on the controls ϕ, ψ in (A.6). □

For a solution $v(w)$ constructed in Lemma A.1, define the optimal policies as

$$
\begin{aligned}
\Big(\phi(w),\psi(w),f(w)\Big) = \arg\max_{\phi,\psi,f}\Big\{&\mu f(1-l)+\Big(\mu\phi f+\lambda\psi\Big)\cdot v'(w)+\mu f\cdot\Big(v\left(w-\phi\right)-v(w)\Big) \\
&+\lambda\cdot\Big(v\left(w-\psi\right)-v(w)\Big)\Big\}
\end{aligned}
$$

subject to the incentive constraints (8) and (9).

Lemma A.2. *Suppose $v(w)$ solves (11) subject to the initial condition $v(\delta)=v_0$. Then both $v(w)$ and $v'(w)$ are increasing in v_0 for each w.*

Proof. Denote $v_1(w)$ and $v_2(w)$ two solutions to (11) such that $v_1(\delta)\le v_2(\delta)$. Following (A.8) in the proof of Lemma A.1 the following inequality holds

$$
(r+\gamma)\cdot\Big(v_2(w)-v_2(w)\Big)\le\Big(v'_2(w)-v'_1(w)\Big)\cdot\Big(\rho w+h+\mu f_2(w)\phi_2(w)+\lambda\psi_2(w)\Big)
$$

$$+\mu f_2(w)\cdot\int_{w-\phi_2(w)}^{w}\Big(v_1'(x)-v_2'(x)\Big)\,dx+\lambda\cdot\int_{w-\psi_2(w)}^{w}\Big(v_1'(x)-v_2'(x)\Big)\,dx.$$

First, consider $w=\delta$. The expression above becomes

$$\overbrace{(r+\gamma)\cdot\Big(v_2(\delta)-v_2(\delta)\Big)}^{>0}\le\Big(v_2'(\delta+)-v_1'(\delta+)\Big)\cdot\Big(\rho w+h+\mu f_2(w)\phi_2(w)+\lambda\psi_2(w)\Big)$$
$$+\mu f_2(w)\cdot\underbrace{\int_{\delta-\phi_2(\delta)}^{\delta}\Big(v_1'(x)-v_2'(x)\Big)\,dx}_{<0}+\lambda\cdot\underbrace{\int_{\delta-\psi_2(\delta)}^{\delta}\Big(v_1'(x)-v_2'(x)\Big)\,dx}_{<0},$$

implying that $v_2'(\delta+)>v_1(\delta+)$.

Next, suppose there exists $\hat{w}=\inf\{w\ge\delta:\,v_2'(\hat{w})=v_1'(\hat{w})\}$. Then

$$\begin{aligned}
0&=\Big(v_2'(\hat{w})-v_1'(\hat{w})\Big)\cdot\Big(\rho\hat{w}+h+\mu f_1(\hat{w})\phi_1(\hat{w})+\lambda\psi_1(\hat{w})\Big)\\
&\ge(r+\gamma)\Big(v_2(\hat{w})-v_1(\hat{w})\Big)+\mu f_1(\hat{w})\Big(v_2(\hat{w})-v_2\left(\hat{w}-\phi_1(\hat{w})\right)-v_1(\hat{w})+v_1\left(\hat{w}-\phi_1(\hat{w})\right)\Big)\\
&\qquad+\lambda\Big(v_2(\hat{w})-v_2\left(\hat{w}-\psi_1(\hat{w})\right)-v_1(\hat{w})+v_1\left(\hat{w}-\psi_1(\hat{w})\right)\Big)\\
&=(r+\gamma)\Big(v_2(\hat{w})-v_1(\hat{w})\Big)+\mu f_2(\hat{w})\int_{\hat{w}-\phi_2(\hat{w})}^{\hat{w}}v_2'(x)\,dx-\mu f_2(\hat{w})\int_{\hat{w}-\phi_2(\hat{w})}^{\hat{w}}v_1'(x)\,dx\\
&\qquad+\lambda\int_{\hat{w}-\psi_2(\hat{w})}^{\hat{w}}v_2'(x)\,dx-\lambda\int_{\hat{w}-\psi_2(\hat{w})}^{\hat{w}}v_1'(x)\,dx\\
&=(r+\gamma)\underbrace{\Big(v_2(\hat{w})-v_1(\hat{w})\Big)}_{>0}+\mu f_1(\hat{w})\underbrace{\int_{\hat{w}-\phi_2(\hat{w})}^{\hat{w}}\Big(v_2'(x)-v_1'(x)\Big)\,dx}_{\ge0}+\lambda\underbrace{\int_{\hat{w}-\psi_2(\hat{w})}^{\hat{w}}\Big(v_2'(x)-v_1'(x)\Big)\,dx}_{\ge0}
\end{aligned}$$

which is positive and implies a contradiction with $v_2'(\hat{w})=v_1'(\hat{w})$. Since $v_2(\delta)>v_1(\delta)$ it also implies that $v_2(w)\ge v_1(w)$ for all $w\ge\delta$. □

Lemma A.3. *There exists a maximal initial condition $v_0=v(\delta)$ to (11) such that there exists a $w=\bar{w}$ such that $v'(\bar{w})=-1$. For this solution, $v'(w)>-1$ for any $w<\bar{w}$.*

Proof. By construction, function $v(w)$ satisfies

$$(r+\gamma)v(w)=\max_{(\psi,\phi,f)}\Big\{\alpha-\lambda l-\mu+\gamma(K-w)+\mu f(1-l)+v'(w)(\rho w+h+\mu f\phi+\lambda\psi)$$
$$+\mu f\big(v(w-\phi)-v(w)\big)+\lambda\big(v(w-\psi)-v(w)\big)\Big\}.$$

Then

$$v'(w)\ge\min_{\psi,\phi,f}\left\{\frac{(r+\gamma)v(w)-\Big(\alpha-\lambda l-\mu+\gamma(K-w)+\mu f(1-l)\Big)+\mu f\big(v(w)-v(w-\phi)\big)+\lambda\big(v(w-\psi)-v(w)\big)}{\rho w+h+\mu f\phi+\lambda\psi}\right\},$$

where the minimum is taken subject to $f \in [0, \bar{f}]$, and $\psi \in [0, w]$, $\phi \in [0, w]$ and subject to incentive constraints (8) and (9). If the initial condition $v_0 > \frac{\alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l)+\gamma K}{r+\gamma} - \delta$, then $v'(w)$ is strictly positive and there does not exist a $\bar{w}$ such that $v'(\bar{w}) = -1$. It implies that for an initial condition $v_0 > \overline{v}_0$ a $\bar{w}$ for which $v'(\bar{w}) = -1$ does not exist. This implies that there exists an upper bound $\overline{v}_0$ on the set of initial conditions for which there still exists a $\bar{w}$ such that $v'(\bar{w}) = -1$. By a similar argument, if $\underline{v}_0$ is very low, then $v'(w) < -1$ for w close to δ. By continuity of the dependence on initial parameters of $v(w)$ satisfying (11), there exists a maximal initial condition v_0 such that there exists a $\bar{w}$ for which $v'(\bar{w}) = -1$, which establishes the first part of Lemma A.3.

The second part of Lemma A.3 states that, if $v(w)$ is the maximal solution, then $v'(w) > -1$ for every $w < \bar{w}$. Suppose, on the contrary, that for the maximal solution $v(w)$ there exists $\hat{w} < \bar{w}$ such that $v'(\hat{w}) < -1$. This means that there exists $\epsilon > 0$ such that the solution to (11) subject to $\hat{v}(\delta) = v(\delta) + \epsilon \in [v(\delta), \overline{v}_0]$ also satisfies $\hat{v}(\bar{w}) = -1$ for some $\bar{w}$ by the continuity of dependence on initial parameters. By Lemma A.2 $\hat{v}(w) > v(w)$ implying the initial solution $v(w)$ is not maximal. Hence, it must be the case that $v'(w) > -1$ for $w \leq \bar{w}$ for the maximal solution $v(\cdot)$. □

A.4.2. Concavity of the Solution to (11)

Lemma A.4. *Suppose parametric restriction* (3)*, generalized to accommodate principal's outside option $K \geq 0$ holds:*

$$\frac{\alpha - \lambda l - \mu - (\rho+\gamma)\delta + \gamma K - h}{r+\gamma} - \frac{h}{\Delta} \geq 1 - l + K. \tag{A.11}$$

Then, $v(w)$ satisfying (11) *subject to* (9) *and* (8) *has a downward kink at $w = \delta$.*

Proof. For $w < \delta$ the value function is linear

$$v(w) = \left(1 - \frac{w}{\delta}\right) \cdot K + \frac{w}{\delta} \cdot v(\delta) \qquad \Rightarrow \qquad v'(w) = \frac{v(\delta) - K}{\delta}. \tag{A.12}$$

Equation (11) at $w = \delta$ is given by

$$(r+\gamma)v(\delta) = \max_{f \in [0,\bar{f}]} \Big\{ \alpha - \lambda l - \mu(1 - f + fl) + \gamma(K - \delta) + v'(\delta)\Big(\rho\delta + h + \mu f \phi + \lambda\delta\Big) + \mu f\Big(v(\delta - \phi) - v(\delta)\Big) + \lambda\Big(v(0) - v(\delta)\Big)\Big\},$$

subject to

$$\delta \leq f \cdot \phi + (1 - f) \cdot w.$$

The sufficient condition for $v'(\delta) \leq \frac{v(\delta)-K}{\delta}$ is

$$(r+\gamma)v(\delta) \leq \max_{f\in[0,\bar{f}]}\Big\{\alpha - \lambda l - \mu(1-f+fl) + \gamma(K-\delta) + \frac{v(\delta)-K}{\delta}\Big(\rho\delta + h + \mu f\phi + \lambda\delta\Big) + \mu f\Big(v(\delta-\phi) - v(\delta)\Big) + \lambda\Big(K - v(\delta)\Big)\Big\}$$

As the objective is linear in f it implies that this condition is equivalent to

$$(r+\gamma)v(\delta) \leq \alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) + \gamma(K-\delta) + (v(\delta)-K)\Big(\rho + \Delta\Big)$$

This condition can be rewritten as

$$(r+\gamma-\rho-\Delta)\cdot v(\delta) \leq \alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) + \gamma(K-\delta) - K(\rho+\Delta)$$
$$(r+\gamma-\rho-\Delta)\cdot (v(\delta)-K) \leq \alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) - rK. \tag{A.13}$$

It follows from (A.11) that $\alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) - rK \geq 0$, implying that the right hand side is positive. Two cases are possible:

(i) $r+\gamma-\rho-\Delta < 0$. Then the left hand side is negative and inequality (A.13) holds.

(ii) $r+\gamma-\rho-\Delta > 0$. Then the left hand side is positive. However, as shown in Lemma A.3 the principal's value $v(\delta)$ is bounded from above by first-best project management, given by

$$\begin{aligned} v(\delta) - K &\leq \frac{\alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) + \gamma K}{r+\gamma} - K \\ &= \frac{\alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) - rK}{r+\gamma} \\ &< \frac{\alpha - \lambda l - \mu(1-\bar{f}+\bar{f}l) - rK}{r+\gamma-\rho-\Delta}. \end{aligned}$$

It follows that (A.13) holds in this case as well.

□

Lemma A.5. *The maximal solution $v(w)$ to* (A.15) *subject to $v'(\bar{w}) = -1$ satisfies the super-contact condition $v''(\bar{w}) = 0$.*

Proof. By construction, $v(w)$ is the largest solution to (A.15) such that $v'(\bar{w}) = -1$. If $v''(\bar{w}) > 0$. Then, it implies that $v''(w) < -1$ for $w < \bar{w}$, which contradicts the result of Lemma A.3. If, on the other hand, $v''(\bar{w}) < 0$ then $v(w)$ is not the maximal solution since, by increasing the initial condition, it is still possible to obtain a solution satisfying $v'(\bar{w}) = -1$ for some $\bar{w}$. □

Lemma A.6. *Suppose $\lambda = 0$. The maximal solution to (11) satisfying $v'(\bar{w}) = -1$ is concave for $w \in [\delta, \bar{w}]$.*

Proof. Suppose the contrary that there exists $w^* \in [\delta, \bar{w}]$ such that

$$w^* = \inf \left\{ w \in [\delta, \bar{w}] : \quad v''(w) \geq 0 \right\}. \tag{A.14}$$

By definition of w^*, it implies that $v''(w) < 0$ for all $w < w^*$ and the incentive compatibility constraint (13) is binding in this region. Function $v(w)$ then satisfies the differential equation

$$\begin{aligned}(r+\gamma)v(w) &= \alpha - \lambda l - \mu(1 - f(w) + f(w)l) + \gamma(K - w) \\ &+ v'(w) \cdot \left(\rho w + h + \mu f(w) \left(w - \frac{w-\delta}{f(w)} \right) \right) + \mu f(w) \cdot \left(v\left(\frac{w-\delta}{f(w)} \right) - v(w) \right).\end{aligned} \tag{A.15}$$

Monitoring intensity $f(w)$ solves

$$f(w) = \underset{f \in [0,\bar{f}]}{\arg\max} \left\{ (1-l)f + v'(w)fw + f\left(v\left(\frac{w-\delta}{f} \right) - v(w) \right) \right\}. \tag{A.16}$$

Three cases are possible.

1. Suppose $f(w^*) = \frac{w^*-\delta}{\delta}$. Equation (A.15) becomes

$$\begin{aligned}(r+\gamma)v(w) &= \alpha - \mu \left(1 - \frac{w-\delta}{\delta} + \frac{w-\delta}{\delta} l \right) + \gamma(K-w) \\ &+ v'(w) \left(\rho w + h + \mu \frac{(w-\delta)^2}{\delta} \right) + \mu f(w) \big(v(\delta) - v(w) \big).\end{aligned}$$

   Differentiating the above with respect to w obtain

$$\begin{aligned}(r+\gamma)v'(w) &= \frac{\mu}{\delta}(1-l) - \gamma + v''(w) \left(\rho w + h + \mu \frac{(w-\delta)^2}{\delta} \right) \\ &+ v'(w) \left(\rho + 2\mu \frac{w-\delta}{\delta} \right) + \frac{\mu}{\delta} \Big(v(\delta) - v(w) \Big) - \mu \frac{w-\delta}{\delta} v'(w)\end{aligned}$$

   which simplifies to

$$\begin{aligned}(\gamma + r - \rho)v'(w) + \gamma &= \frac{\mu}{\delta}(1-l) + v''(w) \left(\rho w + h + \mu \frac{(w-\delta)^2}{\delta} \right) \\ &+ v'(w)\mu \frac{w-\delta}{\delta} + \frac{\mu}{\delta} \big(v(\delta) - v(w) \big).\end{aligned} \tag{A.17}$$

At $w = w^*$ we have $v''(w^*) \geq 0$ and the above expression becomes

$$\begin{aligned}
(\gamma + r - \rho)v'(w) + \gamma &\geq \frac{\mu}{\delta}(1-l) + v'(w)\mu\frac{w-\delta}{\delta} + \frac{\mu}{\delta}\big(v(\delta) - v(w)\big) \\
(\gamma + r - \rho)v'(w) + \gamma - \frac{\mu}{\delta}(1-l) &\geq \mu\left(v'(w)\frac{w-\delta}{\delta} + \frac{1}{\delta}\big(v(\delta) - v(w)\big)\right) \\
(w-\delta)\left((\gamma + r - \rho)v'(w) + \gamma - \frac{\mu}{\delta}(1-l)\right) &\geq \lambda(w-\delta)\left(v'(w)\frac{w-\delta}{\delta} + \frac{1}{\delta}\big(v(\delta) - v(w)\big)\right) \\
(w-\delta)\left((\gamma + r - \rho)v'(w) + \gamma - \frac{\mu}{\delta}(1-l)\right) &\geq v'(w)\mu\frac{(w-\delta)^2}{\delta} + \mu\frac{w-\delta}{\delta}\big(v(\delta) - v(w)\big)
\end{aligned}$$

Substituting the above expression into (A.15) at $w = w^*$ we obtain

$$\begin{aligned}
(r+\gamma)v(w) &\leq \alpha + \mu\left(1 - \frac{w-\delta}{\delta} + \frac{w-\delta}{\delta}l\right) + \gamma(K-w) + v'(w)(\rho w + h) \\
&\quad + (w-\delta)\left((\gamma + r - \rho)v'(w) + \gamma - \frac{\mu}{\delta}(1-l)\right) \\
(r+\gamma)v(w) &\leq \alpha - \mu + \gamma(K-w) + v'(w)(\rho w + h) + (w-\delta)\Big((\gamma + r - \rho)v'(w) + \gamma\Big) \\
(r+\gamma)v(w) &\leq \alpha - \mu + \gamma(K-\delta) + v'(w)(\rho w + h) + (w-\delta)(\gamma + r - \rho)v'(w) \\
(r+\gamma)v(w) &\leq \alpha - \mu + \gamma(K-\delta) + v'(w)\big(\rho w + h + (\gamma + r - \rho)(w-\delta)\big).
\end{aligned}$$

Because $v(w)$ is strictly concave for $w < w^*$ we have

$$v(\delta) + v'(w)(w-\delta) \leq v(w).$$

where the inequality is strict if $w > \delta$. Thus

$$\begin{aligned}
(r+\gamma)\Big(v(\delta)+v'(w)(w-\delta)\Big) &< \alpha-\mu+\gamma(K-\delta)+v'(w)\Big(\rho w+h+(\gamma+r-\rho)(w-\delta)\Big) \\
(r+\gamma)v(\delta) &< \alpha-\mu+\gamma(K-\delta)+v'(w)\Big(\rho w+h+(\gamma+r-\rho)(w-\delta)-(r+\gamma)(w-\delta)\Big) \\
(r+\gamma)v(\delta) &< \alpha-\mu+\gamma(K-\delta)+v'(w)(\rho w+h-\rho(w-\delta)) \\
(r+\gamma)v(\delta) &< \alpha-\mu+\gamma(K-\delta)+v'(w)(\rho\delta+h) \qquad \text{(A.18)}
\end{aligned}$$

Inequality (A.18), however, contradicts the fact that function $v(w)$ satisfies (A.15) at $w = \delta$ if $w^* > \delta$ or $v''(w^*) > 0$.

The remaining case to be considered is $w^* = \delta$ and $v''(\delta) = 0$. Taking subsequent derivatives of (A.17) obtain that $\frac{d}{dw^k}v(w)|_{w=\delta} = 0$ for any $k \in \mathbb{N}$. This implies $v(w)$ must be a linear function for $w \geq \delta$ which is weakly concave. Moreover, in this case the optimal monitoring rule $f(w)$ is always given by $\max\left(\frac{w-\delta}{\delta}, \bar{f}\right)$.

2. Suppose the first order monitoring optimality condition (A.16) is satisfied exactly at w^*, i.e.,

$f(w)$ solves

$$1-l+v\left(\frac{w-\delta}{f(w)}\right)-\frac{w-\delta}{f(w)}\cdot v'\left(\frac{w-\delta}{f(w)}\right)=v(w)-wv'(w). \tag{A.19}$$

at $w=w^*$. Differentiating with respect to w obtain

$$-\frac{w-\delta}{f(w)}\cdot\left(\frac{1}{f(w)}-\frac{w-\delta}{f(w)^2}f'(w)\right)\cdot v''\left(\frac{w-\delta}{f(w)}\right)=-wv''(w). \tag{A.20}$$

At $w=w^*$ it implies that

$$f'(w^*)=\frac{f(w^*)}{w^*-\delta}. \tag{A.21}$$

By envelope theorem taking the derivative with respect to w of (A.15) we obtain

$$\begin{aligned}(r+\gamma)v'(w)=&-\gamma+v''(w)\left(\rho w+h+\mu\big(f(w)w-w+\delta\big)\right)\\&+v'(w)\big(\rho+\mu(f(w)-1)\big)+\mu v'\left(\frac{w-\delta}{f(w)}\right)-\mu f(w)v'(w).\end{aligned}$$

Simplifying terms obtain

$$(r+\gamma+\mu-\rho)v'(w)=-\gamma+v''(w)\Big(\rho w+h+\mu\big(f(w)w-w+\delta\big)\Big)+\mu v'\left(\frac{w-\delta}{f(w)}\right). \tag{A.22}$$

At $w=w^*$ equation (A.22) becomes

$$(r+\gamma-\rho+\mu)v'(w^*)+\gamma=\mu v'\left(\frac{w^*-\delta}{f(w^*)}\right). \tag{A.23}$$

This implies

$$(r+\gamma-\rho)v'(w^*)+\gamma\geq 0.$$

Substituting (A.19) into the above we obtain

$$(r+\gamma-\rho+\mu)v'(w^*)+\gamma=\mu\cdot\frac{1-l+v\left(\frac{w-\delta}{f(w)}\right)-v(w)+wv'(w)}{\frac{w-\delta}{f(w)}} \tag{A.24}$$

Substitute the delay term $v\left(\frac{w-\delta}{f(w)}\right)-v(w)$ from (A.15) to obtain

$$(r+\gamma-\rho+\mu)v'(w^*)+\gamma=\mu f(w^*)\frac{1-l+w^*v'(w^*)}{w^*-\delta}+$$
$$\frac{(r+\gamma)v(w^*)-\alpha-\mu(1-f(w^*)+f(w^*)l)-\gamma(K-w^*)-v'(w^*)(\rho w^*+h+\mu(f(w^*)w^*-w^*+\delta))}{w^*-\delta}$$

Simplifying terms

$$(r+\gamma-\rho+\mu)v'(w^*)+\gamma = \frac{\mu f(w^*)w^*v'(w^*)}{w^*-\delta} + \\ \frac{(r+\gamma)v(w^*)-\alpha+\mu-\gamma(K-w^*)-v'(w^*)\left(\rho w^*+h+\mu(f(w^*)w^*-w^*+\delta)\right)}{w^*-\delta}.$$

Simplifying further

$$\begin{aligned}
(r+\gamma-\rho+\mu)v'(w^*) &= \frac{(r+\gamma)v(w^*)-\alpha+\mu-\gamma(K-\delta)-v'(w^*)\left(\rho w^*+h+\mu(-w^*+\delta)\right)}{w^*-\delta} \\
(r+\gamma-\rho)v'(w^*) &= \frac{(r+\gamma)v(w^*)-\alpha+\mu-\gamma(K-\delta)-v'(w^*)\left(\rho w^*+h\right)}{w^*-\delta} \\
(r+\gamma)v'(w^*) &= \frac{(r+\gamma)v(w^*)-\alpha+\mu-\gamma(K-\delta)-v'(w^*)\left(\rho\delta+h\right)}{w^*-\delta} \\
(r+\gamma)v'(w^*)(w^*-\delta) &= (r+\gamma)v(w^*)-\alpha+\mu-\gamma(K-\delta)-v'(w^*)\left(\rho\delta+h\right).
\end{aligned}$$

Rearranging terms obtain

$$(r+\gamma)(v(w^*)-v'(w^*)(w^*-\delta)) = \alpha-\mu+\gamma(K-\delta)+v'(w^*)(\rho\delta+h).$$

Note that $v(w)$ is concave over $[\delta, w^*]$ implying that

$$\begin{aligned}
(r+\gamma)v(\delta) &< (r+\gamma)(v(w^*)-v'(w^*)(w^*-\delta)) \\
&= \alpha-\mu+\gamma(K-\delta)+v'(w^*)(\rho\delta+h) \\
&< \alpha-\mu+\gamma(K-\delta)+v'(\delta)(\rho\delta+h) \\
&= (r+\gamma)v(\delta)
\end{aligned}$$

This is a contradiction with the fact that $v''(w^*)=0$ and $f(w^*)$ is an interior monitoring solution satisfying (A.19).

3. $f(w^*)=\bar{f}$. Then $f'(w)=0$. Taking the derivative of (A.22) becomes

$$(r+\gamma+\mu-\rho)v'(w) = -\gamma+v''(w)\Big(\rho w+h+\mu\big(\bar{f}w-w+\delta\big)\Big)+\mu v'\left(\frac{w-\delta}{\bar{f}}\right). \qquad (A.25)$$

Because $f(w)=\bar{f}$, we can differentiate (A.25) with respect to w to obtain

$$(r+\gamma+\mu-\rho)v''(w) = -\gamma+v'''(w)\Big(\rho w+h+\mu\big(\bar{f}w-w+\delta\big)\Big)+v''(w)\Big(\rho+\mu(\bar{f}-1)\Big)+\frac{\mu}{\bar{f}}v''\left(\frac{w-\delta}{\bar{f}}\right).$$

By definition of $w = w^*$, $v\left(\frac{w-\delta}{\bar{f}}\right) < 0$ which implies that

$$v'''(w) = -\frac{\frac{\mu}{\bar{f}} v''\left(\frac{w-\delta}{\bar{f}}\right)}{\rho w + h + \mu\big(\bar{f}w - w + \delta\big)} > 0 \tag{A.26}$$

since $v''\left(\frac{w-\delta}{\bar{f}}\right) < 0$. Define

$$\tilde{w} = \inf\left\{w > w^* : \quad v''(w) \leq 0\right\}.$$

Note that it may be the case that $\tilde{w} = \bar{w}$. Because $v(w)$ is convex over $[w^*, \tilde{w}]$, it implies that the monitoring rule $f(w)$ cannot satisfy the first order condition as long as $\frac{\tilde{w}-\delta}{f(\tilde{w})} > w^*$ as the principal would benefit both from more monitoring, as well as from exploiting the convexity of $v(w)$ for $w \in [w^*, \tilde{w}]$. Since at w^* it must be the case that $f(w^*) = \bar{f}$, it implies that $f(\tilde{w}) = \bar{f}$ for any $w \in [w^*, \tilde{w}]$

$$v(\tilde{w}) - \tilde{w}v'(\tilde{w}) < v(w^*) - w^*v'(w^*).$$

and $\tilde{w} \in [w^*, \tilde{w}]$. This implies that the optimal penalty $\phi(w)$ is continuous over $[w^*, \tilde{w}]$. By applying the Envelope theorem to (11) with respect to w, we, then, obtain that $v''(w)$ is continuous over $[w^*, \tilde{w}]$. It implies that $v''(\tilde{w}) = 0$ and $v'''(\tilde{w}) \leq 0$. Given that $f(\tilde{w}) = \bar{f}$, the analogue of (A.26) implies

$$-\mu v''\left(\frac{\tilde{w}-\delta}{\bar{f}}\right) = v'''(\tilde{w})\Big(\rho\tilde{w} + h + \mu\Big(\tilde{w}\bar{f} - \tilde{w} + \delta\Big)\Big). \tag{A.27}$$

It implies that $\frac{\tilde{w}-\delta}{\bar{f}} \geq w^*$ since, otherwise, $v'''(\tilde{w}) > 0$ which is inconsistent with the definition of $\tilde{w}$. Substituting $v''(\tilde{w}) = 0$ into (A.25) obtain

$$\big(r + \gamma + \mu - \rho\big)v'(\tilde{w}) + \gamma = \mu v'\left(\frac{\tilde{w}-\delta}{\bar{f}}\right).$$

Since $\frac{\tilde{w}-\delta}{\bar{f}} \geq w^*$, it implies that $v'(\tilde{w}) > v'\left(\frac{\tilde{w}-\delta}{\bar{f}}\right)$, which leads to

$$(r + \gamma - \rho)v'(\tilde{w}) + \gamma \leq 0.$$

Then, following similar derivations to Biais, Mariotti, Rochet, and Villeneuve (2010), obtain

$$\begin{aligned}
&(\rho w + h) + \mu\big(\bar{f}w - w + \delta\big)\big((\rho - r - \gamma)v'(w) - \gamma + 1\big)\\
\leq&\big(\rho w + h + \mu\big(\bar{f}w - w + \delta\big)\big)\big((\rho - r - \gamma)v'(w) - \gamma + 1\big)\\
=&\big(\rho w + h + \mu\big(\bar{f}w - w + \delta\big)\big)(1 - \gamma) + (\rho - r - \gamma)\big(\rho w + h + \mu(\bar{f}w - w + \delta)\big)v'(w)\\
=&\big(\rho w + h + \mu\big(\bar{f}w - w + \delta\big)\big)(1 - \gamma)
\end{aligned}$$

$$+(\rho - r - \gamma)\left((r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma(K-w) + \mu\bar{f}\left(v(w) - v\left(\frac{w-\delta}{\bar{f}}\right)\right)\right)$$

(a) Suppose that $\rho - r - \gamma > 0$. Then, because of convexity of $v(w)$ in $[w^*, \tilde{w}]$,

$$\begin{aligned}&(\rho w + h) + \mu(\bar{f}w - w + \delta)((\rho - r - \gamma)v'(w) - \gamma + 1)\\ \leq&\Big(\rho w + h + \mu(\bar{f}w - w + \delta)\Big)(1-\gamma)\\ +&(\rho - r - \gamma)\Big((r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma(K-w) + \lambda(\bar{f}w - w + \delta)v'(w)\Big).\end{aligned}$$

Simplifying terms obtain

$$\begin{aligned}&(\rho w + h)\gamma + (\rho - r - \gamma)\mu\big(\bar{f}w - w + \delta\big)v'(w)\\ \leq&(\rho - r - \gamma)\Big((r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma(K-w) + \mu(\bar{f}w - w + \delta)v'(w)\Big).\end{aligned}$$

Simplifing further

$$\begin{aligned}(\rho w + h)\gamma &\leq (\rho - r - \gamma)\Big((r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma(K-w)\Big)\\ (r+\gamma)w + \gamma h &\leq (\rho - r - \gamma)\Big((r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma K\Big)\\ 0 \leq \frac{(r+\gamma)w + \gamma h}{\rho - r - \gamma} &\leq (r+\gamma)v(w) - \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma K.\end{aligned}$$

The above implies that

$$v(w) \geq \frac{\alpha - \mu(1-\bar{f}+\bar{f}l) + \gamma K}{r+\gamma}$$

which exceeds the first-best payoff that can be obtained by the principal leading to a contradiction.

(b) Suppose that $\rho - r - \gamma < 0$. In this case

$$(\rho - r - \gamma)v'(\tilde{w}) - \gamma \geq 0 \qquad \Rightarrow \qquad v'(\tilde{w}) \leq -\frac{\gamma}{\gamma - (\rho - r)} < -1,$$

which is a contradiction with Lemma A.3 and $\tilde{w} \leq \bar{w}$.

□

Because value function $v(w)$ satisfies (11), and is concave, the standard optimality verification argument, which can be found in Sannikov (2008), implies that monitoring policy $f(w)$ corresponds to an optimal contract.

A.5 Optimality of High Effort until Termination

Lemma A.7 (Optimality of High Effort.)**.** *Suppose λ is sufficiently low, $\gamma \geq \rho - r$, and parametric condition* (3) *holds.*[2] *Then*

(i) it is optimal to liquidate the project rather than employ the agent with low effort until retirement;

(ii) the optimal contract implements high effort from the agent up until termination.

Proof. Suppose $\lambda = 0$ – the argument then extends to a sufficiently low λ by continuity. Define the flow profit obtained by the principal under effort $a \in \{0, 1\}$ and monitoring $f \in [0, \bar{f}]$:

$$\pi(a, f) \overset{def}{=} \alpha - (1 - a) \cdot \Delta \cdot l - \mu \cdot (1 - l + l \cdot f).$$

The proof proceeds in two steps. First, I show that for $w \geq \delta$ implementing low effort is dominated by implementing high effort and not monitor the agent. Second, I show that for $w < \delta$ it is optimal to randomize the agent's continuation utility between 0 and δ.

Case $w \geq \delta$. The principal's maximization (B.51) is amended for a choice of high effort and monitoring with some intensity and no effort with full monitoring. It is formally written as

$$(r + \gamma)v(w) + \gamma w =$$
$$\max\Big\{\max_f \Big[\pi(1, f) + v'(w)\Big(\rho w + h + \mu f \phi(w, f)\Big) + \mu f \cdot \Big(v(w - \phi(w, f)) - v(w)\Big)\Big],\ \pi\left(0, \bar{f}\right) + v'(w) \cdot \rho w\Big\}.$$

Note that assuming there is no pay-for-performance sensitivity is without loss since the principal's value function $v(w)$ is weakly concave. Based on the above, high effort with no monitoring ($a = 1$ and $f = 0$) dominates low effort with full monitoring ($a = 0$ and $f = \bar{f}$) if and only if

$$\begin{aligned} \pi(1, 0) + v'(w) \cdot (\rho w + h) &\geq \pi\left(0, \bar{f}\right) + v'(w) \cdot \rho w \\ \alpha - \mu + v'(w) \cdot h &\geq \alpha - \Delta \cdot l - \mu \cdot \left(1 - l + l \cdot \bar{f}\right). \end{aligned} \tag{A.28}$$

Note that since $v(w)$ is weakly concave it follows that $v'(w) \geq -1$ since, otherwise, the principal could make a transfer to the agent as discussed in Section 3.5 and B.3. This implies that in order for (A.28) to be satisfied, it is sufficient to check that

$$\alpha - \Delta \cdot l - \mu \cdot \left(1 - l + l \cdot \bar{f}\right) \leq \alpha - \mu - h,$$

which follows from (3) if $\lambda = 0$.

[2] I provide this proof for outside option $K = 0$ since for $K > 0$ the principal has strictly greater incentives to terminate the agent rather than letting him shirk.

Case $w < \delta$**.** In this case it is impossible to satisfy incentive constraints (9) and (8) for any monitoring policy $f \in [0, \bar{f}]$ implying that the agent cannot be incentivized to exert effort. The principal decides whether to randomize the agent's continuation value or employ him with low effort. If the agent is employed under low effort, i.e., we are in a suspension contract of Zhu (2013), then $v(w)$ must satisfy

$$(r+\gamma)v(w) = \alpha - \mu(1-\bar{f}+\bar{f}l) - \gamma w + v'(w)\cdot \rho w.$$

Differentiating both sides with respect to w obtain

$$(r+\gamma)\cdot v'(w) = -\gamma + v'(w)\cdot \rho + v''(w)\cdot \rho w,$$
$$(r-\rho+\gamma)\cdot v'(w) + \gamma = v''(w)\cdot \rho w.$$

Note that $v'(w) > 0$ for $w \le \delta$ since $v(w)$ must be concave and $v(\delta) > 0$ by parametric condition (3). □

A.6 Proof of Proposition 3 (general case)

Suppose $\lambda = 0$[3] and the following three parametric conditions hold:

(i) $1-l+K \le \frac{\alpha-\lambda l-\mu-h+(\gamma+\lambda)K-(\rho+\gamma)\delta}{r+\lambda+\gamma}$, meaning that it is optimal to employ the agent under high effort but no monitoring in perpetuity relative to fixing a single project and letting him go;

(ii) $\rho - r + \Delta - \gamma \ge 0$, meaning that the marginal value of the agent's effort is sufficiently high;

(iii) $\rho - r \le \gamma$, meaning that the principal wishes to implement $f(w) = \frac{w-\delta}{\delta}$ for $w \le \delta + h/\mu$.

Lemma A.8. *Suppose* $1-l+K \le v(\delta) - \delta\cdot v'(\delta+)$*. Then* $f(w) \le \frac{w-\delta}{\delta}$*.*

Proof. Suppose $\frac{w-\delta}{f} < \delta$. Then, due to linearity of $v(w)$ for $w \in [0,\delta]$ it follows that

$$v\left(\frac{w-\delta}{f}\right) - \frac{w-\delta}{f}\cdot v'\left(\frac{w-\delta}{f}\right) = K.$$

It follows from (15) that monitoring intensity $f > \frac{w-\delta}{\delta}$ is suboptimal if and only if

$$1-l+K \le v(w) - w\cdot v'(w). \tag{A.29}$$

Since $v(w)$ is concave, $v(w) - w\cdot v'(w)$ is increasing in w. Then, (A.29) is satisfied for $w \ge \delta$ if it is satisfied at $w = \delta$. In this case, it cannot be optimal for $\frac{w-\delta}{f} < \delta$, which concludes the proof. □

Lemma A.9. *Suppose* $\rho - r + \Delta - \gamma \ge 0$ *and* $1-l+K < \frac{\alpha-\lambda l-\mu-h+(\gamma+\lambda)K-(\rho+\gamma)\delta}{r+\lambda+\gamma}$*. Then,* $1-l+K < v(\delta) - \delta\cdot v'(\delta)$*.*

[3] Necessary for concavity of the proof of the concavity of the principal's value function $v(w)$. By continuity, the argument extends to λ being sufficiently small.

Proof. The principal's HJB (11) at $w = \delta$ implies

$$(r+\gamma)v(\delta) \geq \alpha - \lambda l - \mu + \gamma(K-\delta) + v'(\delta)\cdot\big(\rho\delta + \lambda\delta + h\big) + \lambda\cdot\big(K - v(\delta)\big)$$
$$v'(\delta) \leq \frac{(r+\gamma)v(\delta) - \Big(\alpha - \lambda l - \mu + \gamma(K-\delta)\Big) - \lambda\cdot\big(K - v(\delta)\big)}{\rho\delta + \lambda\delta + h}$$

since the principal can always set the monitoring intensity to 0. Then

$$v(\delta) - \delta\cdot v'(\delta) \geq v(\delta) - \delta\cdot\frac{(r+\gamma)v(\delta) - \Big(\alpha - \lambda l - \mu + \gamma(K-\delta)\Big) - \lambda\cdot\big(K - v(\delta)\big)}{\rho\delta + \lambda\delta + h}.$$

Then, a sufficient condition for $1 - l + K \leq v(\delta) - \delta\cdot v'(\delta)$ to hold is

$$1 - l + K \leq v(\delta) - \frac{\delta}{\rho\delta + \lambda\delta + h}\cdot\Big((r+\gamma)v(\delta) - \alpha + \lambda l + \mu - \gamma(K-\delta) + \lambda(v(\delta) - K)\Big)$$
$$1 - l + K \leq \frac{v(\delta)\cdot(\rho\delta + \lambda\delta + h - (r+\gamma+\lambda)\delta)}{\rho\delta + \lambda\delta + h} + \frac{\delta}{\rho\delta + \lambda\delta + h}\cdot\Big(\alpha - \lambda l - \mu + \gamma(K-\delta) + \lambda K\Big)$$
$$1 - l + K \leq \frac{v(\delta)\cdot(\rho - r + \Delta - \gamma)\delta}{\rho\delta + \lambda\delta + h} + \frac{\delta}{\rho\delta + \lambda\delta + h}\cdot\Big(\alpha - \lambda l - \mu + \gamma(K-\delta) + \lambda K\Big). \tag{A.30}$$

If $\rho - r + \Delta - \gamma \geq 0$, then a sufficient condition for (A.30) to be satisfied is if it is satisfied for a lower bound of $v(\delta)$ obtained under no monitoring until the agent's retirement

$$1 - l + K \leq \frac{\frac{\alpha - \lambda l - \mu - h - (\rho+\gamma)\delta + (\gamma+\lambda)K}{r+\lambda+\gamma}\cdot(\rho - r + \Delta - \gamma)\delta}{\rho\delta + \lambda\delta + h} + \frac{\delta}{\rho\delta + \lambda\delta + h}\cdot\Big(\alpha - \lambda l - \mu - h + \gamma(K-\delta) + \lambda K\Big)$$
$$1 - l + K \leq \frac{\alpha - \lambda l - \mu - h - (\rho+\gamma)\delta + (\gamma+\lambda)K}{r+\lambda+\gamma}\cdot\left(\frac{(\rho - r + \Delta - \gamma)\delta}{\rho\delta + \lambda\delta + h} + \frac{\delta(r+\lambda+\gamma)}{\rho\delta + \lambda\delta + h}\right) + \frac{\rho\delta^2}{\rho\delta + \lambda\delta + h}$$
$$1 - l + K \leq \frac{\alpha - \lambda l - \mu - h - (\rho+\gamma)\delta + (\gamma+\lambda)K}{r+\lambda+\gamma} + \frac{\rho\delta}{\rho\delta + \lambda\delta + h}\cdot\Big(\alpha - \lambda l - \mu - h + \gamma(K-\delta)\Big),$$

which is, in turn, always satisfied if it is individually rational for the principal to employ the agent under high effort but no monitoring in perpetuity. □

Lemma A.10. *Suppose $\lambda = 0$ and $\rho - r \leq \gamma$. Then, the principal optimally implements $f(w) = \frac{w-\delta}{\delta}$ for $w \leq \delta + h/\mu$.*

Proof. For w close to δ, it is optimal to implement maximum monitoring capacity $f(w) = \frac{w-\delta}{\delta}$. Denote by $w^* \leq (1+\bar{f})\delta$ to be the first time when the maximum monitoring intensity is not binding, but is interior:

$$w^* \stackrel{def}{=} \inf\Big\{w \geq \delta : \quad 1 - l + v(\delta) - \delta\cdot v'(\delta) = v(w) - w\cdot v'(w)\Big\}. \tag{A.31}$$

The principal's HJB (A.15) at $w = w^*$ is

$$(r+\gamma)\cdot v(w^*) = \alpha - \mu + \mu(1-l)\cdot\frac{w^*-\delta}{\delta} + \gamma(K - w^*) + v'(w^*)\cdot\left(\rho w^* + h + \mu\frac{(w^*-\delta)^2}{\delta}\right)$$

$$+\mu\cdot\frac{w^*-\delta}{\delta}\cdot\Big(v(\delta)-v(w^*)\Big).$$

Substitute $v(w^*)$ from (A.31) into the principal's HJB (A.15) at $w=w^*$

$$\begin{aligned}&(r+\gamma)\cdot\Big(1-l+v(\delta)+w^*\cdot v'(w^*)-\delta\cdot v'(\delta)\Big)\\ =&\alpha-\mu+\mu(1-l)\cdot\frac{w^*-\delta}{\delta}+\gamma(K-w^*)+v'(w^*)\cdot\Big(\rho w^*+h+\mu\frac{(w^*-\delta)^2}{\delta}\Big)\\ +&\mu\cdot\frac{w^*-\delta}{\delta}\cdot\Big(v(\delta)-v(\delta)+\delta v'(\delta)-w^*v'(w^*)-(1-l)\Big).\end{aligned}$$

Simplify terms obtain

$$\gamma(w^*-\delta)+(\rho-r-\gamma)\Big(\delta v'(\delta)-w^*v'(w^*)\Big)+(r+\gamma)(1-l)=\Big(v'(\delta)-v'(w^*)\Big)\Big(\mu(w^*-\delta)-h\Big)$$

$$\gamma(w^*-\delta)+(\rho-r-\gamma)\Big(\delta v'(\delta)-\delta v'(w^*)-(w^*-\delta)v'(w^*)\Big)+(r+\gamma)(1-l)=\Big(v'(\delta)-v'(w^*)\Big)\Big(\mu(w^*-\delta)-h\Big)$$

$$\gamma(w^*-\delta)+(\rho-r-\gamma)\Big(1-l+v(\delta)-v(w^*)\Big)+(r+\gamma)(1-l)=\Big(v'(\delta)-v'(w^*)\Big)\Big(\mu(w^*-\delta)-h\Big)$$

$$\gamma(w^*-\delta)+(\rho-r-\gamma)\Big(v(\delta)-v(w^*)\Big)+\rho(1-l)=\Big(v'(\delta)-v'(w^*)\Big)\Big(\mu(w^*-\delta)-h\Big).$$

From the concavity of $v(w)$ and $v'(w)\geq -1$, it follows that $v(w)\geq v(\delta)-(w-\delta)$. Suppose $\rho-r-\gamma\leq 0$. Then

(i) Case $v(\delta)-v(w^*)<0$. Then

$$\gamma(w^*-\delta)+(\rho-r-\gamma)\cdot(v(\delta)-v(w^*))+\rho(1-l)>0.$$

(ii) Case $v(\delta)-v(w^*)>0$. Then, by concavity of the value function, $v'(w)<0$. Then

$$\gamma(w^*-\delta-v(\delta)+v(w^*))+(\rho-r)\cdot(v(\delta)-v(w^*))+\rho(1-l)\geq 0.$$

For $\rho-r-\gamma\leq 0$, it follows that $\underbrace{\Big(v'(\delta)-v'(w^*)\Big)}_{\geq 0}\cdot\Big(\mu(w^*-\delta)-h\Big)\geq 0$, implying that $w^*-\delta\geq\frac{h}{\mu}$. □

Lemma A.11. *Suppose $\rho-r+\Delta-\gamma\geq 0$. Then, the optimal monitoring intensity $f(w)$ is weakly increasing in w, while the optimal pay-for-performance sensitivity $\phi(w)$ is increasing for $f(w)<\bar{f}$.*

Proof. The binding incentive compatibility condition is $\delta=f\cdot\phi+(1-f)\cdot w$ can be rewritten as $f=\frac{w-\delta}{w-\phi}$ and $w-\phi=\frac{w-\delta}{f}$. The first order condition (15) with respect to f (expressed via ϕ) is

$$1-l+v(w-\phi)-(w-\phi)v'(w-\phi)=v(w)-wv'(w) \tag{A.32}$$

Monotonicity of $f(w)$. We can rewrite (A.32) via monitoring intensity f to obtain

$$1-l+v\left(\frac{w-\delta}{f}\right)-\frac{w-\delta}{f}v'\left(\frac{w-\delta}{f}\right)=v(w)-wv'(w). \tag{A.33}$$

Differentiating (A.33) with respect to w obtain

$$\frac{f(w)-f'(w)(w-\delta)}{f(w)^2}\cdot v''\left(\frac{w-\delta}{f}\right)=\frac{wv''(w)}{\frac{w-\delta}{f(w)}} \tag{A.34}$$

For $f'(w)\geq 0$ it is sufficient that $w\cdot v''(w)$ to be increasing, as can be seen from

$$\frac{f(w)}{f(w)^2}\cdot v''\left(\frac{w-\delta}{f(w)}\right)\leq\frac{wv''(w)}{\frac{w-\delta}{f(w)}}\qquad\Leftrightarrow\qquad\frac{1}{f(w)}\cdot\frac{w-\delta}{f(w)}\cdot v''\left(\frac{w-\delta}{f(w)}\right)\leq wv''(w).$$

Substituting into the Hamilton-Jacobi-Bellman equation. If $f(w)$ is an interior solution, then the envelope theorem implies

$$(r+\gamma)v'(w)=-\gamma+v'(w)(\rho+\mu(f-1))+v''(w)\cdot\Big(\rho w+h+\mu(\delta+(f-1)w)\Big)+\mu v'(w-\phi)-\mu f v'(w)$$

$$(r+\gamma-\rho)v'(w)=-\gamma+v'(w)\cdot\mu(f-1))+v''(w)\cdot\Big(\rho w+h+\mu(\delta+(f-1)w)\Big)+\mu v'(w-\phi)-\mu f v'(w)$$

$$(r+\gamma-\rho)v'(w)=-\gamma+v''(w)\cdot\Big(\rho w+h+\mu\Big(\delta+(f-1)w\Big)\Big)+\mu v'\left(\frac{w-\delta}{f}\right)-\mu v'(w).$$

Rewriting this expression using punishment ϕ obtain

$$(r+\gamma-\rho+\mu)v'(w)=-\gamma+v''(w)\big(\rho w+h+\mu f\phi\big)+\mu v'(w-\phi).$$

The same expression can also be evaluated at ϕ, given the monitoring intensity $\hat{f}$ and punishment $\hat{\phi}$ corresponding to continuation value $\hat{w}=w-\phi$:

$$(r+\gamma-\rho+\mu)v'(w-\phi)=-\gamma+v''(w-\phi)\big(\rho(w-\phi)+h+\mu\hat{f}\hat{\phi}\big)+\mu v'(w-\phi-\hat{\phi}).$$

Suppose $v''(w)w=v''(w-\phi)(w-\phi)$. Then

$$(r+\gamma-\rho+\mu)v'(w)=-\gamma+\frac{v''(w-\phi)(w-\phi)}{fw}\big(\rho w+h+\mu f\phi\big)+\mu v'(w-\phi)$$
$$(r+\gamma-\rho+\mu)v'(w-\phi)=-\gamma+v''(w-\phi)\big(\rho(w-\phi)+h+\mu\hat{f}\hat{\phi}\big)+\mu v'(w-\phi-\hat{\phi})$$

Differentiating it again with respect to w we get

$$(r+\gamma-\rho)v''(w)=v'''(w)\Big(\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)\Big)+v''(w)\Big(\rho+\mu\Big(f(w)-1+f'(w)w\Big)\Big)$$

$$+\mu\frac{f(w)-(w-\delta)f'(w)}{f(w)^2}v''\left(\frac{w-\delta}{f(w)}\right)-\mu v''(w)$$

$$\begin{aligned}(r+\gamma-\rho)v''(w) &= v'''(w)\Big(\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)\Big)+v''(w)\Big(\rho+\mu\Big(f(w)-2+f'(w)w\Big)\Big)\\ &\quad+\mu\frac{f(w)-(w-\delta)f'(w)}{f(w)^2}v''\left(\frac{w-\delta}{f(w)}\right).\end{aligned}$$

Substituting (A.34) into the above expression obtain and rearranging terms obtain

$$\begin{aligned}0 &= v'''(w)\Big(\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)+\lambda\delta\Big)+v''(w)\Big(2\rho-\gamma-r+\mu\Big(f(w)-2+f'(w)w\Big)\Big)\\ &\quad+\mu\frac{wf(w)}{w-\delta}v''(w)+\lambda(v''(w-\delta)-v''(w)).\end{aligned}$$

Rearranging terms obtain

$$\begin{aligned}0 &= v'''(w)\Big(\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)\Big)\\ &\quad+v''(w)\left(2\rho-\gamma-r+\mu\big(f(w)-2+f'(w)w\big)+\mu\frac{wf(w)}{w-\delta}\right).\end{aligned}\tag{A.35}$$

Verification that $w\cdot v''(w)$ is increasing for $w>\delta$. Suppose there exists a $\tilde{w}$ such that

$$\tilde{w}=\inf\big\{w>\delta:\ v''(\tilde{w})+\tilde{w}v'''(\tilde{w})\le 0\big\}.$$

Then $v'''(w)$ is continuous from (A.35). Thus $v'''(\tilde{w})\le-\frac{v''(\tilde{w})}{\tilde{w}}$. Then at $w=\tilde{w}$ equation (A.35) becomes

$$\begin{aligned}0 &\le -\frac{v''(w)}{w}\Big(\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)\Big)\\ &\quad+v''(w)\left(2\rho-\gamma-r+\mu\left(f(w)-2+f'(w)w+\frac{wf(w)}{w-\delta}\right)\right).\end{aligned}$$

Dividing both sides by $v''(w)<0$ obtain

$$0\ge-\frac{\rho w+h+\mu\Big(\delta+(f(w)-1)w\Big)}{w}+\left(2\rho-\gamma-r+\mu\left(f(w)-2+f'(w)w+\frac{wf(w)}{w-\delta}\right)\right)$$

$$0\ge-\frac{h+\mu\delta+\lambda\delta}{w}+\rho-r-\gamma+\mu(f'(w)w-1)+\mu\frac{wf(w)}{w-\delta}.$$

Note that $v''(w-\delta)\cdot(w-\delta)\le v''(w)\cdot w\le 0$ for $w<\tilde{w}$ implying that $\frac{v''(w-\delta)}{v''(w)}\ge\frac{w}{w-\delta}$. Hence

$$0\ge-\frac{\frac{h}{\mu}+\delta}{w}+\frac{\rho-r-\gamma}{\mu}+(f'(w)w-1)+\frac{wf(w)}{w-\delta}$$

$$0\ge-\frac{\delta+h/\mu}{w}+\frac{\rho-r-\gamma}{\mu}+f'(w)w-1+\frac{wf(w)}{w-\delta}$$

$$0 \geq -\frac{\delta + h/\mu}{w} + \frac{\rho - r - \gamma}{\mu} + f'(w)w - 1 + \frac{wf(w)}{w - \delta}$$

$$0 \geq -\frac{h}{\mu w} + \frac{\rho - r - \gamma}{\mu} - \frac{\delta}{w} + f'(w)w - 1 + \frac{wf(w)}{w - \delta}. \tag{A.36}$$

Differentiate the optimal incentive compatibility condition with respect to w to obtain

$$\begin{aligned}
0 &= f'(w)\phi(w) + f(w)\phi'(w) - f'(w)w + 1 - f(w) \\
0 &= f'(w)(\phi(w) - w) + 1 - f(w) + f(w)\phi'(w) \\
0 &= -f'(w)\frac{w - \delta}{f(w)} + 1 - f(w) + f(w)\phi'(w) \\
f'(w) &= \frac{1 - f(w) + f(w)\phi'(w)}{\frac{w-\delta}{f(w)}} \geq \frac{1 - f(w)}{\frac{w-\delta}{f(w)}} = \frac{f(w)(1 - f(w))}{w - \delta}
\end{aligned}$$

Substituting the above lower bound for $f'(w)$ given by $\frac{(1-f(w))f(w)}{w-\delta}$ into (A.36) a sufficient condition for the contradiction with the existence of $\tilde{w}$ is that

$$\begin{aligned}
0 &\leq -\frac{h}{\mu w} + \frac{\rho - r - \gamma}{\mu} - \frac{\delta}{w} + \frac{f(w)(1 - f(w))}{w - \delta} \cdot w - 1 + \frac{wf(w)}{w - \delta} \\
0 &\leq -\frac{h}{\mu w} + \frac{\rho - r - \gamma}{\mu} - \frac{\delta}{w} + f(w)(2 - f(w)) \cdot \frac{w}{w - \delta} - 1 \\
0 &\leq -\frac{h}{\mu w} + \frac{\rho - r - \gamma}{\mu} + f(w)(2 - f(w)) \cdot \frac{w}{w - \delta} - 1 - \frac{\delta}{w} \\
0 &\leq -\frac{h}{\mu}(w - \delta) + \frac{\rho - r - \gamma}{\mu} w(w - \delta) + f(w)(2 - f(w))w^2 - w(w - \delta) - \delta(w - \delta) \\
0 &\leq -\frac{h}{\mu}(w - \delta) + \frac{\rho - r - \gamma}{\mu} w(w - \delta) + f(w)\big(2 - f(w)\big) \cdot w^2 - w^2 + \delta^2
\end{aligned} \tag{A.37}$$

The right hand side of (A.37) is increasing in $f(w)$. This means that if it is satisfied for the smallest $f(w)$, then it is satisfied for other $f(w)$. Note that $\delta = f(w)\phi(w) + (1 - f(w))w \quad \Rightarrow \quad f(w) \geq \frac{w-\delta}{w}$. Substituting $f(w) = \frac{w-\delta}{w}$ into (A.37) obtain

$$\begin{aligned}
0 &\leq -\frac{h}{\mu}(w - \delta) + \frac{\rho - r - \gamma}{\mu} w(w - \delta) + \frac{w - \delta}{\delta}\left(2 - \frac{w - \delta}{\delta}\right) \cdot w^2 - w^2 + \delta^2 \\
0 &\leq -\frac{h}{\mu}(w - \delta) + \frac{\rho - r - \gamma}{\mu} w(w - \delta) + \frac{w - \delta}{\delta} \cdot \frac{w + \delta}{\delta} \cdot w^2 - w^2 + \delta^2 \\
0 &\leq \frac{(\rho - r - \gamma)w - h}{\mu}(w - \delta) + \frac{(w^2 - \delta^2)^2}{\delta^2} \\
0 &\leq \frac{(\rho - r - \gamma)w - h}{\mu} + \frac{(w - \delta)(w + \delta)^2}{\delta^2}
\end{aligned} \tag{A.38}$$

The derivative of the right hand side of (A.38) respect to w is equal to

$$\frac{d}{dw}\left[\frac{(\rho-r-\gamma)w-h}{\mu}+\frac{(w-\delta)(w+\delta)^2}{\delta^2}\right]=\frac{(\rho-r-\gamma)}{\mu}+\frac{(w-\delta)^2+2(w^2-\delta^2)}{\delta^2}$$
$$\overset{(i)}{\geq}-\frac{\Delta}{\mu}+\frac{(w-\delta)(3w+2\delta)}{\delta^2}\overset{(ii)}{>}0,$$

where (i) holds for $\rho-r+\Delta-\gamma\geq 0$ and (ii) holds for $w-\delta\geq\frac{h}{\mu}$. Condition (A.38), thus, holds if it holds at $w=\delta+\frac{h}{\mu}$:

$$\begin{aligned}
0 &\leq \frac{(\rho-r-\gamma)(\delta+\frac{h}{\mu})-h}{\mu}\cdot\frac{h}{\mu}+\left(\frac{h}{\mu}\right)^2\cdot\frac{(2\delta+\frac{h}{\mu})^2}{\delta^2}\\
0 &\leq \frac{(\rho-r-\gamma)(\delta+\frac{h}{\mu})-h}{\mu}+\frac{h}{\mu}\cdot\frac{(2\delta+\frac{h}{\mu})^2}{\delta^2}\\
0 &\leq \frac{(\rho-r-\gamma)(\delta+\frac{h}{\mu})}{\mu}+\frac{h}{\mu}\cdot\frac{(2\delta+\frac{h}{\mu})^2-\delta^2}{\delta^2}\\
0 &\leq \frac{\rho-r-\gamma}{\mu}+\frac{h}{\mu}\cdot\frac{3\delta+\frac{h}{\mu}}{\delta^2}\\
0 &\leq \rho-r-\gamma+\frac{h}{\delta}\cdot\frac{3\delta+\frac{h}{\mu}}{\delta}\\
0 &\leq \rho-r+\Delta-\gamma+\Delta\cdot\frac{2\delta+\frac{h}{\mu}}{\delta}.
\end{aligned}$$

The above sufficient condition is satisfied whenever $\rho-r+\Delta-\gamma\geq 0$. □

Lemma A.12. *Suppose $\rho=r$ and $r+\gamma\geq\mu\cdot\bar{f}$. Then, the optimal monitoring is given by* (17).

Proof. The first order condition (16) is given by

$$1-l+v'(w)w-v(w)+v\left(\frac{w-\delta}{f}\right)-\frac{w-\delta}{f}\cdot v'\left(\frac{w-\delta}{f}\right)\geq 0.$$

We need to check that for r sufficiently large the maximum amount of monitoring $f(w)$ and the implied penalty $\phi(w)$ hit the corner solution

$$1-l+v'(w)w-v(w)+v\left(\frac{w-\delta}{f(w)}\right)-\frac{w-\delta}{f(w)}\cdot v'\left(\frac{w-\delta}{f(w)}\right)\geq 0.$$

Consider the two cases:

(i) Case $w\leq\left(1+\bar{f}\right)\cdot\delta$. Thus $f(w)=\frac{w-\delta}{\delta}$. The maximum monitoring is optimal if and only if

$$1-l+wv'(w)+v(\delta)-v(w)-\delta v'(\delta)\geq 0.$$

The derivative of the above expression with respect to w is given by $w\cdot v''(w)\leq 0$.

(ii) Case $w > \left(1+\bar{f}\right)\cdot\delta$. The maximum monitoring intensity $f(w)=\bar{f}$ is optimal if

$$1 - l + v'(w)w - v(w) + v\left(\frac{w-\delta}{\bar{f}}\right) - \frac{w-\delta}{\bar{f}}\cdot v'\left(\frac{w-\delta}{\bar{f}}\right) \geq 0. \tag{A.39}$$

The derivative of the above expression with respect to $\bar{f}$ is $\frac{(w-\delta)^2}{\bar{f}^3}\cdot v''\left(\frac{w-\delta}{\bar{f}}\right) \leq 0$. If (A.39) holds at $\tilde{f} = \frac{w-\delta}{\delta}$,[4] then the maximum monitoring intensity $f(w)=\bar{f}$ is sufficient.

Define the auxiliary function $g(w) \stackrel{def}{=} w\cdot v'(w) - v(w)$. It is sufficient to show that

$$1 - l + wv'(w) - v(w) + v(\delta) - \delta\cdot v'(\delta) \geq 0 \quad \Leftrightarrow \quad 1 - l + g(w) - g(\delta) \geq 0.$$

Function $g(\cdot)$ is decreasing since $g'(w) = wv''(w) \leq 0$. Thus, if it holds at $w = \bar{w} = \frac{\delta}{1-\bar{f}}$, it holds for all $w \in [\delta, \bar{w}]$. At the upper threshold we have

$$g(\bar{w}) = \bar{w}\cdot v'(\bar{w}) - v(\bar{w}) = -\bar{w} - \frac{\alpha - \mu(1-\bar{f}+l\bar{f}) - h + \gamma K}{r+\gamma} + \bar{w} = -\frac{\alpha - \mu(1-\bar{f}+l\bar{f}) - h + \gamma K}{r+\gamma}.$$

To evaluate $g(\delta) = \delta\cdot v'(\delta) - v(\delta)$, note that at $w = \delta$ the principal's value function satisfies the differential equation

$$\begin{aligned}
(r+\gamma)\cdot v(\delta) &= \alpha - \mu + \gamma(K-\delta) + v'(\delta)\cdot(r\delta + h) \\
(r+\gamma)v(\delta) - (r+\gamma)\delta v'(\delta) &= \alpha - \mu + \gamma(K-\delta) + \left(1 - \frac{\gamma}{\Delta}\right)\cdot h\cdot v'(\delta) \\
-(r+\gamma)\cdot g(\delta) &= \alpha - \mu + \gamma(K-\delta) + \left(1 - \frac{\gamma}{\Delta}\right)\cdot h\cdot v'(\delta).
\end{aligned}$$

Then the sufficient condition for maximum monitoring to be optimal is

$$\begin{aligned}
1 - l + g(\bar{w}) - g(\delta) &= 1 - l - \frac{\alpha - \mu(1-\bar{f}+l\bar{f}) - h + \gamma K}{r+\gamma} + \frac{\alpha - \mu + \gamma(K-\delta) + \left(1-\frac{\gamma}{\Delta}\right)h\cdot v'(\delta)}{r+\gamma} \\
&= 1 - l + \frac{\mu(1-\bar{f}+l\bar{f}) + h}{r+\gamma} + \frac{-\mu - \gamma\delta + \left(1-\frac{\gamma}{\Delta}\right)h\cdot v'(\delta)}{r+\gamma} \\
&= 1 - l + \frac{-\mu(1-l)\bar{f} + h}{r+\gamma} + \frac{-\gamma\delta + \left(1-\frac{\gamma}{\Delta}\right)h\cdot v'(\delta)}{r+\gamma} \\
&= \frac{(r+\gamma)(1-l) - \mu(1-l)\bar{f} + h}{r+\gamma} + \frac{-\frac{\gamma}{\Delta}h + \left(1-\frac{\gamma}{\Delta}\right)h\cdot v'(\delta)}{r+\gamma} \\
&= \frac{(r+\gamma-\mu\bar{f})(1-l) + h\cdot\left(1-\frac{\gamma}{\Delta}\right)(v'(\delta)+1)}{r+\gamma}
\end{aligned}$$

Since $v'(\delta) \geq -1$, it implies a sufficient condition for maximum monitoring to be given by

$$r + \gamma \geq \mu\cdot\bar{f} \qquad \Rightarrow \qquad 1 - l + g(\bar{w}) - g(\delta) \geq 0. \tag{A.40}$$

[4] Even if it is infeasible, it is useful to consider whether even for such a high monitoring intensity, it is optimal to increase the monitoring intensity further.

□

A.7 Proof of Proposition 4 (renegotiation-proof contract)

Denote by $v_R(w)$ the principal's value function under the optimal renegotiation-proof contract. It must be the case that $v_R(w) \leq K$ and $v_R'(w) \leq 0$. The dynamics of the agent's continuation value follow (7).

Case $\lambda = 0$. Suppose $v_R(h/\Delta) \leq 0$. Then the optimal contract is renegotiation-proof due to concavity of $v_R(w)$ for $w > h/\Delta$. Suppose $v_R(h/\Delta) > 0$. Then define $\underline{w} > h/\Delta$ such that $v_R'(\underline{w}) = 0$ and $v_R(\underline{w}) = v(h/\Delta)$. Under a renegotiation-proof contract the value function of the principal, $v_R(w)$ follows (11) for interior continuation values. If μ is sufficiently large, then the optimal monitoring $f(w)$ is bounded above by $\frac{w-\delta}{\delta}$ implying that the agent's continuation value w_t does not drop below δ. This implies that, even though $v'(w) > 0$ for $w \in [0, \delta)$, the optimal contract needs not to be renegotiated since the agent's continuation value does not enter this region on-path. Since termination needs not be induced, the principal does not wish to renegotiate with the agent.

Case $\lambda > 0$. Suppose $v_R'(0) \leq 0$. Then the original contract is renegotiation-proof. Suppose that $v_R'(0) > 0$. If $\lambda > 0$ it implies that process $w_t = (w_t)_{t \geq 0}$ has full support due to the possibility of multiple negative shocks. Denote by $\underline{w}$ the lowest value such that $v_R'(\underline{w}) = 0$. Due to the concavity of the solution it implies that $v_R'(w) < 0$ for $w > \underline{w}$ which means the contract is renegotiation-proof.

A.8 Proof of Proposition 5 (observability of Y in a renegotiation-proof contract)

Define by $v_R(w)$ the value function of the principal under the optimal renegotiation-proof contract. The social surplus is given by $b(w) + w$ and must be concave in w. Otherwise the principal can use public randomization to generate a strict improvement.

Lemma A.13. *Under any optimal renegotiation-proof contract $\mathcal{C}^R$ it must be the case that*

$$h \geq \Delta \cdot \Big(f_t \cdot \phi_t + (1 - f_t) \cdot w_t \Big). \tag{A.41}$$

Proof. Suppose there exists an optimal, incentive compatible, renegotiation-proof contract $\mathcal{C}^R$ for every initial continuation value w for the agent. For each w define the set of effort levels $\mathcal{A}(w)$ such that

$$\mathcal{A}(w) = \left\{ \hat{a} = \{\hat{a}_t\}_t \geq 0 : \mathrm{E}_{\hat{a}} \left[e^{-\rho\tau} C_\tau - \int_0^\tau e^{-\rho t} \hat{a}_t h \, dt \right] = \mathrm{E}_{\bar{a}} \left[e^{-\rho\tau} C_\tau - \int_0^\tau e^{-\rho t} h \, dt \right] \right\}.$$

Then define

$$A(w) = \inf_{\hat{a} \in \mathcal{A}(w)} \mathrm{E} \left[\int_0^\tau e^{-\rho t} \hat{a}_t h \, dt \right] \tag{A.42}$$

Function $A(w)$ is the minimum expected amount of effort the agent needs to exert to be indifferent between his deviation and high effort in the incentive compatible contract $\mathcal{C}^R$. Note that it is without loss of generality to think for $A(\cdot)$ being indexed by agent's continuation utilities. It would be equivalent to index it by the associated public history of the contract. The only relevant property is that it is non-negative.

Define $W(w,q)$ to be the continuation value of the agent if the probability that the expected compensation C_τ is $q \leq 1$. If the agent exerts high effort then, along the equilibrium path, $q = 1$. However if $q < 1$, then the agent might wish to deviate to an alternative effort level:

$$W(w,q) = \sup_{\hat{a}} \mathrm{E}_{\hat{a}} \left[\pi \cdot e^{-\rho\tau} C_\tau - \int_0^\tau e^{-\rho t} \hat{a}_t h \, dt \right].$$

According to the envelope theorem:

$$\frac{\partial W}{\partial q}(w,q) = \mathrm{E}_{\hat{a}} \left[e^{-\rho\tau} C_\tau \right] = w + A(w)$$

where $A(w)$ is defined in (A.42). This is necessary since, otherwise, by following effort $A(w)$ the agent could achieve a higher expected payout than $W(w,q)$ in some neighborhood $q \in (1-\varepsilon, 1)$, which would contradict the definition of $W(w,q)$. Following Proposition 1 of Sannikov (2014), the necessary local incentive compatibility condition is given by:

$$\delta \leq f(w) \cdot \phi(w) + (1 - f(w)) \cdot \frac{\partial W}{\partial q}(w,q). \tag{A.43}$$

This is based on Lemma 3 which proves that the agent is only compensated in the high state $U_\tau = 0$ in the renegotiation-proof contract in which Y is not privately observed by the agent.

In a renegotiation-proof contract the principal cannot commit to delayed punishments if such incentives are not dynamically credible. Every period the principal can offer a new "forward-looking" contract which does not condition on information prior to that period. This implies that, when choosing (f_t, ϕ_t), in period t, the principal does not take into account the effect this had on incentive compatibility in periods $s < t$. Note that this issue does not arise in contracts in which the agent directly observes performance since the sensitivity of agent's compensation to effort at time s is resolved at time s, and has no implication on incentives at time t.

Using the above argument I show below that (A.43) is binding under $\mathcal{C}^R$. From the contrary, suppose (A.43) is strict:

$$\delta < f(w) \cdot \phi(w) + (1 - f(w)) \cdot (w + A(w)).$$

By arguments similar to the proof of Proposition 2 the value function of the principal under the optimal renegotiation-proof contract must satisfy

$$(r+\gamma)v(w) = \max_{\phi,f} \Big\{\alpha - \mu\cdot(1-f+fl) + \gamma(K-w) + v'(w)\cdot\Big(\rho w + h + \mu f\phi\Big) + \mu f\cdot\Big(v(w-\phi) - v(w)\Big)\Big\}.$$

For a given level of monitoring f the first order condition with respect to ϕ is given by

$$\mu\cdot f\cdot v'(w) - \mu\cdot f\cdot v'(w-\phi) = 0.$$

This implies that $v'(w-\phi) = v'(w)$. Then taking derivatives of the left and right hand sides using the envelope theorem:

$$(r+\gamma)\cdot v'(w) = -\gamma + v''(w)\cdot\Big(\rho w + h + \mu f\phi\Big) + v'(w)\rho + \mu f\cdot\Big(v'(w-\phi) - v'(w)\Big)$$

$$\gamma\cdot(v'(w)+1) + (r-\rho)\cdot v'(w) = v''(w)\cdot\Big(\rho w + h + \mu f\phi\Big).$$

Note that $v''(w) \le 0$, while $v'(w) \ge -1$ and $v'(w) \le 0$. This implies that if the local incentive compatibility constraint is not binding, then $v'(w) = -1$. However this implies that $w \ge \bar{w}$ which is a contradiction if either $\rho > r$ and/or $\bar{f} = 1$. Thus, the local incentive compatibility condition must be binding under the optimal renegotiation-proof contract. Thus

$$h = \Delta\cdot\Big(f_t\cdot\phi_t + (1-f_t)\cdot(w_t + A(w_t))\Big)$$

This implies that

$$h \ge \Delta\cdot\Big(f_t\cdot\phi_t + (1-f_t)\cdot w_t\Big)$$

which is strict whenever $A(w_t) > 0$. □

Lemma A.14. *Contract $\mathcal{C}^R$ satisfying* (A.41) *is incentive compatible if and only if* (A.41) *holds with equality with probability* 1.

Proof. Consider an agent's relaxed best response to a contract problem in which agent's effort cost if he deviates is given by

$$h_t(a) \stackrel{def}{=} q_t\cdot h < h$$

where

$$q_t \stackrel{def}{=} e^{-\int_0^t \Delta(1-f_s)(1-a_s)\,ds}.$$

Function q_t is a decreasing function in time and $\dot{q}_t \le 0$ if and only if $(1-f_t)(1-a_t) > 0$. In such a relaxed problem the agent's effort cost permanently decreases after a deviation to low effort. This makes deviations more profitable. The payoff of the agent under such an effort cost out of equilibrium is given by:

$$\max_a \; \mathrm{E}\left[e^{-\rho\tau} \cdot \mathbb{1}\left\{U_\tau = 0\right\} \cdot w_\tau - \int_0^\tau e^{-\rho t} h_t(a_t)\, dt\right] \geq \max_a \mathrm{E}\left[e^{-\rho\tau} \cdot \mathbb{1}\left\{U_\tau = 0\right\} \cdot w_\tau - \int_0^\tau e^{-\rho t} h a_t \, dt\right].$$

The agent's continuation value along the equilibrium path is w_t. The agent's continuation value if he deviates to alternative effort profiles is then given by $\hat{W}(w, q) = q \cdot w$. The incentives to exert effort are thus independent of the history. Necessary and sufficient incentive compatibility conditions are given by

$$q_t \cdot \delta \leq q_t \cdot f_t \cdot \phi_t + q_t \cdot (1 - f_t) \cdot w_t,$$

$$\delta \leq f_t \cdot \phi_t + (1 - f_t) \cdot w_t.$$

This implies that if (A.41) holds, then one of the agent's best responses is to not exert effort. In the latter case however the effort costs are 0 since he does not exert any effort and, the fact that the problem was relaxed, does not matter. Contradiction. □

This sequence of lemmas imply that the optimal renegotiation-proof contract sets incentive constraint (13) to be binding. The optimality of bounded monitoring is given by Lemma A.9. Since $v'(\delta) \leq 0$ for a renegotiation-proof contract, then $1 - l + K \leq v(\delta)$ is sufficient to limit monitoring to reduce termination risks.

A.9 Numerical Implementation of the Model

Define by L to be the loss associated with a bad project. The main model features a normalized value of $L = -1$. In this context, divestment cost l best captures a percentage cost. Equation (11) is given by

$$\begin{aligned}(r + \gamma) \cdot v(w) = \max_{f,\,\phi,\,\psi} \Big\{ & \alpha - \lambda \cdot l \cdot L - \mu \cdot (1 - f + fl) \cdot L \\ & + \gamma \cdot (K - w) + v'(w) \cdot \Big(\rho w + h + \mu f \phi + \lambda \psi\Big) \\ & + \mu \cdot f \cdot \Big(v(w - \phi) - v(w)\Big) + \lambda \cdot \Big(v(w - \psi) - v(w)\Big) \Big\}\end{aligned}$$

subject to the incentive compatibility conditions

$$\begin{cases} \psi \leq f \cdot \phi + (1 - f) \cdot w, \\ \delta \leq \psi. \end{cases} \tag{A.44}$$

Under the optimal contract $\psi = \delta$ and the truth-telling incentive compatibility condition is binding implying that

$$f \cdot \phi = \delta - (1-f) \cdot w = \delta + (f-1) \cdot w \qquad \Rightarrow \qquad \phi = w - \frac{w-\delta}{f}.$$

This implies the principal's Hamilton-Jacobi-Bellman equation (A.15) can be written as

$$\begin{aligned}(r+\gamma) \cdot v(w) = \max_f \Big\{ & \alpha - \lambda \cdot l \cdot L - \mu \cdot (1-f+fl) \cdot L \\ & + \gamma \cdot (K-w) + v'(w) \cdot \Big[\rho w + h + \mu(\delta + (f-1)w)\Big] \\ & + \mu \cdot f \cdot \left[v\left(\frac{w-\delta}{f}\right) - v(w)\right] + \lambda \cdot \Big[v(w-\delta) - v(w)\Big]\Big\}.\end{aligned}$$

The first order condition with respect to f is given by (A.19) given by

$$v(w) - w \cdot v'(w) = L \cdot (1-l) + v\left(\frac{w-\delta}{f}\right) - \frac{w-\delta}{f} \cdot v'\left(\frac{w-\delta}{f}\right)$$

subject to $\frac{w-\delta}{f} \in \left[\delta,\ \frac{w-\delta}{\bar{f}}\right]$. If $v(w)$ is concave, then there is a unique solution to the above first-order-condition. Can rewrite the first order condition via ϕ as

$$v(w) - w \cdot v'(w) = L \cdot (1-l) + v(w-\phi) - (w-\phi) \cdot v'(w-\phi) \tag{A.45}$$

with monitoring rate f being pinned down by ϕ via $f = \min\left\{\frac{w-\delta}{w-\phi},\ \bar{f}\right\}$. Can rewrite the ordinary differential equation for $v(w)$ as

$$\begin{aligned}(r+\gamma) \cdot v(w) = \alpha - \lambda \cdot L \cdot l - \mu \cdot (1-f+fl) \cdot L + \gamma \cdot (K-w) + v'(w) \cdot \big[\rho w + h + \mu f \phi + \lambda \delta\big] \\ + \mu f \cdot \big[v(w-\phi) - v(w)\big] + \lambda \cdot \big[v(w-\delta) - v(w)\big].\end{aligned}$$

Can express the value of the derivative of the value function given the pair (f, ϕ) as

$$v'(w) = \frac{(r+\gamma)v(w) - \alpha + \lambda l L + \mu(1-f+fl)L - \gamma(K-w) - \mu f \cdot \big[v(w-\phi) - v(w)\big] - \lambda \cdot \big[v(w-\delta) - v(w)\big]}{\rho w + h + \mu f \phi + \lambda \delta}.$$

Substituting this into (A.45) obtain an expression for (f, ϕ) that does not depend on $v'(w)$

$$v(w) - w \cdot \frac{(r+\gamma)v(w) - \alpha + \lambda l L + \mu(1-f+fl)L - \gamma(K-w) - \mu f \cdot \big[v(w-\phi) - v(w)\big] - \lambda \cdot \big[v(w-\delta) - v(w)\big]}{\rho w + h + \mu f \phi + \lambda \delta}$$

$$= L \cdot (1-l) + v(w-\phi) - (w-\phi) \cdot v'(w-\phi)$$

Note that the pair (f, ϕ) in the above expression satisfies (A.45) at w. Multiplying both sides of the above expression by the denominator, obtain

$$
\begin{aligned}
&v(w)\cdot\big(\rho w+h+\mu f\phi+\lambda\delta\big)\\
&-w\cdot\Big[(r+\gamma)v(w)-\alpha+\lambda lL+\mu(1-f+fl)L-\gamma(K-w)-\mu f\cdot\big(v(w-\phi)-v(w)\big)-\lambda\cdot\big(v(w-\delta)-v(w)\big)\Big]\\
&=\Big[L(1-l)+v(w-\phi)-v'(w-\phi)(w-\phi)\Big]\cdot\big(\rho w+h+\mu f\phi+\lambda\delta\big).
\end{aligned}
$$

Rearranging terms

$$
\begin{aligned}
&v(w)\cdot\Big[(\rho-r-\gamma)w+h+\mu f\phi+\lambda\delta\Big]\\
&-w\cdot\Big[-\alpha+\lambda lL+\mu(1-f+fl)L-\gamma(K-w)-\mu f\cdot\big(v(w-\phi)-v(w)\big)-\lambda\cdot\big(v(w-\delta)-v(w)\big)\Big]\\
&=\Big[L(1-l)+v(w-\phi)-(w-\phi)\cdot v'(w-\phi)\Big]\cdot\Big[\rho w+h+\mu f\phi+\lambda\delta\Big].
\end{aligned}
$$

Collecting the terms by $v(w)$ in the top equation obtain

$$
\begin{aligned}
&v(w)\cdot\Big[(\rho-r-\gamma)w+h+\mu f\phi+\lambda\delta-w\mu f-w\lambda\Big]\\
&-w\cdot\Big[-\alpha+\lambda lL+\mu(1-f+fl)L-\gamma(K-w)-\mu f\cdot v(w-\phi)-\lambda\cdot v(w-\delta)\Big]\\
&=\Big[L(1-l)+v(w-\phi)-(w-\phi)\cdot v'(w-\phi)\Big]\cdot\Big[\rho w+h+\mu f\phi+\lambda\delta\Big].
\end{aligned}
$$

Note that $f\cdot\phi=\delta+(f-1)\cdot w$. Thus

$$
\begin{aligned}
&v(w)\cdot\Big[(\rho-r-\gamma)w+h+\mu(\delta+(f-1)w)+\lambda\delta-w\mu f-w\lambda\Big]\\
&-w\cdot\Big[-\alpha+\lambda l\cdot L+\mu\cdot(1-f+fl)\cdot L-\gamma(K-w)-\mu f\cdot v(w-\phi)-\lambda\cdot v(w-\delta)\Big]\\
&=\Big[L(1-l)+v(w-\phi)-(w-\phi)\cdot v'(w-\phi)\Big]\cdot\Big[\rho w+h+\mu\cdot(\delta+(f-1)\cdot w)+\lambda\delta\Big].
\end{aligned}
$$

Simplifying the top equation obtain

$$
\begin{aligned}
&v(w)\cdot\Big[(\rho-r-\gamma)w+h+(\mu+\lambda)(\delta-w)\Big]\\
&-w\cdot\Big[-\alpha+\lambda lL+\mu(1-f+fl)L-\gamma(K-w)-\mu f\cdot v(w-\phi)-\lambda\cdot v(w-\delta)\Big]\\
&=\Big[L(1-l)+v(w-\phi)-(w-\phi)\cdot v'(w-\phi)\Big]\cdot\Big[\rho w+h+\mu(\delta+(f-1)w)+\lambda\delta\Big].
\end{aligned}
$$

Moving $L(1-l)\cdot\mu(f-1)\cdot w$ from the right hand side to the left hand side obtain

$$
v(w)\cdot\Big[(\rho-r-\gamma-\lambda-\mu)\cdot w+h+(\mu+\lambda)\cdot\delta\Big]
$$

$$-w \cdot \Big[-\alpha + \lambda l L + \mu L \cdot \Big(1 - f + fl + (f-1)(1-l)\Big) - \gamma(K-w) - \mu f \cdot v(w-\phi) - \lambda \cdot v(w-\delta)\Big]$$
$$= \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] \cdot \Big[\rho w + h + \mu \cdot \Big(\delta + (f-1) \cdot w\Big) + \lambda\delta\Big]$$
$$+ L(1-l) \cdot \Big[\rho w + h + (\mu+\lambda) \cdot \delta\Big].$$

Simplifying the second term of the left hand side obtain

$$\begin{aligned}
&v(w) \cdot \Big[(\rho - r - \gamma - \lambda - \mu) \cdot w + h + (\mu+\lambda) \cdot \delta\Big] \\
&-w \cdot \Big[-\alpha + (\mu+\lambda) \cdot lL - \gamma \cdot (K-w) - \mu f \cdot v(w-\phi) - \lambda \cdot v(w-\delta)\Big] \\
&= \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] \cdot \Big[\rho w + h + \mu\Big(\delta + (f-1) \cdot w\Big) + \lambda\delta\Big] \\
&+ L(1-l) \cdot \Big[\rho w + h + (\mu+\lambda) \cdot \delta\Big].
\end{aligned}$$

Splitting the right hand side of the above identity obtain

$$\begin{aligned}
&v(w) \cdot \Big[(\rho - r - \gamma - \lambda - \mu) \cdot w + h + (\mu+\lambda) \cdot \delta\Big] \\
&-w \cdot \Big[-\alpha + (\mu+\lambda) \cdot lL - \gamma(K-w) - \mu f \cdot v(w-\phi) - \lambda \cdot v(w-\delta)\Big] \\
&= \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] \cdot \Big[\rho w + h + \mu(\delta - w) + \lambda\delta\Big] \\
&+ \mu f \cdot w \cdot \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] + L(1-l) \cdot \Big[\rho w + h + (\mu+\lambda) \cdot \delta\Big].
\end{aligned}$$

Canceling out $\mu f \cdot w \cdot v(w-\phi)$ terms on both sides and moving the second term from the right hand side to the left hand side obtain

$$\begin{aligned}
&v(w) \cdot \Big[(\rho - r - \gamma - \lambda - \mu) \cdot w + h + (\mu+\lambda) \cdot \delta\Big] \\
&+ w \cdot \Big[\alpha - (\mu+\lambda) \cdot lL + \gamma \cdot (K-w) + \lambda \cdot v(w-\delta)\Big] - L(1-l) \cdot \Big[\rho w + h + (\mu+\lambda) \cdot \delta\Big]. \\
&= \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] \cdot \Big[\rho w + h + \mu(\delta - w) + \lambda\delta\Big] - \mu f \cdot w \cdot (w-\phi) \cdot v'(w-\phi).
\end{aligned}$$

Note that $\frac{w-\delta}{f} = w - \phi$ implies that $f = \frac{w-\delta}{w-\phi}$. This yields further simplification

$$\begin{aligned}
&v(w) \cdot \Big[(\rho - r - \gamma - \lambda - \mu) \cdot w + h + (\mu+\lambda) \cdot \delta\Big] \\
&+ w \cdot \Big[\alpha - (\mu+\lambda) \cdot lL + \gamma \cdot (K-w) + \lambda \cdot v(w-\delta)\Big] - L(1-l) \cdot \Big[\rho w + h + (\mu+\lambda) \cdot \delta\Big] \\
&= \Big[v(w-\phi) - (w-\phi) \cdot v'(w-\phi)\Big] \cdot \Big[(\rho - \mu) \cdot w + h + (\mu+\lambda) \cdot \delta\Big] - \mu \cdot w \cdot (w-\delta) \cdot v'(w-\phi)
\end{aligned} \tag{A.46}$$

Define

$$LHS \stackrel{def}{=} v(w) \cdot \Big[(\rho - r - \gamma - \lambda - \mu) \cdot w + h + (\mu + \lambda) \cdot \delta\Big]$$
$$+ w \cdot \Big[\alpha - (\mu + \lambda) \cdot lL + \gamma \cdot (K - w) + \lambda \cdot v(w - \delta)\Big] - L(1 - l) \cdot \Big[\rho w + h + (\mu + \lambda) \cdot \delta\Big]$$

$$RHS(\phi) \stackrel{def}{=} \Big[v(w - \phi) - (w - \phi) \cdot v'(w - \phi)\Big] \cdot \Big[(\rho - \mu) \cdot w + h + (\mu + \lambda) \cdot \delta\Big] - \mu \cdot w \cdot (w - \delta) \cdot v'(w - \phi). \tag{A.47}$$

The LHS is independent of ϕ. The derivative of $RHS(\phi)$ with respect to ϕ is

$$\begin{aligned} \frac{d}{d\phi} RHS(\phi) &= (w - \phi) \cdot v''(w - \phi) \cdot \Big[\rho w + h + \mu(\delta - w) + \lambda\delta\Big] + \mu \cdot w \cdot (w - \delta) \cdot v''(w - \phi) \\ &= v''(w - \phi) \cdot \Big[(w - \phi) \cdot \Big(\rho w + h + \mu(\delta - w) + \lambda\delta\Big) + \mu \cdot w \cdot (w - \delta)\Big] \\ &= \underbrace{v''(w - \phi)}_{\leq 0} \cdot \underbrace{\Big[(w - \phi) \cdot \Big(\rho w + h + \lambda \cdot \delta\Big) + \lambda \cdot \phi \cdot (w - \phi)\Big]}_{>0} \leq 0, \end{aligned}$$

which implies that there exists a unique solution to (A.46) which pins down the optimal ϕ. The resulting ODE can be computed as

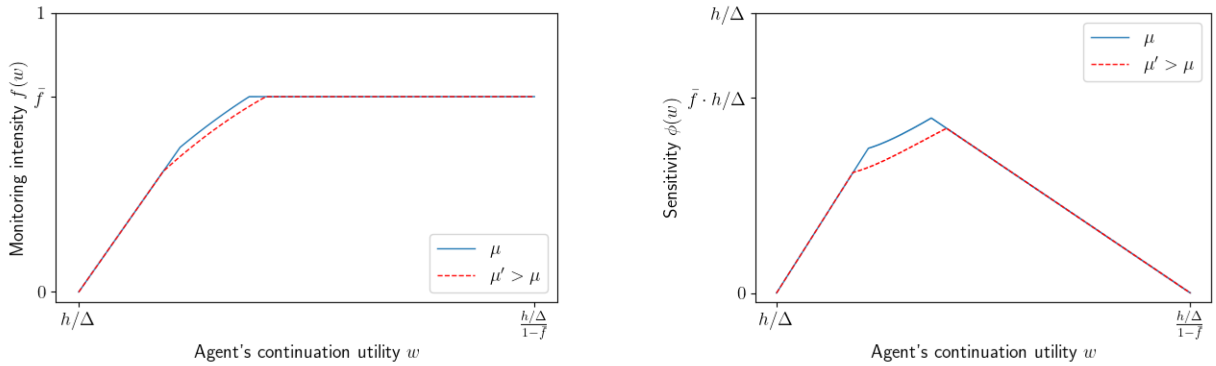
$$v'(w) = \frac{(r+\gamma)v(w) - \alpha + \lambda lL + \mu(1-f+fl)L - \gamma(K-w) - \mu f \cdot \big[v(w-\phi) - v(w)\big] - \lambda \cdot \big[v(w-\delta) - v(w)\big]}{\rho w + h + \mu f\phi + \lambda\delta}.$$

A.10 Numerical Comparative Statics

The optimal policies obtained via (A.46) allow for a numerical implementation of the model. I explore what happens as arrival intensity μ, discount rate r, monitoring capacity $\bar{f}$, and the arrival intensity Δ. The baseline parameters of the model are chosen to satisfy the sufficient parametric condition (3), but the illustrations are representative of a broader set of parameters.

Figure 1 shows that a higher arrival intensity μ reduces both the monitoring intensity $f(w)$ and the associated pay-for-performance sensitivity $\phi(w)$. The intuition is that a higher μ increases the riskiness of the agent's promised utility and, consequently, the probability that the monitoring intensity $f(w)$ will be constrained in order to avoid terminating the agent. In order to mitigate these increased fluctuations in the agent's promised utility, the principal monitors the agent less and backloads more of the pay-for-performance sensitivity to the agent's retirement date.

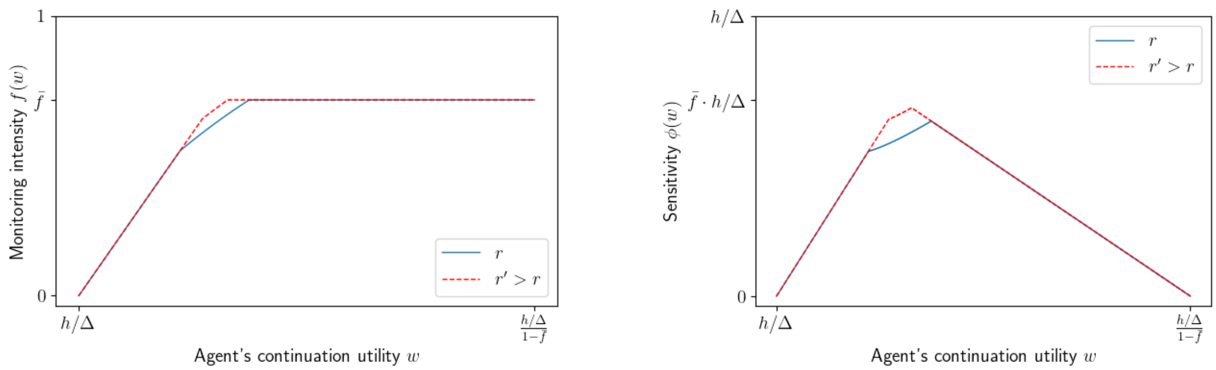
Figure 2 shows that the monitoring intensity increases as the agents become less patient, i.e., discount rate r is higher. The intuition is that a more impatient principal discounts more value of intervening in the future, thus preferring to intervene in bad projects early at the cost of a reduction of future interventions in the future.



(a) Optimal monitoring intensity $f(w)$.

(b) Optimal sensitivity $\phi(w)$.

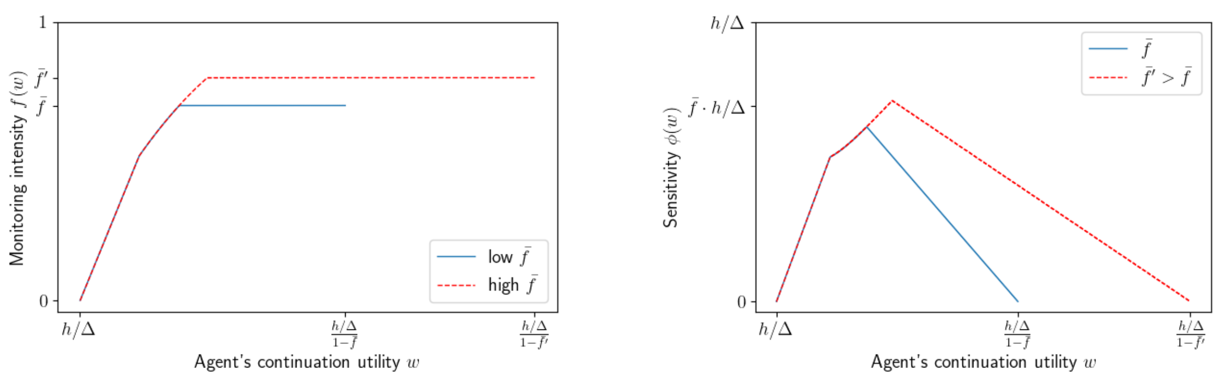
Figure 1: Comparison of optimal policies for arrival intensities μ and $\mu' > \mu$. Parameters: $h = 0.05$, $\mu = 1$, $\mu' = 1.2$, $r = 0.01$, $\lambda = 0$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.3$, $l = 0.85$, $L = 1$, $\bar{f} = 0.7$, $K = 0$.



(a) Optimal monitoring intensity $f(w)$.

(b) Optimal sensitivity $\phi(w)$.

Figure 2: Comparison of optimal policies for discount rates r and $r' > r$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $r' = 0.1$, $\lambda = 0$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.3$, $l = 0.85$, $L = 1$, $\bar{f} = 0.7$, $K = 0$.

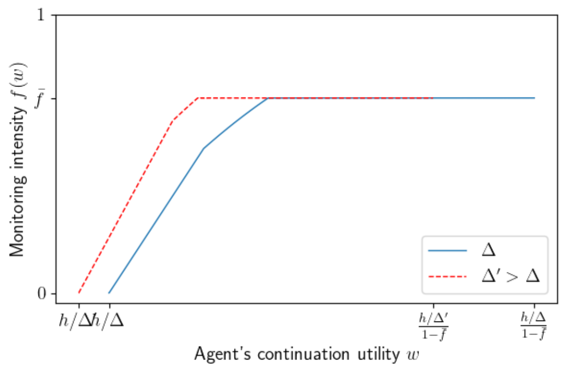


(a) Optimal monitoring intensity $f(w)$.

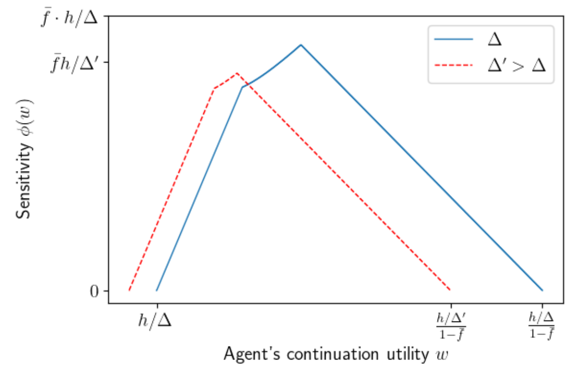
(b) Optimal sensitivity $\phi(w)$.

Figure 3: Comparison of optimal policies for monitoring capacities $\bar{f}$ and $\bar{f}' > \bar{f}$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $\lambda = 0$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.3$, $l = 0.85$, $L = 1$, $\bar{f} = 0.7$, $\bar{f}' = 0.8$, $K = 0$.

Figure 3 shows that a higher monitoring capacity increases overall monitoring intensity. The intuition is that an increase in $\bar{f}$ does not change the expected agency value of intervening early, while reducing the capacity constraint of intervening more frequently when the agent accumulates a high continuation value.



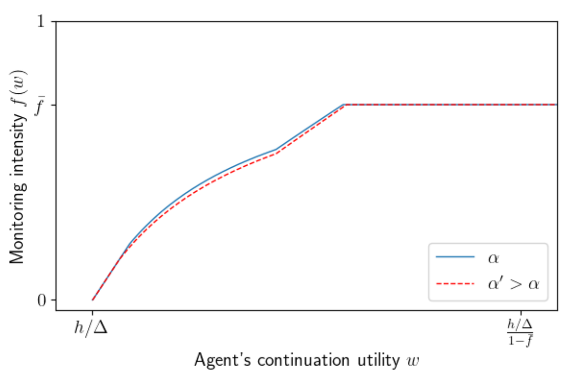
(a) Optimal monitoring intensity $f(w)$.



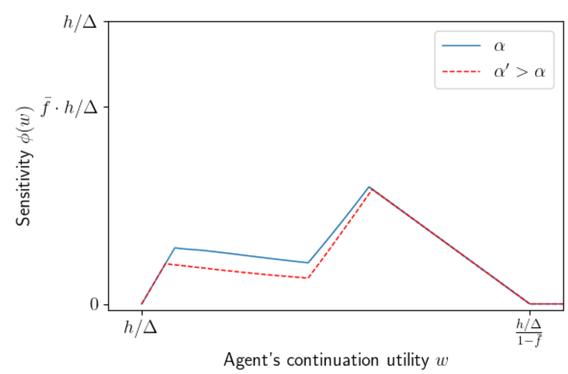
(b) Optimal sensitivity $\phi(w)$.

Figure 4: Comparison of optimal policies for arrival intensities Δ and $\Delta' > \Delta$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $\lambda = 0$, $\Delta = 0.5$, $\Delta' = 0.6$, $\gamma = 0.01$, $\alpha = 1.3$, $l = 0.85$, $L = 1$, $\bar{f} = 0.7$, $K = 0$.

Figure 4 shows that a higher arrival intensity Δ increases the monitoring intensity $f(w)$. The intuition is that a higher Δ increases the detection rate of the agent if he shirks, thus reducing the agency costs of implementing effort.



(a) Optimal monitoring intensity $f(w)$.

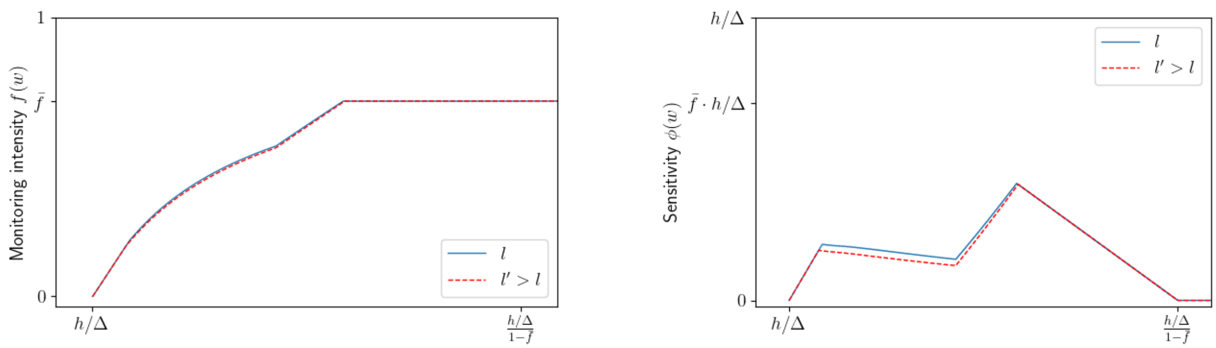


(b) Optimal sensitivity $\phi(w)$.

Figure 5: Comparison of optimal policies for profit flows α and $\alpha' > \alpha$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $\lambda = 0.03$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.15$, $\alpha' = 1.26$, $l = 0.85$, $L = 1$, $\bar{f} = 0.7$, $K = 0$.

I also consider how optimal policies respond to changes in the profit flow of good projects α, the cost L of a bad project to the principal, as well as the percentage intervention cost l. Interestingly, if $\lambda = 0$ and the set of parameters satisfies restriction (3), the project is operated efficiently and never liquidated. This implies that the principal's value function can be expressed as $v(w) = \frac{\alpha}{r+\gamma} + L \cdot \hat{v}(w)$, where $\hat{v}(w)$ is independent of either γ or L. This implies that the value of α and L cancel out in the determination of optimal monitoring (16) resulting in the optimal policy being independent of these parameters. If, however, $\lambda > 0$, then the project can be terminated prematurely and the optimal monitoring is affected by the profitability of the project to the principal.[5]

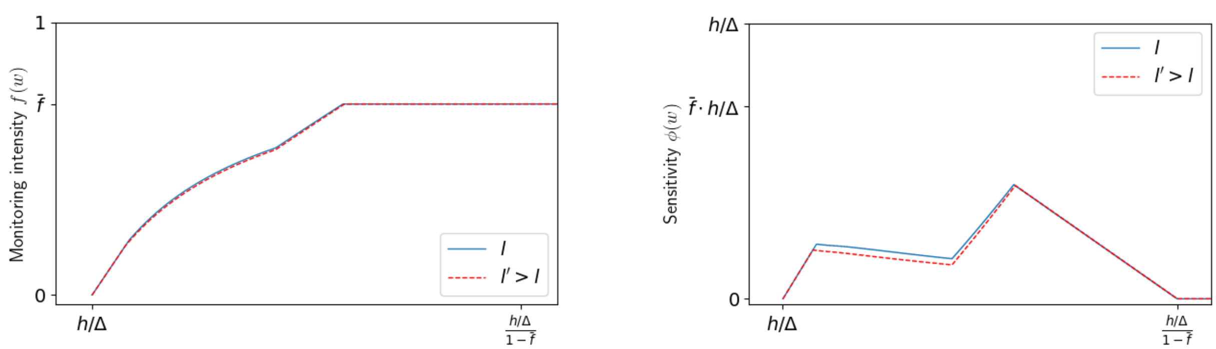
A positive arrival intensity $\lambda > 0$ means that the principal cannot isolate herself from the possibility of distress completely. A higher profit flow α means that such termination is more costly for the principal. Figure 5 shows that a higher profit α reduces monitoring intensity $f(w)$, and the associated pay-for-performance sensitivity $\phi(w)$. Conversely, an increase in the disaster arrival cost L or intervention cost l, reduces the termination costs for the principal and increases the optimal monitoring intensity $f(w)$ and pay-for-performance sensitivity $\phi(w)$, as can be seen in Figures 6 and 7.



(a) Optimal monitoring intensity $f(w)$.

(b) Optimal sensitivity $\phi(w)$.

Figure 6: Comparison of optimal policies for bad project costs L and $L' > L$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $\lambda = 0.03$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.15$, $l = 0.85$, $L = 1$, $L' = 1.05$, $\bar{f} = 0.7$, $K = 0$.



(a) Optimal monitoring intensity $f(w)$.

(b) Optimal sensitivity $\phi(w)$.

Figure 7: Comparison of optimal policies for intervention costs l and $l' > l$. Parameters: $h = 0.05$, $\mu = 1$, $r = 0.01$, $\lambda = 0.03$, $\Delta = 0.5$, $\gamma = 0.01$, $\alpha = 1.15$, $l = 0.85$, $l' = 0.9$, $L = 1$, $\bar{f} = 0.7$, $K = 0$.

[5] Setting $\lambda > 0$ in Figures 1-4 does not substantially change the comparative statics presented earlier.

A.11 Change of Measure for Poisson Processes

Lemma A.15. *Suppose* $\lambda = (\lambda_t)_{t \geq 0}$ *is an intensity of arrival of a Poisson process. Denote the associated process by* N_t*. Suppose* $\hat{\lambda} = (\hat{\lambda}_t)_{t \geq 0}$ *is an arrival intensity of process* $\hat{N}_t$ *such that* $\hat{\lambda}_t \geq \lambda_t$ *for each* t*. Then process* N_t *is a Poisson process with intensity* $\hat{\lambda}$ *under the change of measure density derivative*

$$q_T = exp\left\{ -\int_0^T \left(\hat{\lambda}_s - \lambda_s \right) ds + \int_0^T \log \left(\hat{\lambda}_s / \lambda_s \right) dN_s \right\}.$$

Proof. Can construct it by using discretization of the intensities and considering product measures of the pasted Poisson processes. It is also derived in Brémaud (1981, Chapter VI, Theorem T3). □

B Online Appendix B: Supplementary Analysis

B.1 Partial Observability of Monitoring Outcomes

The agent may observe monitoring outcomes due to the nature of the monitoring technology, observability of the principal's actions, or information leakage within the firm.[6] A natural way to account for partial observability of monitoring outcomes by the agent is to assume that he observes them with probability $\pi \leq 1$. The intensity of the agent seeing a bad project uncovered by monitoring along the path is then $\mu \cdot \pi \cdot f_t$. The continuation value dynamics account for this lower arrival intensity of monitoring penalties and are given by

$$dw_t = rw_t\,dt + a_t h\,dt + \phi_t \cdot \Big(\mu\pi f_t\,dt - dM_t\Big) + \psi_t \cdot \Big(\big(\lambda + \Delta(1-a_t)\big)\,dt - dR_t\Big),$$

quite similar to (7). The truthful-reporting incentive compatibility condition (8) is similarly updated to

$$\psi_t \leq \pi f_t \cdot \phi_t + (1 - \pi f_t) \cdot w_t,$$

while the effort provision constraint (9) remains unchanged. When monitoring results are partially observed, the first-order condition (16) pinning down the optimal monitoring intensity f becomes

$$1 - l = \pi \cdot \left(v(w) - wv'(w) - v\left(\frac{w - h/\Delta}{\pi f}\right) + \frac{w - h/\Delta}{\pi f} \cdot v'\left(\frac{w - h/\Delta}{\pi f}\right)\right), \qquad \text{(B.48)}$$

Partial observability of monitoring outcomes translates into a lower monitoring intensity πf from the perspective of the agent, but a higher marginal benefit of monitoring $\frac{1-l}{\pi}$ from the perspective of the principal. Both channels point to monitoring being more valuable to the principal if π is lower. The extreme case when the agent does not observe monitoring corresponds to $\pi = 0$. In this case, the principal monitors the agent with the maximum intensity $\bar{f}$. As π increases, monitoring becomes more costly, and the principal begins to monitor poorly-performing agents less to reduce the probability of inefficient termination.

B.2 Finite Horizon Model

In the main model, it was assumed that the agent retires at an exponentially distributed time η in order to avoid the necessity to keep track of time as an additional state variable. This allows establishing properties of the optimal contract analytically. In reality, as highlighted in Marinovic and Varas (2018), many employment relationships feature a fixed deadline. In this section I extend

[6] Partial leakage of performance results is already incorporated in the model via the agent observing bad projects stemming from process Y.

the baseline model and assume the agent must retire by a fixed date T.[7] In this setting, the agent's incentives are still pinned down by (8) and (9), however, characterizing the optimal monitoring policy requires solving a two-dimensional optimal control problem. The principal's value function $v(w,t)$, now a function of the agent's continuation utility w and, also, the remaining time $t \leq T$, satisfies

$$(r+\gamma)v(w,t) = \max_{(\phi,f,\psi)}\Big[\alpha - \lambda l - \mu(1-f+fl) - \gamma w + \Big(\rho w + h + \mu f\phi + \lambda\psi\Big)\frac{\partial}{\partial w}v(w,t) \\ - \frac{\partial}{\partial t}v(w,t) + \mu f\Big(v(w-\phi,t) - v(w,t)\Big) + \lambda\Big(v(w-\psi,t) - v(w,t)\Big)\Big]. \tag{B.49}$$

At time T the relationship ends and the principal pays the agent his promised utility, resulting in a boundary condition $v(w,0) = -w$. If a solution exists, then Lemma B.16 highlights that the main features of the infinite horizon optimal contract hold both for a finite t and in the limit as $t \to \infty$.

The formulation of (B.49) is standard. I do not prove the global concavity of the solution, but it can be verified numerically. For T sufficiently large, a continuity based argument guarantees that the solution to the optimal control problem (B.49) converges to (11). Appendix B provides a procedure how to implement the solution numerically.

Lemma B.16. *There exists a threshold t^* such that for every $t > t^*$ the optimal monitoring intensity is bounded from above by $\frac{w-h/\Delta}{h/\Delta}$. Moreover, the optimal policies converge to the infinite horizon model as the horizon t increases to infinity*

$$\lim_{t\to\infty} f(w,t) = f(w), \qquad \lim_{t\to\infty} \phi(w,t) = \phi(w).$$

The optimal monitoring intensity changes discontinuously at $t = t^*$. As the expected duration of the agent's employment increases, the principal becomes more averse to terminating the agent prematurely meaning that she lowers monitoring intensity $f(w,t)$ for low continuation value states. The relationship must have a sufficiently long horizon, in order for the principal to limit monitoring intensity in favor of the lower probability of inefficient termination. Moreover, the retirement intensity γ may be set to 0, even under equal patience. Then, as $t \to \infty$, then there still exists a well-defined limit $\lim_{t\to\infty} f(w,t)$, while the limiting contract may be defined only if $\rho > r$ and/or $\lambda = 0$.

B.2.1. Numerical Implementation of the Finite Horizon Model

Denote by $v(w,t)$ the value function of the principal. Then

$$rv(w,t) = \max_{\phi,f}\Big\{\alpha - \lambda l - \mu(1-f+fl) + \frac{\partial}{\partial t}v(w,t) + \Big(\rho w + h + \mu f\phi + \lambda\delta\Big)\frac{\partial}{\partial w}v(w,t)$$

[7] To make the model directly comparable to the one analyzed in Section 3 I allow the agent to retire with a constant intensity γ between 0 and T. One can set $\gamma = 0$ to obtain the case of the pure finite horizon model.

$$+\mu f\Big(v(w-\phi,t)-v(w,t)\Big)+\lambda\Big(v(w-\delta,t)-v(w,t)\Big)\Big\}.$$

The boundary condition is $v(w,T)=K-w$. Denote by $\phi(w,t)$ the optimal control given by

$$\phi(w,t)=\max\left\{\phi>0:\quad 1-l+v(w-\phi,t)-(w-\phi)\cdot\frac{\partial}{\partial w}v(w-\phi,t)\geq v(w,t)-w\cdot\frac{\partial}{\partial w}v(w,t)\right\}.$$

The corresponding monitoring rule is given by

$$f(w,t)=\frac{w-\delta}{w-\phi(w,t)}.$$

Note that

$$f(w,t)\leq\bar{f}\qquad\Leftrightarrow\qquad\phi(w,t)\leq w-\frac{w-\delta}{\bar{f}}.$$

Discrete scheme. Use the following discrete scheme

$$\begin{aligned}\frac{\partial}{\partial w}v(w,t)&=\frac{v(w,t)-v(w-dw,t)}{dw}\\ \frac{\partial}{\partial t}v(w,t)&=\frac{v(w,t)-v(w,t-dt)}{dt}\end{aligned}$$

Substituting into this discrete scheme obtain

$$\begin{aligned}rv(w,t)=\alpha-\lambda l-\mu(1-f(w,t)+f(w,t)l)+\frac{v(w,t)-v(w,t-dt)}{dt}\\ +\frac{v(w,t)-v(w-dw,t)}{dw}\Big(\rho w+h+\mu(f(w,t)w-w+\delta)+\lambda\delta\Big)\\ +\mu f(w,t)\Big(v(w-\phi(w,t),t)-v(w,t)\Big)+\lambda\Big(v(w-\delta,t)-v(w,t)\Big).\end{aligned}\tag{B.50}$$

This permits us to back out $v(w,t-dt)$ from (B.50) solving for $v(w,t)$ via backward induction.

B.3 Impatient Agent

Suppose the agent is relatively impatient and discounts future cash flows at a rate $\rho>r$. As shown in DeMarzo and Sannikov (2006) and Biais, Mariotti, Rochet, and Villeneuve (2010), if the agent is relatively impatient, the principal finds it expensive to defer compensation and optimally pays the agent when his continuation value is sufficiently high. Such interim transfers pose a challenge for the optimal contract derived in Section 3.2 since, if Lemma 3 were to hold, the agent can only be paid at time t if $U_t=0$. This poses a problem since if the agent observes that $U_t>0$, then he knows that he will not be compensated in the future. To resolve this conundrum, the principal can communicate additional performance feedback to the agent. While such communication may take many forms, it is sufficient to focus on the principal *probabilistically* disclosing that latent performance is good, i.e., that

$U_t = 0$.[8] The optimal contract rewards the agent in this event, and he receives a discrete bonus which is immediately paid out.[9] As a result, bonuses are optimally awarded when the principal discloses good latent performance. Optimal communication preserves the efficiency of delayed incentives as derived in Lemma 3 resulting in incentive compatibility constraints (8) and (9) being necessary and sufficient under the optimal contract.

Proposition 1. *Suppose* (3) *holds, λ is sufficiently small, and $\rho - r + \Delta \geq \gamma \geq \rho - r$. The principal optimally implements high effort as long as the agent is employed. The optimal contract is characterized by a payment threshold $\bar{w} < \frac{h/\Delta}{1-\bar{f}}$. For $w \in [h/\Delta, \bar{w}]$ the principal's value function satisfies the Hamilton-Jacobi-Bellman equation pinning down the optimal dynamic monitoring policy $f(w)$ and penalties $\phi(w), \psi(w)$ given by*

$$\begin{aligned}(r+\gamma)v(w) = \max_{(\phi,f,\psi)} \Big[&\alpha - \lambda l - \mu(1-f+fl) - \gamma w + v'(w)\big(\rho w + h + \mu f \phi + \lambda \psi\big) \\ &+ \mu f \big(v(w-\phi) - v(w)\big) + \lambda \big(v(w-\psi) - v(w)\big)\Big]\end{aligned} \tag{B.51}$$

where $f \in [0, \bar{f}]$ and $\phi, \psi \in [0, w]$ satisfy incentive constraints (8) *and* (9)*. For $w < h/\Delta$, the principal terminates the agent with probability $1 - \frac{w}{h/\Delta}$ and pays him nothing, while with probability $\frac{w}{h/\Delta}$ she retains him with a continuation value h/Δ resulting in $v(w)$ given by* (12)*. For $w > \bar{w}$, the principal probabilistically discloses good performance and pays the agent, resulting in the principal's value function being given by $v(w) = v(\bar{w}) - (w - \bar{w})$. Moreover, function $v(w)$ is the maximal solution to* (11) *and* (12) *satisfying the boundary condition $v'(\bar{w}) = -1$ for some $\bar{w}$.*

Proof. The proof for the principal wishing to implement high effort in the optimal contract is in Lemma A.7. Then, based on Lemma 3, the most efficient incentives are achieved when the agent does not get compensated if $U_t > 0$. Consider the relaxed problem of the principal in which incentives are captured by (8) and (9), while the principal can still make interim transfers to the agent. The proof of Proposition 2 shows that the solution to (B.51) is concave under the appropriate parameters. The implementation of this contract is done described in the main text. □

Proposition 1 pins down the principal's value function $v(w)$ and optimal policies $f(w)$, $\phi(w)$, and $\psi(w) \equiv h/\Delta$. Consider the following implementation of this contract. Denote by $D = (D_t)_{t \geq 0}$ to be a counting process with arrival intensity

$$\mathrm{E}_t\left[dD_t\right] = \kappa \cdot \frac{p_t}{1-p_t} \cdot \mathbb{1}\left\{w_t = \bar{w}\right\} \qquad \text{(disclosure intensity)}. \tag{B.52}$$

where $p_t = \mathrm{P}_t(U_t = 0)$ and $\kappa > 0$.[10] Process D can be taken to represent the principal's disclosure

[8] E.g., if the principal discloses $\{U_t = 0\}$ with probability 1/2, then $\{U_t = 0\}$ is still possible even if not disclosed.

[9] A positive transfer is made at time t only conditional on $\{U_t = 0\}$ being disclosed by the principal.

[10] As is often the case in mechanism design, the optimal contract is not unique. Here, the payoff to the principal can be sustained for an arbitrary $\kappa > 0$.

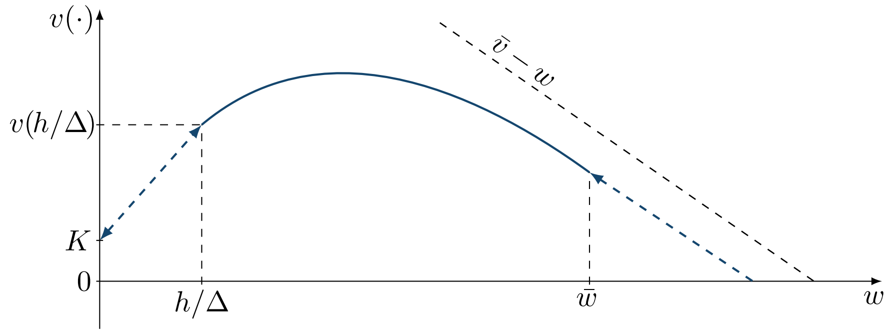


Figure 8: Principal's value function $v(\cdot)$ is the solid line if the agent is impatient.

decisions: the principal discloses that $U_t = 0$ at time t if $dD_t = 1$. The agent's belief process about $U_t = 0$ is then given by

$$dp_t = -\mu(1 - f_t)p_t\,dt + (1 - p_t) \cdot (dD_t - \mathrm{E}\,[dD_t]) = -(\mu(1 - f_t) + \kappa) \cdot p_t\,dt + (1 - p_t)\,dD_t.$$

The agent's belief p_t jumps to 1 if $dD_t = 1$, as it corresponds to the principal verifiably disclosing that $U_t = 0$. If $dD_t = 0$, however, the agent marginally revises his belief downwards, becoming more pessimistic that $U_t = 0$ and reducing his expected continuation value. Under such a disclosure process it is always the case that $p_t = \mathrm{P}_t(U_t = 0) > 0$ after every history, permitting the principal to compensate the agent only if no bad project received investment.

The agent's continuation value reflects at $\bar{w}$ similar to Biais, Mariotti, Rochet, and Villeneuve (2010) and DeMarzo and Sannikov (2006). In order to achieve this, the contract pays the agent a bonus b_t if $dD_t = 1$, or reduces his continuation value if $dD_t = 0$. Formally, the agent's continuation value follows

$$dw_t = \rho w_t\,dt - dC_t + h\,dt + \phi_t \cdot \big(\mu f_t\,dt - dM_t\big) + \psi_t \cdot \big(\lambda\,dt - dR_t\big) + b_t \cdot \left(dD_t - \frac{\kappa p_t}{1 - p_t}\,dt\right).$$

The total compensation C_t experiences a discrete jump $dC_t = b_t$ whenever good performance is disclosed and a bonus is paid out. In order for the agent's continuation value to reflect at $\bar{w}$, bonus b is chosen such that $dw_t = 0$ at $w_t = \bar{w}$ if the principal does not communicate good performance

$$b_t = \frac{\rho\bar{w} + h + \phi(\bar{w}) \cdot \mu \cdot f(\bar{w}) + \lambda \cdot h/\Delta}{\kappa \cdot p_t/(1 - p_t)} \qquad \text{(optimal bonus).} \tag{B.53}$$

The principal probabilistically discloses that $U_t = 0$ and rewards the agent with bonus b_t only if the agent's continuation value is sufficiently high, i.e., $w_t = \bar{w}$.

This construction hinges on the optimal monitoring intensity $f(w)$ and penalty $\phi(w)$ obtained as solutions to (B.51). By considering a relaxed problem in which the principal makes a state-contingent transfer to the agent, but the agent's belief does not respond to the changes in the probability that $U_t = 0$, we can obtain an upper bound on the principal's value under an optimal contract. This allows the optimal contract to preserve the backloaded incentive structure derived in Lemma 3 and implement value function $v(w)$, which solves (B.51), via a disclosure process $D = (D_t)_{t\geq 0}$ and a bonus process $b = (b_t)_{t\geq 0}$ defined in (B.52) and (B.53) respectively.

B.4 Two- and Three- Period Versions of the Model

Complete characterization of the optimal contract if the agent does not observe Y is beyond the scope of the main model. The technical difficulty lies in understanding the agent's optimally binding global incentive compatibility constraint after different histories and under the information structures implied by the principal's monitoring and communication policies. It is possible to construct examples of optimal contracts where the agent is initially indifferent between working and shirking, yet incentives are slack in all subsequent periods, meaning the agent *strictly* prefers to exert effort even if he deviates once, as long as this deviation occurs later in the contracting relationship.

In this simplified setting, the magnitude of the monitoring intensity determines whether the single- or double- deviation incentive constraints are optimally binding in the first period if the agent does not observe Y. If the agent were to observe Y, then the incentive constraint is, qualitatively, an "average" of a single- and a double-deviation constraint resulting in (13) leading to a tractable characterization. I also analyze a three-period version of this binary model numerically. The optimal compensation for a given monitoring rule is determined as a solution to a linear program with 128 global incentive constraints. The optimal monitoring intensity balances the benefits of investment efficiency and the expected compensation costs. Numerical examples confirm that the optimal monitoring is limited for low continuation values and is higher after good performance as predicted by the main model.

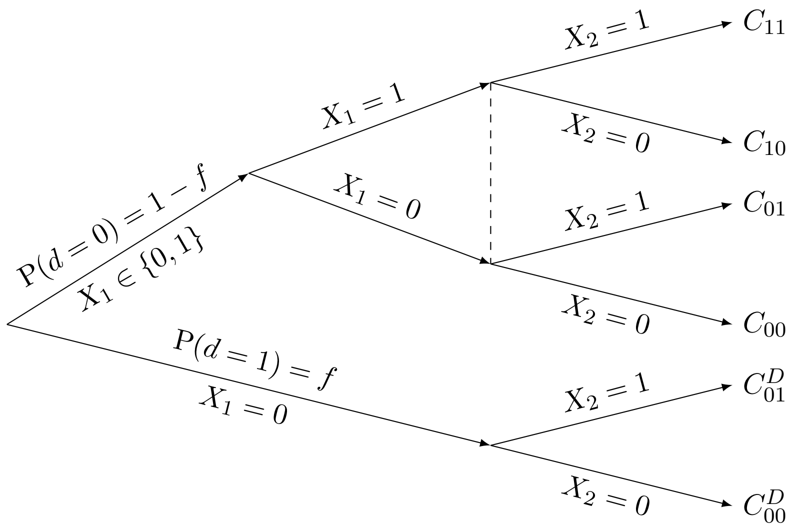
This section provides a simple two-period example for why information available to the agent matters for optimal transfers. As in the main section of the paper, principal and agent are risk neutral and equally patient. For simplicity I assume there is no discounting across periods. The agent exerts efforts a_1 and a_2. Suppose the principal monitors the agent with frequency f in the first period. In the second period performance is fully revealed.

Two Period Contract and Asymmetric Information Disclosure

Lemma B.17. *Suppose $f < 1$. There exists an optimal contract satisfying $C_{10} = C_{01} = C_{00} = C^D_{00} = 0$.*

Proof. Suppose contract $\mathcal{C}$ is incentive compatible. Define

$$\hat{C}^D_{01} = \frac{pC^D_{01} + (1-p)C^D_{00}}{p}.$$



Conditional on no disclosure of performance at $t = 1$ the probability is

$$\mathrm{P}(X_1 = 1 \,|\, N) = \frac{p}{p + (1-f)(1-p)}.$$

Then define

$$\hat{C}_{11} = \frac{\frac{p}{p+(1-f)(1-p)}(pC_{11} + (1-p)C_{10}) + \frac{(1-f)(1-p)}{p+(1-f)(1-p)}(pC_{01} + (1-p)C_{00})}{\frac{p^2}{p+(1-f)(1-p)}}.$$

Contract $\hat{\mathcal{C}}$ is still incentive compatible. □

Lemma B.18 states that if f is not very large the optimal contract provides strict incentives to the agent to work in period $t = 2$ along the path of effort.

Lemma B.18. *Suppose $f < \frac{\Delta}{p(p+1-\Delta)}$. In the uniquely optimal incentive compatible contract the agent is initially indifferent between working and shirking in both periods. Conditional on working in $t = 1$ the agent strictly prefers to work in $t = 2$.*

Proof. The contract thus constitutes of C_{11} and C_{01}^D. The contract is incentive compatible if and only if

$$\begin{cases} p^2 C_{11} + f(1-p)pC_{01}^D - 2h \geq (p-\Delta)pC_{11} + f(1-p+\Delta)pC_{01}^D - h, \\ p^2 C_{11} + f(1-p)pC_{01}^D - 2h \geq (p-\Delta)(p-\Delta)C_{11} + f(1-p+\Delta)pC_{01}^D - fh. \end{cases}$$

This can be simplified to

$$\begin{cases} C_{11} - fC_{01}^D \geq \dfrac{h}{\Delta p}, \\ (2p-\Delta)C_{11} - fpC_{01}^D \geq (2-f)\dfrac{h}{\Delta}. \end{cases}$$

From the above constraints it is efficient to set C_{01}^D to be the smallest possible such that the agent is still willing to exert effort. Thus $C_{01}^D = \frac{h}{\Delta}$. Substituting into the above equations we obtain

$$\begin{cases} C_{11} - f\dfrac{h}{\Delta} \geq \dfrac{h}{\Delta p} \\ (2p-\Delta)C_{11} - fp\dfrac{h}{\Delta} \geq (2-f)\dfrac{h}{\Delta} \end{cases}$$

which can be rewritten as

$$\begin{cases} C_{11} \geq \dfrac{h}{\Delta} \cdot \dfrac{1+fp}{p} & \text{(single deviation)}, \\ C_{11} \geq \dfrac{h}{\Delta} \cdot \dfrac{2-f+fp}{2p-\Delta} & \text{(double deviation)}. \end{cases}$$

The double deviation constraint is binding if and only if

$$\begin{aligned} \frac{1+fp}{p} &\leq \frac{2-f+fp}{2p-\Delta} \\ 2p - \Delta + 2fp^2 - \Delta fp &\leq 2p - fp + fp^2 \\ -\Delta + fp^2 - \Delta fp &\leq -fp \\ fp(p+1-\Delta) &\leq \Delta \\ f &\leq \frac{\Delta}{p(p+1-\Delta)} \end{aligned}$$

It is also worth noting that introducing any positive payments C_{01}, C_{10}, C_{00} lead to a strict ex-ante increase in the expected compensation required by the agent. □

Corollary B.1. *The optimal contract implements one of three feedback frequencies* $f \in \left\{0, \frac{\Delta}{p(p+1-\Delta)}, \bar{f}\right\}$.

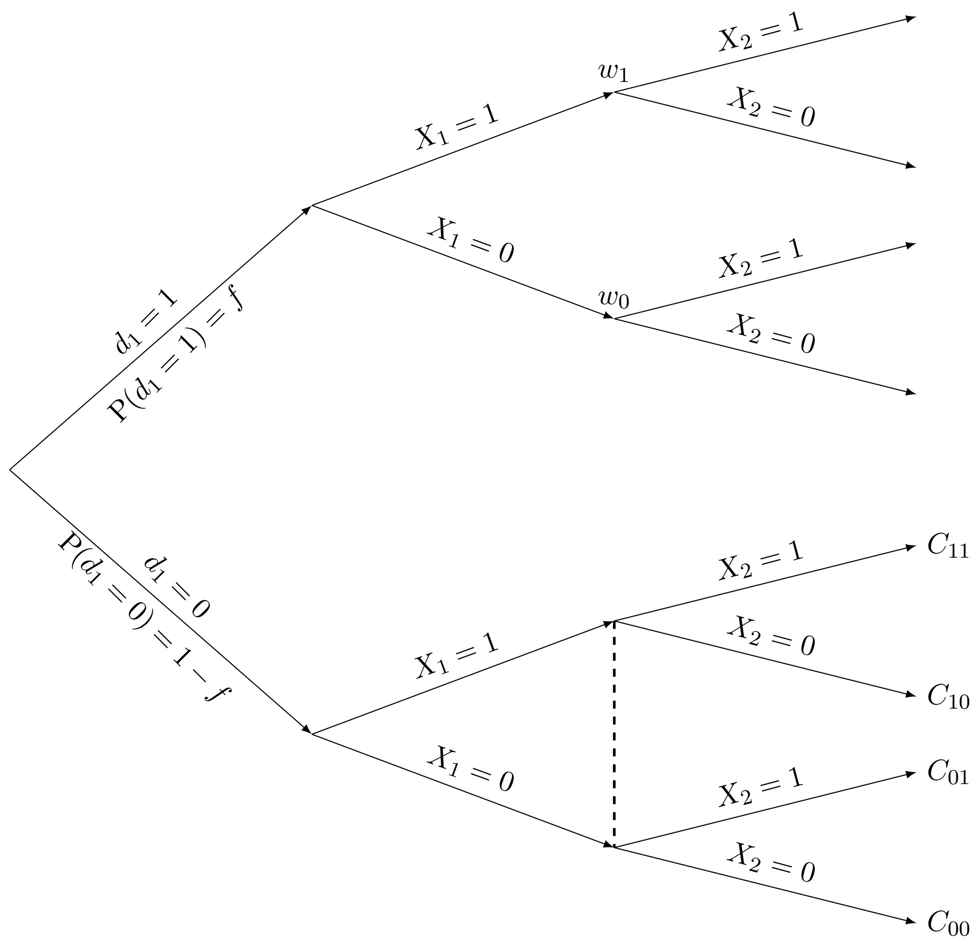
B.4.1. Two Period Contract with Symmetric Monitoring and Slack Incentives

Lemma B.19. *There exists an optimal contract such that* $C_{10} = C_{01} = C_{00}$. *Moreover under this contract the agent strictly prefers to work in the second period conditional on having worked in the first period and not having been monitored.*

Proof. Suppose contract $\mathcal{C} = \{w_0, w_1, C_{00}, C_{01}, C_{10}, C_{11}\}$ is incentive compatible. Define

$$\hat{C}_{11} = \frac{p^2 C_{11} + p(1-p)C_{10} + (1-p)pC_{01} + (1-p)^2 C_{00}}{p^2} \tag{B.54}$$

Consider the contract $\hat{\mathcal{C}} = \{w_0, w_1, 0, 0, 0, \hat{C}_{11}\}$. If the agent exerts effort the expected payoffs to the principal and the agent are the same under contracts $\mathcal{C}$ and $\hat{\mathcal{C}}$. We need to verify incentive compatibility of $\hat{\mathcal{C}}$ for the agent to exert effort.


$d_1 = 1$
$P(d_1 = 1) = f$
$d_1 = 0$
$P(d_1 = 0) = 1 - f$
$X_1 = 1$
$X_1 = 0$
$X_1 = 1$
$X_1 = 0$
w_1
w_0
$X_2 = 1$
$X_2 = 0$
$X_2 = 1$
$X_2 = 0$
$X_2 = 1$
$X_2 = 0$
$X_2 = 1$
$X_2 = 0$
C_{11}
C_{10}
C_{01}
C_{00}

- If the agent has worked in the first period it is incentive compatible for him to work in the second period under $\hat{C}$. Since $\mathcal{C}$ was incentive compatible it must be the case that

$$p\Delta(C_{11} - C_{10}) + (1-p)\Delta(C_{01} - C_{00}) \geq h.$$

For contract $\hat{C}$ the constraint is given by

$$\begin{aligned}
p\Delta\hat{C}_{11} &= p\Delta\frac{p^2C_{11} + p(1-p)C_{10} + (1-p)pC_{01} + (1-p)^2C_{00}}{p^2} \\
&= p\Delta C_{11} + (1-p)\Delta C_{01} + (1-p)\Delta C_{10} + \frac{(1-p)^2}{p}\Delta C_{00} \\
&\geq p\Delta(C_{11} - C_{10}) + (1-p)\Delta(C_{01} - C_{00}) \geq h.
\end{aligned}$$

- Exerting effort in both periods must be better than exerting effort only in the second period. The original contract satisfied

$$\begin{aligned}
&f(pw_1 + (1-p)w_0) + (1-f)(p^2C_{11} + p(1-p)C_{10} + (1-p)pC_{01} + (1-p)^2C_{00}) - (2-f)h \geq \\
&f((p-\Delta)w_1 + (1-p+\Delta)w_0) + (1-f)((p-\Delta)pC_{11} + (p-\Delta)(1-p)C_{10} \\
&+ (1-p+\Delta)pC_{01} + (1-p+\Delta)(1-p)C_{00}) - (1-f)h
\end{aligned}$$

To show that $\mathcal{C}$ is incentive compatible it is sufficient to show that

$$\begin{aligned}
\Delta p\hat{C}_{11} &\geq \Delta pC_{11} + \Delta(1-p)C_{10} - \Delta pC_{01} - \Delta(1-p)C_{00} \\
p\hat{C}_{11} &\geq pC_{11} + (1-p)C_{10} - pC_{01} - (1-p)C_{00}
\end{aligned}$$

Substituting (B.54) into the above expression we see that

$$\begin{aligned}
\frac{p^2C_{11} + p(1-p)C_{10} + (1-p)pC_{01} + (1-p)^2C_{00}}{p} &\geq pC_{11} + (1-p)C_{10} - pC_{01} - (1-p)C_{00} \\
\frac{(1-p)pC_{01} + (1-p)^2C_{00}}{p} &\geq 0 \geq -pC_{01} - (1-p)C_{00}
\end{aligned}$$

- Exerting effort in both periods must be better than shirking in both periods

$$\begin{aligned}
\Big(p^2 - (p-\Delta)^2\Big)\hat{C}_{11} \geq \Big(p^2 - (p-\Delta)^2\Big)C_{11} &+ \Big(p(1-p) - (p-\Delta)(1-p+\Delta)\Big)(C_{10} + C_{01}) \\
&+ \Big((1-p)^2 - (1-p+\Delta)^2\Big)C_{00}
\end{aligned}$$

Simplifying terms

$$\Delta(2p-\Delta)(\hat{C}_11 - C_{11}) \geq \Delta(1-2p+\Delta)(C_{10}+C_{01}) - \Delta(2-2p+\Delta)C_{00}$$
$$(2p-\Delta)(\hat{C}_11 - C_{11}) \geq (1-2p+\Delta)(C_{10}+C_{01}) - (2-2p+\Delta)C_{00}$$

Substituting (B.54) we get

$$(2p-\Delta)\left(\frac{1-p}{p}(C_{10}+C_{01}) + \frac{(1-p)^2}{p^2}C_{00}\right) \geq (1-2p+\Delta)(C_{10}+C_{01}) - (2-2p+\Delta)C_{00}$$

Arranging terms

$$(2p-\Delta)\frac{(1-p)^2}{p^2}C_{00} \geq \left((2p-\Delta)\frac{1-p}{p} - (1-2p+\Delta)\right)(C_{10}+C_{01}) - (2-2p+\Delta)C_{00}$$
$$(2p-\Delta)\frac{(1-p)^2}{p^2}C_{00} \geq 0 \geq -\frac{p-\Delta}{p}(C_{10}+C_{01}) - (2-2p+\Delta)C_{00} \tag{B.55}$$

Thus, contract $\hat{\mathcal{C}}$ is incentive compatible. □

Lemma B.20. *Suppose under the optimal contract the double-deviation constraint is binding, but the single-shot deviation at $t=1$ is not binding. Then it must be the case that $C_{01}=C_{10}=C_{00}$.*

Proof. Proof from the contrary. Suppose this is not the case. Then (B.55) implies that it is possible to transition to an alternative contract under which both incentive constraints in the first period are slack. By construction, transitioning to $\hat{C}$ preserves features slack incentive compatibility in the second period. This implies the original contract was not optimal. □

Lemma B.21. *Suppose that $f=0$. There exists a uniquely optimal contract implementing effort in periods $t=0$ and $t=1$. In the first period the agent is indifferent between working and shirking in both periods. In the second period, along the path of effort, the agent strictly prefers to work, rather than shirk.*

Proof. There exists an optimal contract in which the agent is only compensated after histories $\{(1,1),(1,0)\}$. Denote the corresponding compensation by $\{C_0, C_1\}$. In order for this contract to be incentive compatible we must have

$$\begin{cases} p^2C_1 + p(1-p)C_0 - 2h \geq (p-\Delta)pC_1 + (p-\Delta)(1-p)C_0 - h \\ p^2C_1 + p(1-p)C_0 - 2h \geq (p-\Delta)^2C_1 + (p-\Delta)(1-p+\Delta)C_0 \\ \quad pC_1 + (1-p)C_0 - h \geq (p-\Delta)C_1 + (1-p+\Delta)C_0 \end{cases}$$

We can simplify these constraints as

$$\begin{cases} C_1 \geq -\dfrac{1-p}{p} \cdot C_0 + \dfrac{h}{\Delta p} \\ C_1 \geq \dfrac{2p-1-\Delta}{2p-\Delta} \cdot C_0 + \dfrac{2h}{\Delta(2p-\Delta)} \\ C_1 \geq C_0 + \dfrac{h}{\Delta} \end{cases}$$

If $\{C_0, C_1\}$ is an optimal contract, then contract $\left\{0, C_1 + \frac{1-p}{p} C_0\right\}$ is incentive compatible and delivers the same expected compensation cost to the principal.

$$\begin{cases} p^2\hat{C}_1 - 2h \geq (p-\Delta)p\hat{C}_1 - h \\ p^2\hat{C}_1 - 2h \geq (p-\Delta)^2\hat{C}_1 \\ \quad p\hat{C}_1 - h \geq (p-\Delta)\hat{C}_1 \end{cases}$$

The optimal contract sets $\hat{C}_1 = \frac{2}{2p-\Delta} \cdot \frac{h}{\Delta}$. Increasing C_0 and reducing C_1 while keeping the compensation cost the same. We can see the same proof graphically in Figure 9.

The plot is as depicted above since the intercepts of the incentive constraints are ranked and the blue line has a greater slope than the red line.

$$\begin{cases} \dfrac{2h}{\Delta(2p-\Delta)} \geq \dfrac{h}{\Delta p} \geq \dfrac{h}{\Delta} \\ \dfrac{2p-\Delta-1}{2p-\Delta} \geq -\dfrac{1-p}{p}. \end{cases}$$

□

Lemma B.22. *For any monitoring frequency f there exists an optimal contract in which at $t=1$ the agent is indifferent between working and shirking in both periods. If $fp < \Delta$ then, under any optimal contract, in period $t=2$ the agent strictly prefers to work if he was not monitored in the first period. If $fp \geq \Delta$ then there exists an optimal contract in which the agent is always indifferent between working and shirking in subsequent periods.*

Proof. Denote by w_0, w_1 conditional on disclosure of performance in period 1. The single deviation incentive constraint requires

$$\begin{aligned} &f(pw_1 + (1-p)w_0) + (1-f)p^2C - (2-f)h \\ \geq &f((p-\Delta)w_1 + (1-p+\Delta)w_0) + (1-f)(p-\Delta)pC - (1-f)h \end{aligned}$$

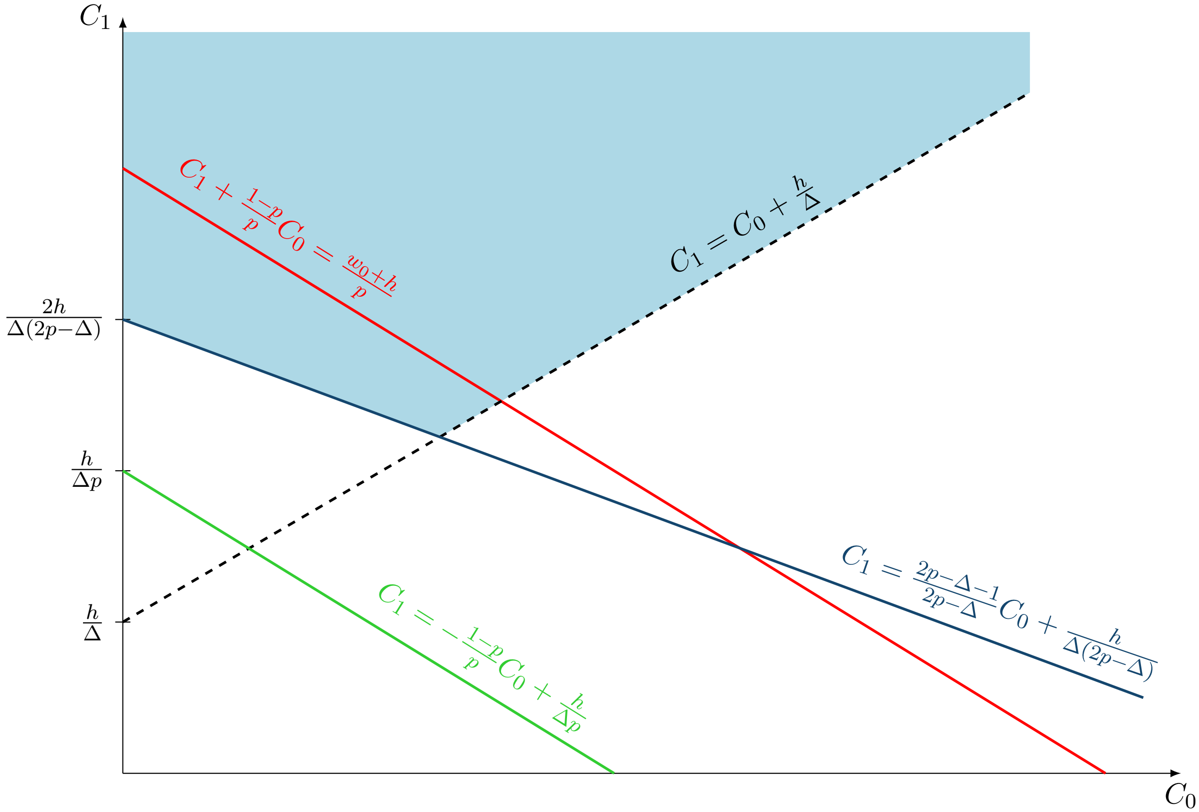


Figure 9: Graphical representation of incentive constraints and compensation costs. The optimal contract is a point on a red line for some w_0 which falls in the blue region. We can see that it has to be $C_0 = 0$ and $C_1 = \frac{2h}{\Delta(2p-\Delta)}$.

The double deviation constraint implies

$$\begin{aligned}
&f(pw_1 + (1-p)w_0) + (1-f)p^2C - (2-f)h \\
\geq &f((p-\Delta)w_1 + (1-p+\Delta)w_0) + (1-f)(p-\Delta)^2C
\end{aligned}$$

These constraints simplify to

$$f(w_1 - w_0) + (1-f)pC \geq \frac{h}{\Delta} \tag{B.56}$$

$$f(w_1 - w_0) + \underbrace{\frac{2p-\Delta}{p}}_{\geq 1} \cdot (1-f)pC \geq (2-f)\frac{h}{\Delta} \tag{B.57}$$

It is without loss to set $w_0 = p \cdot \frac{h}{\Delta} - h$. There exists an optimal contract in which $w_1 = w_0$ and

$$C = \frac{1}{1-f}\frac{1}{p}\frac{h}{\Delta} \cdot \max\left[1, \quad \frac{p(2-f)}{2p-\Delta}\right]$$

- Suppose $fp \geq \Delta$. Then there exists an optimal contract that sets $C = \frac{1}{1-f}\frac{1}{p}\frac{h}{\Delta}$. Expected compensation cost of this contract is

$$\begin{aligned}
V(f) &= f\left(p \cdot p\frac{h}{\Delta} + (1-p) \cdot p\frac{h}{\Delta}\right) + (1-f)\left(p^2\frac{1}{1-f}\frac{1}{p}\frac{h}{\Delta}\right) \\
&= (fp^2 + f(1-p)p + p)\frac{h}{\Delta} \\
&= (fp^2 + fp - fp^2 + p) \cdot \frac{h}{\Delta} = (1+f)p \cdot \frac{h}{\Delta}.
\end{aligned}$$

The marginal cost in compensating the agent for the principal is

$$V'(f) = p\frac{h}{\Delta}.$$

Consider an alternative contract in which the agent is also compensated with a payment C_0 conditional on success in period 1 and failure in period 0. The necessary incentive compatibility condition is

$$\begin{cases}
p\Big(p\hat{C}_1 + (1-p)\hat{C}_0\Big) = pC \\
p\Big(p\hat{C}_1 + (1-p)\hat{C}_0\Big) - h = p\Big((p-\Delta)\hat{C}_1 + (1-p+\Delta)\hat{C}_0\Big)
\end{cases}$$

In this case the agent is indifferent between working in each period or shirking for the remaining periods. We can rewrite these conditions as

$$\begin{cases} \hat{C}_0 + \dfrac{h}{\Delta} = \dfrac{1}{1-f}\dfrac{h}{p\Delta} \\ \hat{C}_1 - \hat{C}_0 = \dfrac{h}{p\Delta} \end{cases}$$

This implies

$$\begin{cases} \hat{C}_0 = \dfrac{h}{\Delta}\left(\dfrac{1}{p(1-f)} - 1\right) \\ \hat{C}_1 = \dfrac{h}{\Delta}\left(\dfrac{1}{p(1-f)} + \dfrac{1}{p} - 1\right) \end{cases}$$

- Suppose $fp < \Delta$. Then

$$C = \frac{1}{1-f}\frac{1}{p}\frac{h}{\Delta} \cdot \frac{p(2-f)}{2p-\Delta}.$$

Under this contract the agent is indifferent between working in both periods and shirking in both periods. However, conditional on having worked in $t = 1$ and not being monitored, the agent strictly prefers working since

$$C = \frac{1}{1-f}\frac{1}{p}\frac{h}{\Delta} \cdot \frac{p(2-f)}{2p-\Delta} > \frac{h}{p\Delta}.$$

In this region $w_1 = w_0$. By the same logic as in Lemma $B.21$ it implies that increasing C_0 is sub-optimal.

The expected compensation cost to the principal is equal to

$$\begin{aligned} V(f) &= fp\frac{h}{\Delta} + (1-f)p^2\frac{1}{1-f}\frac{1}{p}\frac{p(2-f)}{2p-\Delta}\frac{h}{\Delta} \\ &= fp\frac{h}{\Delta} + p^2\frac{2-f}{2p-\Delta}\frac{h}{\Delta} \\ &= p\frac{h}{\Delta}\left(f + \frac{(2-f)p}{2p-\Delta}\right) \\ &= p\frac{h}{\Delta}\frac{(2p-\Delta)f + (2-f)p}{2p-\Delta} \\ &= p\frac{h}{\Delta}\frac{(p-\Delta)f + 2p}{2p-\Delta} \end{aligned}$$

The marginal compensation cost for the principal is

$$V'(f) = p\frac{h}{\Delta}\frac{p-\Delta}{2p-\Delta} < p\frac{h}{\Delta}.$$

□

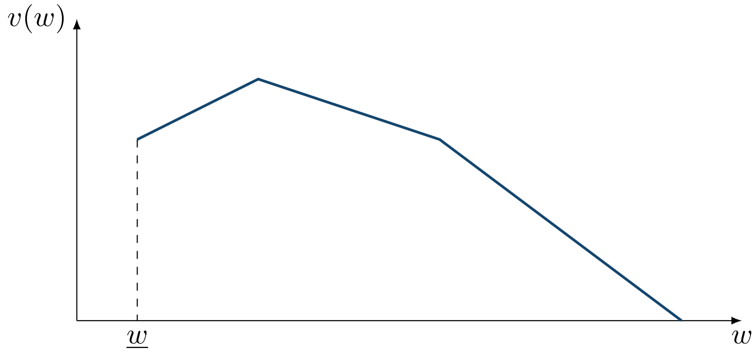
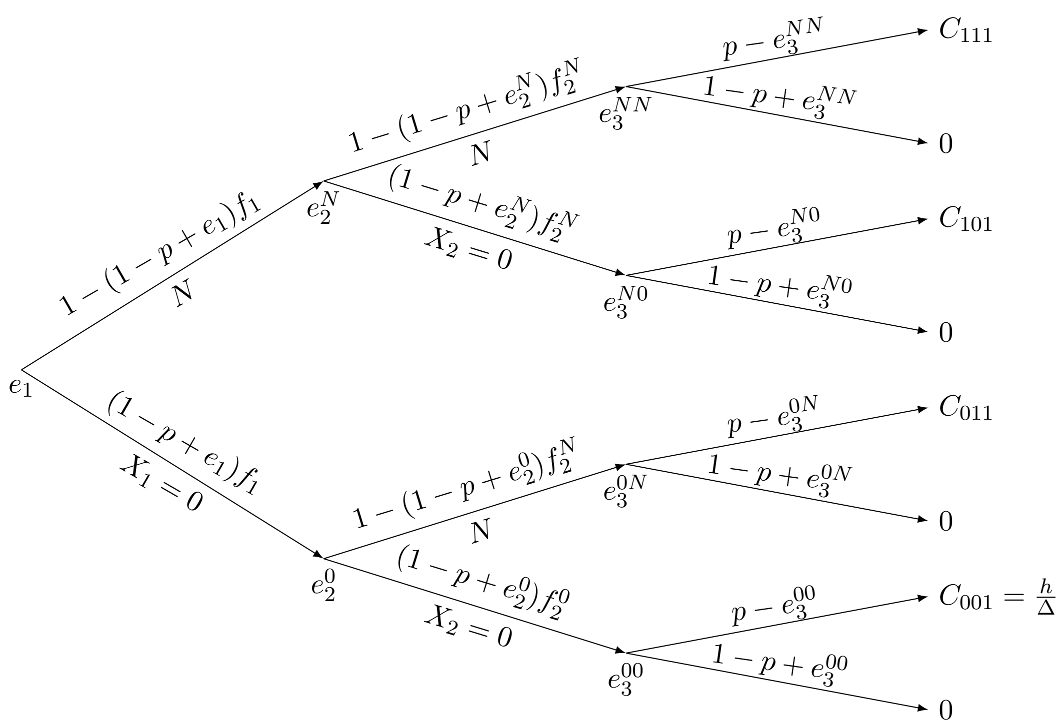


Figure 10: Two Period Value Function of the Principal for parameters in which $f^* = \frac{\Delta}{p}$. Note that $\underline{w} = \frac{2hp^2}{\Delta(2p-\Delta)} - 2h$.

Lemma B.23. *The optimal two-period contract either implements one of three levels of monitoring:* $f \in \{0,\ \Delta/p,\ \bar{f}\}$.

Proof. See above. □

B.4.2. Three Period Contract, Asymmetric Monitoring For notational convenience define $e = \Delta(1-a)$



Global incentive compatibility can be formally written down using 128 incentive constraints corresponding to 7 effort choices $\left\{e_1, e_2^N, e_2^0, e_3^{NN}, e_3^{N0}, e_3^{0N}, e_3^{00}\right\}$

$$
\begin{aligned}
& p^3 \cdot C_{111} + p^2(1-p) \cdot f_2^N C_{101} + (1-p)p^2 \cdot f_1 C_{011} + (1-p)^2 p \cdot f_1 f_2^0 C_{001} \\
\geq & (p-e_1)(p-e_2^N)(p-e_3^{NN}) \cdot C_{111} \\
& +(p-e_1)(1-p+e_2^N)(p-e_3^{N0}) \cdot f_2^N C_{101} \\
& +(1-p+e_1)(p-e_2^0)(p-e_3^{0N}) \cdot f_1 C_{011} \\
& +(1-p+e_1)(1-p+e_2^0)(p-e_3^{00}) \cdot f_1 f_2^0 C_{001} \\
& +\frac{h}{\Delta}e_1 + +(1-(1-p+e_1)f_1) \cdot \frac{h}{\Delta}e_2^N + (1-p+e_1)f_1 \cdot \frac{h}{\Delta}e_2^0 \\
& +(1-(1-p+e_1)f_1)(1-(1-p+e_2^N)f_2^N) \cdot \frac{h}{\Delta}e_3^{NN} \\
& +(1-(1-p+e_1)f_1)(1-p+e_2^N)f_2^N \cdot \frac{h}{\Delta}e_3^{N0} \\
& +(1-p+e_1)f_1(1-(1-p+e_2^0)f_2^0) \cdot \frac{h}{\Delta}e_3^{0N} \\
& +(1-p+e_1)f_1(1-p+e_2^0)f_2^0 \cdot \frac{h}{\Delta}e_3^{00}
\end{aligned}
$$

It should be possible to simply generate the matrix of constraints and plug into python. Then maximize over $\{f_1, f_2^N, f_2^0\}$. In the numerical examples

$$f_2^0 < f_1 < f_2^N.$$

Three Period Contract, Symmetric Monitoring The objective of the family should also include the continuation utility $f(w_0)$ and $f(w_1)$. In reality, we're maximizing over 4 elements and can then look at the optimization. Hence can solve this linear program like this.

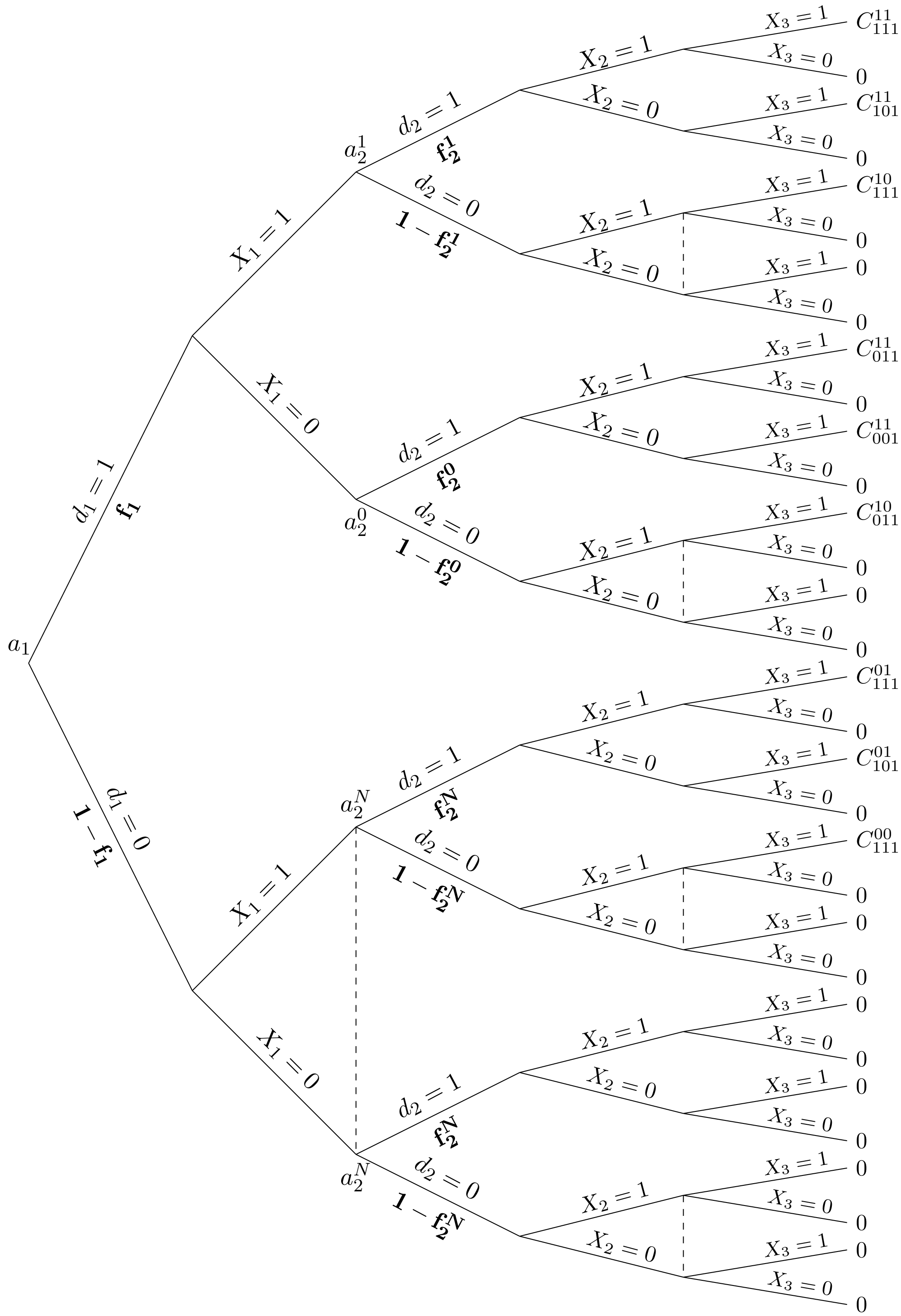


Figure 11: Three Period Contract Extensive Form

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