Design of Macro-prudential Stress Tests*

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Abstract

We study the design of macro-prudential stress tests and bank capital requirements. The stress test provides information about systemic risk in banks' portfolios and imposes contingent capital requirements that create a buffer against future losses. We find that the optimal stress test discloses information partially: when risk is low, capital requirements reflect full information and set mild restrictions on dividend issuance; when risk is high, the regulator pools information and requires all banks to hold precautionary liquidity. Weak banks determine the level of transparency and hold efficient capital, while strong banks end up overcapitalized. The optimal dynamic disclosure stress test alleviates this distortion by forcing weak banks raise capital first. Once they are less exposed to systemic risk, the regulator can reveal additional information leading to efficient recapitalization of stronger banks. In the presence of interbank trade, it may be optimal to prohibit strong banks from issuing dividends in order to maximize the capital cushion of the whole system and support the weak banks.

Keywords: stress tests, capital requirements, systemic risk, macro-prudential regulation, dynamic disclosure.

1 Introduction

Systemically important financial institutions (SIFIs) are at the core of the financial system and are integral to macroeconomic stability. Due to their size and interconnectedness, a fault with any one

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of these banks imposes large externalities on the rest of the economy and they may be considered "Too Big to Fail". Bank regulators try to identify sources of risks to the financial system in a forward-looking way and regulate financial institutions, including the SIFIs, to prevent potential crises. Stress tests are the main forward-looking regulatory tool to monitor SIFIs and maintain stability of the financial system.

We model a stress test as a combination of an adverse scenario and contingent capital regulation. The aim of the adverse scenario is to discover and disclose information about the level of systemic risk. The role of capital requirements is to require banks to respond to this new information in a socially optimal way. For example, the regulator might set limits on the banks dividend payouts and may force the banks that fail the test to raise (liquid) capital in a precautionary way to strengthen the financial system and avoid the need for future bailouts. We provide a normative analysis of how stress tests should be conducted (i.e. what should constitute passing and failing), under what conditions their results should be disclosed, and how much capital the banks should be required to raise upon learning the stress test outcomes. The optimal supervision solves the trade-off between allocative efficiency of risky assets within the system and the ability of financial institutions to raise sufficient capital to remain safe.

In our baseline model the banks own correlated risky assets financed by legacy debt. This exposes them to default risk. In order to comply with capital requirements banks sell some of the risky assets to the market in a precautionary way. To identify how much capital needs to be raised, the regulator runs an adverse scenario and sets contingent capital requirements based on its results. This is the link between information uncovered in a stress test and contingent capital regulation: from an informativeness perspective, a very soft scenario passed by all banks, and a very adverse scenario which fails all banks, are equivalent. The real difference is that in the first case banks are allowed to pay dividends, while in the second they are prohibited from issuing dividends and may be forced to raise additional capital.

The market price of the assets depends on their underlying risk. The regulator requires banks to sell these assets exactly in states of the world where they carry too much risk for banks alone to bear. Such information disclosure hurts the banks' ability to raise capital. In the extreme, when the markets know the assets are very risky, prices are too low for the banks to recapitalize safely. It is important that the stress test uncovers the level of systemic risk and, thus, applies to all of the banks simultaneously. The regulator cannot reveal the very worst levels of systemic risk as many of the large financial institutions would be at risk of default simultaneously. This leads to a fully transparent stress test being suboptimal. The regulator takes this into account when determining how much information to disclose to the market, i.e., what adverse scenarios to run, and the corresponding capital requirements to be set. The optimal macro-prudential stress test solves the trade-off between allocative efficiency of risky assets and the ability of financial institutions to remain solvent if the risks materialize in the future.

Our first result is that optimal information disclosure is partial and asymmetric. For low to medium levels of risk the regulator is transparent. For high levels of risk, disclosure takes the form of a pooling message. Capital requirements are contingent on the information disclosed. If systemic risk is low, banks either pay out dividends or raise capital depending on whether the amount of risk is below or above its expectation. If the risk is high, banks ability to raise capital is compromised and they are required to sell a significant fraction of their assets in order to create a liquidity buffer. Pooling information for high levels of systemic risk is important to ensure that the banks' assets are sufficiently liquid. Under the optimal test the banks are penalized for holding large quantities of risk and, as we show, end up choosing an efficient quantity of the asset ex-ante.

Our second result shows that dynamic information disclosure and capital requirement policy dominates a static one in the presence of heterogeneous banks. The static test is inefficiently opaque if there is a weak bank in the system since it must maintain asset prices high enough for this bank to recapitalize. This limits the allocative efficiency for stronger banks that end up holding excess liquidity. The optimal dynamic stress test, which is characterized by the regulator disclosing stress test results and adjusting capital requirements over time, is welfare improving. If weak banks raise capital first, they become more resilient to incremental disclosures about the level of systemic risk. The regulator can then reveal additional information and implement efficient capital requirements for strong banks. Improvement in allocative efficiency under dynamic disclosure comes from the added ability to further refine the coarse information structure needed to keep the weak banks safe after they are recapitalized and strong banks are yet to raise capital.

We further develop the model to allow for interbank trade of risky assets. We show that a higher ex-ante level of the strong bank's capital allows the strong bank to take over more of the risky assets of the weak bank thus increasing overall welfare. In the extreme case it corresponds to the sale of the weak bank to the strong bank once a large level of systemic risk is disclosed. The regulator subsequently imposes efficient capital requirements on the merged bank. The strong bank is strictly better off in this scenario. Dynamic information disclosure and capital requirements are crucial for the regulator to have the necessary flexibility to implement such allocations. If the strong bank has private incentives to pay out dividends, the regulator may restrict its dividend policy to focus it on acquiring the assets of the weak bank.

Most of our analysis is focused on the case in which banks hold correlated cash flows and the regulator discloses information about the potential downside of these assets. In this setting we are able to characterize the optimal dynamic stress test. In Section 5 we micro-found the relation between portfolio correlation and downside risk by considering a stylized intermediary asset-pricing model in which higher portfolio correlation endogenously causes greater downside risk of the asset. We show that our main results are unchanged and that dynamic stress tests improve welfare relative to the static stress tests because they allow for greater risk-sharing between banks.

1.1 Related Literature

The main results of our paper speak to optimal information disclosure of stress test results to the market. Goldstein and Sapra (2014) and Leitner (2014) present overviews of the costs and benefits of such disclosures. Our focus is on the optimal disclosure of information related to systemic risk. When the financial system is relatively safe, we trust the markets to act as a representative agent who benefits from additional information. As the regulator sets capital requirements, banks cannot abuse information in our model. In this way we differ from Alvarez and Barlevy (2015) who argue that information should not be disclosed in good times as it does not lead to improvements in allocative efficiency.

Faria-e Castro, Martinez, and Philippon (2015) study a related model of disclosure in which there is uncertainty about the fraction of good and bad banks in the system. The regulator faces a trade-off in conducting individual bank tests as it exposes her to aggregate uncertainty and, consequently, bailouts. The paper only considers information structures which reveal the aggregate state, but allows for differing precisions of signals about individual banks. The main contribution of Faria-e Castro, Martinez, and Philippon (2015) is the rich analysis of possible fiscal interventions by the regulator as a function of the outcome of the test. We complement it by analyzing unrestricted information policies that can be used by the regulator, while allowing her to commit to contingent capital requirements imposed on the banks as to keep the system default-free. We limit the channels through which banks issue capital to asset sales, but the argument follows through if we let banks issue new equity at some cost. For an in depth analysis of optimal recapitalization see Philippon and Schnabl (2013).

Goldstein and Leitner (2015) study an information design problem by the regulator who wishes to facilitate banks of heterogeneous quality to raise funds in markets plagued by adverse selection. Under some regularity assumptions the optimal test reveals a fraction of bad quality banks who cannot raise funds, while allowing the remaining banks to raise capital at a pooling price. Like in our model, banks raise funds by selling their tradable asset. The information design in question concerns idiosyncratic asset quality which is quite different from disclosure of aggregate uncertainty as the regulator is exposed to aggregate risk in the latter case. Williams (2017) extends the analysis of Goldstein and Leitner (2015) to an endogenous portfolio choice of the banks showing, surprisingly, that banks hold less liquidity under the optimal stress test. In our model capital requirements are designed jointly with stress tests such that in equilibrium no bank defaults. In our opinion, this makes our paper more relevant to regulation of SIFIs, while Goldstein and Leitner (2015) and Williams (2017) are better suited at explaining the optimal tests conducted by FDIC on smaller banks which can be unwound or sold without creating market panic and contagion.[1]

Gick and Pausch (2013) and Shapiro and Skeie (2015) consider settings where the regulator balances incentives of investors to withdraw funds from the banking system and risk taking by the banks. The first paper focuses on information design, while the latter on reputation formed by the regulator during two consecutive bank interventions. Both papers result in partially informative scenarios which limit the volatility arising from their actions. The mechanism in our paper differs since the optimal stress test trades off efficient allocation of risk in economy with the costs of recapitalizing weak banks.

To demonstrate our mechanism we write down a stylized asset pricing model in which aggregate risk faced by the banks matters for asset pricing in both periods. The style of the model is similar to Geanakoplos (2010) and Harrison and Kreps (1978) in which the fluctuating wealth of the marginal investor (bank) creates volatility in asset markets. While we do not model asset choice directly, the inefficiencies arise from illiquidity of the bank's long-term assets when their balance sheets are hit by an aggregate shock. The latter arises from a maturity mismatch between bank's assets and liabilities, a friction studied in Allen and Gale (2000) and Diamond and Dybvig (1983).

Our model features incomplete asset markets. This serves two purposes. First, it does not allow SIFIs to buy insurance against systemic risk. This friction was pointed out in Krishnamurthy (2003) and Allen and Gale (2004) as one driving many of the intermediary asset pricing models.[2] Second,

[1] For failed banks entering into FDIC receivership see: https://www.fdic.gov/bank/individual/failed/banklist.html.

[2] For an excellent overview of intermediary asset pricing literature see Benoit, Colliard, Hurlin, and Pérignon

market incompleteness prevents markets from learning the amount of aggregate risk through prices. This puts the regulator in an advantageous position of being able to aggregate individual bank signals. Marin and Rahi (2000) provide an analysis of the costs and benefits of information structures generated by incomplete markets. Flannery, Hirtle, and Kovner (2017), Petrella and Resti (2013), and Peristiani, Morgan, and Savino (2010) confirm empirically that stress test disclosures in the U.S. in 2009 and Europe in 2011 contained new information manifested through abnormal returns and trading volume.

There are several empirical measures of systemic risk. Acharya, Pedersen, Philippon, and Richardson (2017b) and Adrian and Brunnermeier (2016) propose market based measures of systemic risk termed Systemic Expected Shortfall (SES) and Conditional Value at Risk (CoVaR) respectively. We show that there is an intuitive mapping between these measures and what we refer to as systemic risk on our model. As the regulator has superior access to bank balance sheet information, she is in a better position to construct forward looking estimates of these risk measures. An interesting distinction is that while papers above use SES and CoVaR to tax institutions based on their contributions to systemic risk (and thus align individual bank's incentives with the incentives of the regulator), we use these measures to set capital requirements which can be viewed as both an incentive tool and a stability tool. See Brunnermeier, Gorton, and Krishnamurthy (2012) and Duffie (2013) for suggestions over what relevant information the regulator should collect from the banks in order to identify sources of systemic risk.

We impose a requirement on policies that banks do not default even after a systemic shock. It is based on an implicit assumption that the shadow cost of government funds is significantly large[3] and that the banks are too big and interconnected to fail. The latter is supported by arguments of financial contagion Allen and Gale (2000), historical examples Wiggins and Metrick (2015), as well as current proposals on managing failing institutions summarized in "Too Big to Fail: The Path to a Solution" (2013) highlighting the need to keep SIFIs operating even under distress. In our model banks raise capital in a precautionary way by selling illiquid assets in order to be solvent in case a systemic shock arrives.

Finally, our paper is related to a broad and growing literature on information design pioneered by Kamenica and Gentzkow (2011). In our setting the stress test is a signal design problem conducted

(2016).

[3]Bailouts both require large sums of funds and they tend to be politically unpopular. We acknowledge that profits for the taxpayers can be made to be the lender of last resort, but the latter is not part of an explicit mandate of the Federal Reserve Banks or other government agencies.

by the regulator in a general equilibrium economy. We provide conditions under which the optimal information structures are monotone partitions of the unknown state. Our analysis is related to Dworczak and Martini (2017) who provide necessary and sufficient conditions for a monotone signal partition in a related class of models.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 contains our main results where we derive the optimal static and dynamic stress tests. Section 4 analyzes the role of bank's ex-ante portfolio choice and the possibility for government bailouts. In Section 5 we microfound the systemic downside risk by considering a stylized model of intermediary asset pricing with portfolio correlation uncertainty. Section 6 concludes.

2 Model

We start with a brief summary of the model. There are three periods $t = 0, 1, 2$. The financial system consists of several systemically important banks who receive an uncertain correlated cash flow at $t = 2$.[4] The magnitude of this shock is unknown to the banks and is a source of systemic risk as it may cause them to default. The regulator can uncover the potential for losses $\tilde{l}$ by performing a stress test. She designs it at $t = 0$, prior to knowing the state of the economy, and banks are required to raise capital in a precautionary manner at $t = 1$, ensuring the solvency of the financial system at $t = 2$. The timing of the game is summarized in Figure 1.

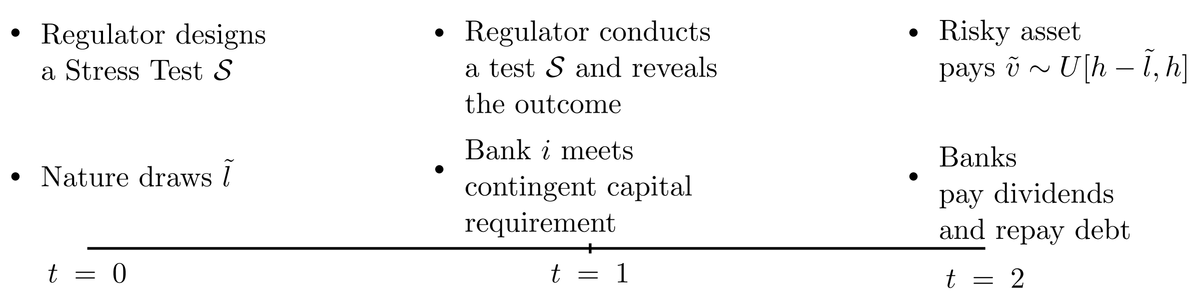

Figure 1: Timing of the Model

Banks. They are I risk-neutral banks. Bank i's portfolio at $t = 0$ consists of $c_i > 0$ units of cash and $n_i > 0$ units of divisible risky asset which pays out $\tilde{v}_i$ at $t = 2$. Bank i also has outstanding

[4] We do not explicitly model the reason why the banks are systemically important, and take as given that the regulator's priority is on keeping them safe.

debt d_i that needs to be repaid in full in period $t = 2$.[5] Without loss we order the banks from weakest to strongest according to their relative funding gap, i.e., $\frac{d_i - c_i}{n_i}$ is increasing in i. The bank is operated in the interests of its shareholders who collect the cash remaining at the end of $t = 2$ subject to limited liability.

Aggregate risk held by the banks manifests itself through the downside potential of their risky assets $\{\tilde{v}_i\}_{i=1}^{I}$. We model this by assuming that $\tilde{v}_i$ is uniformly distributed on $[h - \tilde{l}, h]$ where $\tilde{l}$ reflects the unknown aggregate risk in the economy: if $\tilde{l}$ it large, then banks may suffer large losses at $t = 2$. Random variable $\tilde{l}$ is realized at $t = 0$ prior to the cash flows and can be uncovered by the regulator during a stress test. It is ex-ante distributed according to a density function $f(\cdot)$ on $[\underline{l}, \bar{l}]$.[6]

Markets. Banks raise liquid capital by selling assets to a competitive market of risk-neutral investors.[7] To capture the inefficiency of reallocating risky assets to investors, we assume that if a unit of asset i is held by the market at $t = 2$, it pays $\delta \cdot \tilde{v}_i$. The resulting market price of the risky asset at $t = 1$ is given by the expected value of $\delta \cdot \tilde{v}_i$ conditional on all available information at the time of the asset sale:

$$p = \delta \cdot \mathrm{E}\left[\tilde{v}_i\right].$$

Absent default considerations, SIFIs are efficient holders of the risky assets. First, they may be better at monitoring these assets and managing the associated idiosyncratic risks. Second, banks serve as broker-dealers and there may be intermediation benefits of holding a larger inventory, e.g. to alleviate search frictions. Finally, capital markets may have their own investment opportunities, and every unit of capital committed to holding risky assets ends up being taken away from other projects, generating a welfare loss for investors.

Stress Test. The regulator designs a stress test to maximize allocative efficiency of risky assets in the economy subject to these systemically important banks being safe. She utilizes her unique

[5] One can think of a bank trying to rollover its debt from period $t = 2$ onwards, however it may be subject to a run by its creditors.

[6] Our findings do not rely on $\tilde{v}_i$ following a uniform distribution. It is sufficient that $\tilde{v}_i = \eta_i - \tilde{l} \cdot \tilde{\xi}_i$ where $\tilde{\xi}_i$ has positive support and expected value. We also do not require much structure on the joint distribution of $\{\tilde{v}_i\}$ beyond them being identically distributed and that $(h - \tilde{l}, h - \tilde{l}, \ldots, h - \tilde{l}) \in \text{support}\left[Law(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_I \mid \tilde{l})\right]$.

[7] We assume that once the asset is sold to the outside markets, liquidity frictions make buying it back from the markets too expensive. In this way, we focus on asset sales from the banks to the markets, rather than asset sales and subsequent buybacks which are, in practice, very costly due to a market for lemons through which the bank would have to be exposed twice.

position in collecting information about $\tilde{l}$ from the cross-section of bank portfolios and, then, credibly communicating it to the capital markets.[8] The regulator then uses this information to update the banks capital requirements to reflect the newly discovered amount of risk in the system.[9] A stress test is, then, a combination of how to disclose information about $\tilde{l}$ to the markets and what capital requirements to impose on the banks as a result. The regulator designs and commits to a stress test at $t = 0$, prior to observing any additional signals about $\tilde{l}$.

Definition. *A static stress test $\mathcal{S} = (\tilde{s}, \{m_i(s)\}_{i=1}^{I})$ is a pair of an adverse scenario $\tilde{s}$ and a collection of capital requirements $m_i(\cdot)$ for each bank $i = 1, \ldots, I$ where*

1. *$\tilde{s}$ is a random variable correlated with $\tilde{l}$ and,*
2. *$m_i(s)$ is the minimum quantity of cash bank i must hold given the outcome of the adverse scenario $\tilde{s} = s$.*

The stress test above involves a single adverse scenario $\tilde{s}$ on the outcome of which contingent capital requirements $m_i(\tilde{s})$ are set. We make the capital requirements contingent on the stress test outcome $\tilde{s} = s$ rather than on the underlying level of risk $\tilde{l} = l$ to ensure that imposed capital requirements do not convey any information to the market above and beyond the stress test outcome. While the regulator could, in principle, simply announce the new capital requirements via a direct mechanism $\{m_i(\tilde{s})\}_{i=1}^{I}$, disclosing the stress test outcome $\tilde{s} = s$ allows the regulator to affect asset prices without changing the capital requirements. In Section 3.2 we introduce the notion of a dynamic stress test in which the regulator can run multiple adverse scenarios and recapitalize banks sequentially.

The realized utility of bank i's shareholders given stress test $\mathcal{S}$ is

$$\tilde{W}_i = \left[m_i(\tilde{s}) + \left(n_i - \frac{m_i(\tilde{s}) - c_i}{\delta \mathrm{E}\left[\tilde{v} \mid \tilde{s}\right]} \right) \cdot \tilde{v} - d_i \right]^+$$

The regulator maximizes the expected allocative efficiency in the economy subject to the banks being solvent at $t = 2$ with certainty. Throughout the paper we maintain the assumption that

[8] In Section 5 we micro-found the ability of the regulator to infer systemic risk from the cross-section of bank portfolios, which, in spirit, follows the proposals of Brunnermeier, Gorton, and Krishnamurthy (2012) in collecting information from the banks. See Goldstein and Sapra (2014) for a discussion of the certification ability of the regulator when conducting stress tests.

[9] In the Stress Testing framework imposed by the Dodd-Frank Act the regulator has authority to set capital requirements even if it violates banks' incentive compatibility. This is an important departure from Faria-e Castro, Martinez, and Philippon (2015) and Goldstein and Leitner (2015) as it alleviates the lemons problem when banks have discretion over which assets to sell.

banks are ex-ante able to raise enough capital to be safe.[10]

Assumption 1. *All banks can raise enough capital if the regulator does not disclose additional information*

$$c_i + n_i \cdot \delta \left(h - \frac{1}{2} \mathrm{E}\left[\tilde{l}\right] \right) \geq d_i \qquad \forall i = 1, 2, \ldots, I.$$

The regulator commits to a stress test ex-ante. This means that at $t = 0$, prior to observing any signals about $\tilde{l}$, she chooses the informativeness of the adverse scenario, i.e. $\tilde{s}$, and contingent capital requirements as a function of $\tilde{s}$.

Definition. *A stress test $\mathcal{S}^* = \left(\tilde{s}^*, \{m_i^*(\cdot)\}_{i=1}^I\right)$ is optimal if it maximizes expected welfare of the banks*

$$\mathcal{S}^* \in \underset{\mathcal{S} \ s.t. \ \{DF_i\}_{i=1}^I}{\arg\max} \ \mathrm{E}\left[\sum_{i=1}^I \left(m_i^*(\tilde{s}) + \left(n_i - \frac{m_i^*(\tilde{s}) - c_i}{\delta \mathrm{E}\left[\tilde{v} \mid \tilde{s}\right]} \right) \cdot \tilde{v}_i - d_i \right) \right]$$

subject to no bank defaulting at $t = 2$:

$$m_i^*(\tilde{s}) + \left(n_i - \frac{m_i^*(\tilde{s}) - c_i}{\delta \mathrm{E}\left[\tilde{v} \mid \tilde{s}\right]} \right) \cdot \tilde{v}_i - d_i \geq 0, \quad \forall i \qquad P - a.s. \tag{DF}$$

The expected welfare of the banks coincides with social welfare since creditors are repaid with certainty and the capital markets always break even. The default-free assumption (DF) deserves a separate mention. As evidenced by the 2007-2009 financial crisis, default costs of a systemically important financial institution are incredibly high and the Too Big to Fail problem has been excessively studied by academics and policymakers.[11] We are explicitly agnostic about the social costs of a default of a large broker-dealer and implicitly assume that the regulator would be forced to bail them out. If the social costs of the bailout are sufficiently high relative to the welfare loss of mis-allocating risk assets, $1 - \delta$, the default-free condition (DF) is natural.[12]

3 Optimal Stress Test

We begin by characterizing the optimal static stress test for a single bank. The optimal adverse scenario reveals the exact level of downside risk only when it is sufficiently small. On the other

[10] In Section 4.1 we consider a stylized model of portfolio choice and show that a default-free stress test always exists since the banks dislike having to sell risky assets to meet tough capital requirements.

[11] See "Too Big to Fail: The Path to a Solution for details" (2013) by the Bipartisan Policy Center and Duffie (2010). For an analysis of the Lehman Bankruptcy see Wiggins and Metrick (2015). Our analysis is qualitatively unchanged if we require each bank to be solvent at any fixed confidence level.

[12] In Section 4.2 we explicitly introduce government interventions and bailouts. Our results are qualitatively unchanged and if the social cost of government intervention is sufficiently high the optimal test is default free.

hand, it imposes partial transparency when the risk is high as to not make the bank assets illiquid prior to raising funds. Such partial transparency is accompanied by severe capital requirements. We proceed to characterize the optimal stress test for multiple banks. We highlight the value of using a dynamic stress test when banks are heterogeneous.

3.1 Optimal Stress Test for a Single Bank

Suppose the banking system consists of just one bank ($I = 1$) which holds c units of cash, n units of risky asset $\tilde{v}$, and debt with face value d. First, we characterize optimal default-free capital requirements if the regulator were to reveal $\tilde{l}$. We also show that if $\tilde{l}$ turns out to be very low, it can significantly depress the market value of the bank's risky assets making it impossible for it to recapitalize safely. We then characterize the optimal adverse scenario and the resulting capital requirements.

If $\tilde{l} = l$ is revealed to the market at $t = 1$, the market price of the risky asset becomes

$$p_1(l) = \delta \cdot \mathrm{E}\left[\tilde{v} \,\middle|\, \tilde{l} = l\right] = \delta \cdot \frac{1}{l} \int_{h-l}^{h} v\, dv = \delta \left(h - \frac{l}{2}\right).$$

Given capital requirement $m(l)$, the bank raises $m(l) - c$ of cash by selling $\frac{m(l)-c}{p_1(l)}$ units of the risky asset at $t = 1$. This leaves the bank with $q(l) = n - \frac{m(l)-c}{p_1(l)}$ units of the risky asset entering the second period. In order for the bank to be solvent with probability 1 under a fully informative test the bank's solvency constraint (DF) must be satisfied for $\tilde{v} = h - l$:

$$m(l) + \left(n - \frac{m(l) - c}{p_1(l)}\right) \cdot (h - l) \geq d. \tag{1}$$

Denote by l^f the worst case realization l such that the bank is safe even if it retains all of its risky assets:

$$c + n \cdot (h - l^f) = d \qquad \Rightarrow \qquad l^f = h - \frac{d - c}{n}.$$

If $\tilde{l} < l^f$ there is little risk in the system and the regulator can simply set $m^*(l) = c$. Similarly, define l^w to be the worst case l such that the bank can only satisfy (DF) by selling all of its assets at their expected value at $t = 1$:

$$c + n \cdot p_1(l^w) = d \qquad \Rightarrow \qquad l^w = 2h - \frac{2}{\delta}\frac{d - c}{n}.$$

If $\tilde{l} \in (l^f, l^w]$ the regulator can require the bank to raise capital by selling a fraction of its assets to satisfy (DF). In this region (1) must be binding under the optimal capital requirement implying that

$$m(l) = c + \frac{d - c - n \cdot (h - l)}{p_1(l) - (h - l)} \cdot p_1(l). \tag{2}$$

Finally, if $\tilde{l} > l^w$, even if the bank sells all of its assets to the market at $t = 1$, their expected value does not cover the amount the bank needs to raise to pay down its debt. Capital requirements reach the limit of their effectiveness and, thus, the regulator cannot reveal $\tilde{l} > l^w$. Even more so, it can never be the case that $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}\right] \geq l^w$ since it would, also, imply defaults with positive probability. The expected welfare to the bank if $\tilde{l} < l^w$ is revealed and the regulator sets binding capital requirements (2) can be written as

$$W(l) = c + n \cdot \frac{l + h}{2} - d - \frac{1 - \delta}{\delta} \cdot (m(l) - c). \tag{3}$$

Function $W(l)$ is depicted in Figure 3 for $l \leq l^w$. The following proposition shows that, in order to keep the system safe for $\tilde{l} \in (l^w, \bar{l}]$, the optimal adverse scenario discloses $\tilde{l}$ only when it is below a certain threshold. When the possible losses exceed this threshold, however, the regulator sends a single pooling message and requires that banks raise $d - c$ units of cash in the markets to satisfy future debt claims.

Proposition 1 (Static Stress Test). *Suppose* $\frac{W(l)}{l^w - \underline{l}} \geq \frac{W(l^p)}{l^w - l^p}$, *where* l^p *solves* $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \geq l^p\right] = l^w$. *The optimal static stress test* $\mathcal{S}^* = (\tilde{s}^*, m^*(\cdot))$ *is characterized by an adverse scenario*

$$\tilde{s}^* = \begin{cases} \tilde{l} & \text{if} \quad \tilde{l} < l^p, \\ l^w & \text{if} \quad \tilde{l} \geq l^p, \end{cases} \tag{4}$$

and contingent capital requirement

$$m^*(s) = c + \left[\frac{d - c - n \cdot h + n \cdot s}{p_1(s) - h + s}\right]^+ \cdot p_1(s). \tag{5}$$

The optimal stress test reveals $\tilde{l}$ perfectly if $\tilde{l} < l^p$ and sets the capital requirement such that the bank has just enough liquidity to survive the worst case scenario $\tilde{v} = h - \tilde{l}$. If, however, $\tilde{l} \geq l^p$, the optimal test does not reveal the specific $\tilde{l}$ and requires the banks to sell all of their assets to the market. This eliminates all bank exposure to $\tilde{v}$ and results in just enough liquidity to pay down d

even when $\tilde{v} = h - \bar{l}$. The bank raises more capital than it could if $\tilde{l} \in (l^w, \bar{l}]$ were revealed, but more capital than is ex-post efficient if $\tilde{l} \in [l^p, l^w)$ were revealed.

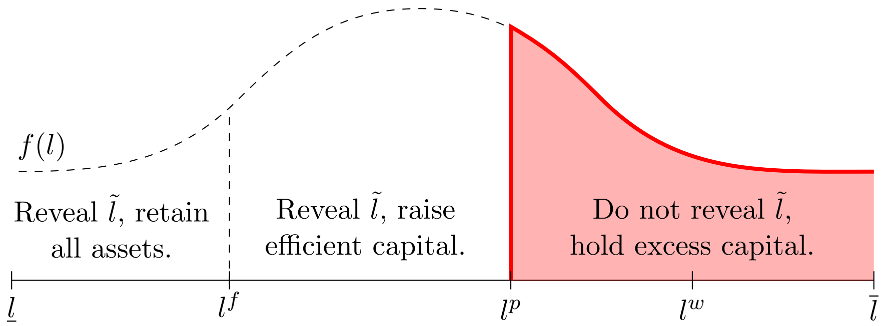

Figure 2: For $\tilde{l} \geq l^p$ the bank holds excess liquidity to safeguard against the worst case scenario of $\tilde{l} = \bar{l}$. In the region $\tilde{l} \in (l^p, l^w)$ the bank could raise less capital, but such information would undermine the bank if $\tilde{l} \in (l^w, \bar{l})$.

The optimal adverse scenario $\tilde{s}^*$ is opaque for $\tilde{l} \geq l^p$ in order to keep states $\tilde{l} \in (l^w, \bar{l}]$ safe. If $\tilde{s}^*$ were to reveal all $\tilde{l} \in [l^p, l^w)$, then investors could detect when $\tilde{l} \in (l^w, \bar{l}]$ leading the price of the risky asset to drop and leaving the bank exposed in those states of the world. Overcapitalizing only the states in $\tilde{l} \in (l^p, l^w)$ is optimal as it implements efficient allocations in the very good states of the economy $\tilde{l} \in [l^p, \bar{l}]$ in which bank's shareholders do especially well.

To illustrate the optimality of pooling realizations in $[l^w, \bar{l}]$ with other relatively risky states of the system consider a simplified version of the model in which $\tilde{l}$ takes one of three values $\{l', l'', \bar{l}\}$, where $l' < l'' < \bar{l}$. The ex-ante probability of $\tilde{l} = \bar{l}$ is π and, for the purpose of this example, we assume it is small. Suppose the regulator uses adverse scenario $\tilde{s}'$ which discloses l' always, but pools l'' with $\bar{l}$ with probability $\pi \frac{l^w - l''}{\bar{l} - l^w}$:

$$\tilde{s}' = \begin{cases} l' & \text{if} \quad \tilde{l} = l', \\ l'' & \text{if} \quad \tilde{l} = l'' \text{ and } \tilde{\eta} > \frac{\pi}{\mathrm{P}(\tilde{l} = l'')} \cdot \frac{\bar{l} - l^w}{l^w - l''}, \\ l^w & \text{if} \quad \tilde{l} = l'' \text{ and } \tilde{\eta} \leq \frac{\pi}{\mathrm{P}(\tilde{l} = l'')} \cdot \frac{\bar{l} - l^w}{l^w - l''}, \text{ or } \tilde{l} = \bar{l}, \end{cases}$$

where $\tilde{\eta}$ is uniformly distributed and not disclosed. It is easy to check that $\mathrm{E}\left[\tilde{l}\,|\tilde{s}\right] = \tilde{s}$. When $\tilde{l}$ is perfectly revealed by $\tilde{s}'$, the regulator implements optimal capital requirements. If, however, $\tilde{s} = l^w$, then the only way to make the bank safe is to sell all of its assets, resulting in 0 welfare.

The expected welfare under adverse scenario $\tilde{s}'$ is then given by

$$W(l') \cdot \mathrm{P}\Big(\tilde{l} = l'\Big) + W(l'') \cdot \left(\mathrm{P}\Big(\tilde{l} = l''\Big) - \pi \cdot \frac{\bar{l} - l^w}{l^w - l''}\right).$$

By pooling state $\tilde{l} = l''$ with $\bar{l}$ the regulator gives up the expected welfare of $W(l'')$ that could have been achieved if this state were disclosed. This is the opportunity cost for the regulator.

Next, we consider an alternative adverse scenario in which the regulator pools outcomes l' with $\bar{l}$:

$$\tilde{s}' = \begin{cases} l' & \text{if} \quad \tilde{l} = l' \text{ and } \tilde{\eta} > \frac{\pi}{\mathrm{P}(\tilde{l}=l')} \cdot \frac{\bar{l}-l^w}{l^w-l'}, \\ l'' & \text{if} \quad \tilde{l} = l'', \\ l^w & \text{if} \quad \tilde{l} = l' \text{ and } \tilde{\eta} \leq \frac{\pi}{\mathrm{P}(\tilde{l}=l')} \cdot \frac{\bar{l}-l^w}{l^w-l'}, \text{ or } \tilde{l} = \bar{l}. \end{cases}$$

The expected welfare corresponding to this adverse scenario is given by

$$W(l') \cdot \left(\mathrm{P}\Big(\tilde{l} = l'\Big) - \pi \cdot \frac{\bar{l} - l^w}{l^w - l'}\right) + W(l'') \cdot \mathrm{P}\Big(\tilde{l} = l''\Big).$$

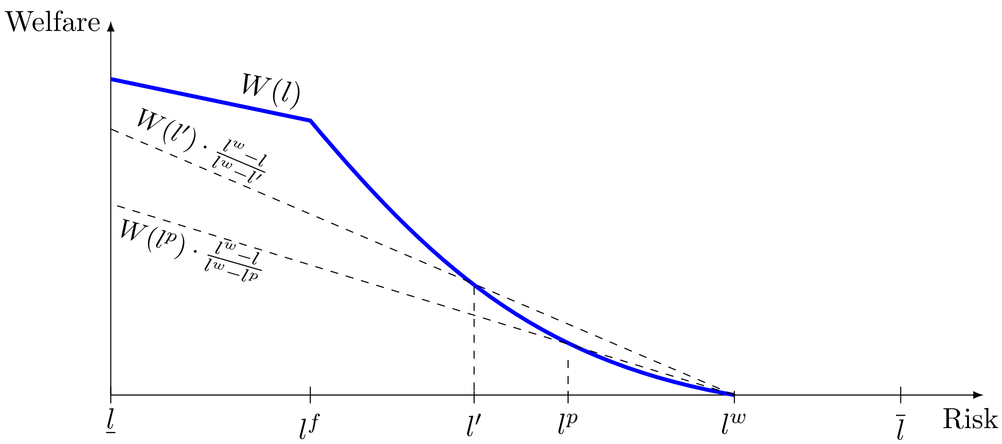

Figure 3: Expected welfare conditional on $\tilde{l} = l$ is depicted in blue. The costs of pooling l'' with 0 corresponds to the slope of the line that passes through $(l^w, 0)$ and $(l'', W(l''))$.

The regulator considers the following trade-off: to pool more risky states of the world l'', which have a low opportunity cost $W(l'')$, or to pool a few safe states of the world l' which have a high opportunity cost $W(l')$. It is more efficient to pool l'' with $\bar{l}$ if and only if the expected loss welfare under signal $\tilde{s}'$ is lower than the welfare loss under signal $\tilde{s}''$:

$$W(l'') \cdot \pi \frac{\bar{l} - l^w}{l^w - l''} \leq W(l') \cdot \pi \frac{\bar{l} - l^w}{l^w - l'}. \tag{6}$$

This condition is shown geometrically in Figure 3. Specifically, if (6) holds for all $l' \leq l^p$ and $l'' \geq l^p$, the regulator prefers to the risky states $[l^p, l^w)$ with $(l^w, \bar{l}]$ generating a threshold adverse scenario $\tilde{s}^*$ in (4). A threshold scenario is optimal even though function $W(l)$ is concave at $l = l^f$.

Given optimal scenario $\tilde{s}^*$ optimal capital requirements $m^*(s)$ are plotted in figure 4 as a function of the realized risk $\tilde{l}$. When risk is low to moderate $\tilde{l} < l^p$ the capital requirements efficiently reflect all information as defined in 5. For $\tilde{l} \geq l^p$ the bank sells all of its assets and, in order for it to be default free, the revenue from this sale must be equal to $d - c$.

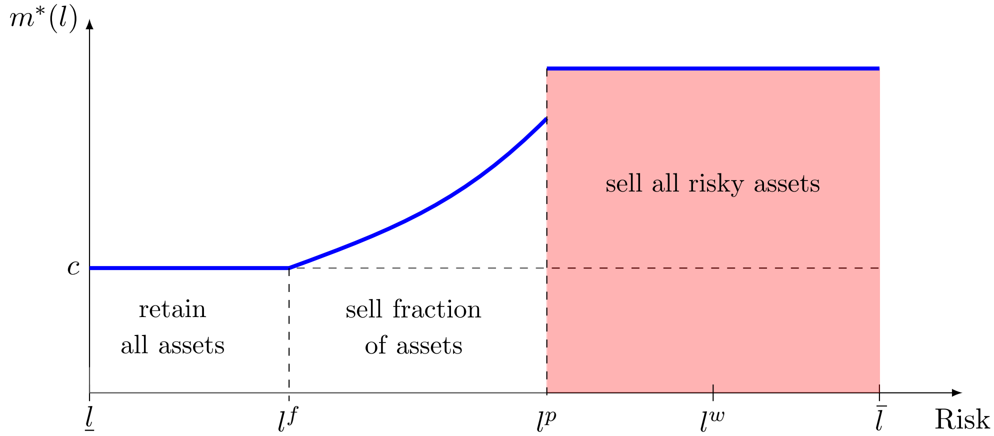

Figure 4: Optimal capital requirements as a function of the underlying state l. Downside loss $\tilde{l}$ is disclosed conditional on $l \geq l^p$.

The optimal stress test can be thought of as two thresholds: $\{l^f,\, l^p\}$. If the magnitude of systemic risk is small, i.e. $\tilde{l} < l^f$, the bank passes the stress test and is allowed to retain all of its assets. If the risk exceeds l^f, then the bank is under-capitalized. Consequently the bank fails the stress test and is required to sell some of its risky assets in order to increase the liquidity cushion. Two cases are possible if the bank fails the stress test. If $\tilde{l} \in (l^f, l^p)$, then $\tilde{l}$ is revealed, and the capital requirement is set at an efficient full-information level (2). When the potential cash flows are very low, however, the regulator imposes very strict capital requirements without providing precise information to the market. This partial transparency is crucial for the ability of the bank to raise enough capital to pay its debt in the second period.

Transparency of the optimal stress test is uniquely captured by the region threshold l^p below which $\tilde{l}$ is revealed. A bank with a higher relative funding gap $\frac{d-c}{n}$ needs to raise more capital, thus, the optimal threshold l^p decreases in order to maintain higher asset prices. Such decrease in the transparency of the stress test sacrifices the efficiency of achieving a fully informed allocation and increases the number of states in which the bank is prepared for the worst outcome of $\tilde{l} = \bar{l}$.

Corollary 1. *Transparency of the bank-optimal stress test is decreasing in the relative funding gap* $\frac{d-c}{n}$ *and is increasing with the liquidity of bank's assets* δ*.*

In what follows we refer to the stress test derived in Proposition 1 as the individual bank-optimal test. With multiple ($I > 1$) heterogeneous (different relative funding gaps $\frac{d_i - c_i}{n_i}$) banks Corollary 1 states that transparency of the bank-optimal test is ranked inversely with the funding gap. We use this property in Section 3.2 to derive the optimal dynamic test for multiple banks.

Stress test implementation. Adverse scenario $\tilde{s}^*$ relies on the ex-post commitment of the regulator to inefficiently recapitalize outcomes $\tilde{l} \in [l^p, l^w)$. If the regulator did not have such commitment power she would disclose these states, thus, compromising the stress test from an ex-ante perspective. One way the regulator can commit to keep information out of the market is by strategically not collecting information from the banks that would reveal that $\tilde{l} \geq l^p$. Specifically, instead of collecting information about the entire distribution of $\tilde{v}$, the regulator can require the bank to report the worst case value of the random variable

$$\tilde{v}^p = \tilde{v} \cdot \mathbb{1}\left\{\tilde{v} > h - l^p\right\}.$$

Since $\tilde{l}$ is realized at $t = 1$, this worst case outcome is equal to $\tilde{l}$ whenever $\tilde{l} < l^p$ and is equal to 0 when $\tilde{l} \geq l^p$. By disclosing this information to the market, the regulator can implement adverse scenario outcome $\tilde{s}^*$. The key is that if this worst case outcome turns out to be 0, the regulator cannot provide finer information to the market since this is not something it collected from the banks. Public information collection can serve as an ex-ante commitment device to remain strategically ignorant when systemic risk is very high. This, in turn, creates additional liquidity for the banks and lets them recapitalize safely.

Matters are more straightforward when it comes to ex-post enforcement of capital requirements. The regulator chooses the weakest ones such that the bank remains solvent. It is incentive compatible for the regulator to enforce them since, otherwise, the systemically important bank could default causing major disruptions in the financial markets.[13]

[13] We do not model the source of systemic importance of the banks. We have in mind a setting similar to AIG in which a default of the institution causes a break in a large number of contracts greatly amplifying uncertainty in the financial markets and, possibly, leading to fire sales, runs, and, consequently, disruption of the real economy.

3.2 Optimal Stress Test for Multiple Banks

Now we turn to the case of multiple banks $I > 1$. The analysis is quite different here since a static stress test that is optimal for bank i is not be optimal for bank i' unless their relative funding gaps are equal ($\frac{d_i - c_i}{n_i} = \frac{d_{i'} - c_{i'}}{n_{i'}}$). The stronger bank always benefits from additional transparency. In order to allow for this flexibility we introduce dynamic stress tests that allow the regulator to release information and implement capital requirements sequentially. In the context of the current stress tests conducted by the Fed, this allows the regulator to first disclose the results of the adverse scenario and force only the very weak banks to raise capital. The Fed could subsequently disclose the results of the severely adverse scenario and impose capital requirements on stronger banks.

Definition. *A dynamic stress test $\mathcal{S}^d$ is a set of stress tests $\left\{\tilde{s}_j, \{m_{i,j}(\cdot)\}_{i=1}^{I}\right\}_{j=1}^{J}$ to be implemented sequentially:*

1. *$\tilde{s}_j$ is an adverse scenario, i.e. a random variable correlated with $\tilde{l}$, disclosed at step j and,*
2. *$\tilde{m}_{i,j} = m_{i,j}(s_1, \ldots, s_j)$ is the minimum quantity of cash that bank i must hold given the outcomes of the first j adverse scenarios $\tilde{s}_1 = s_1, \ldots, \tilde{s}_j = s_j$.*

For any ex-post allocation of risky assets and cash, we focus on the dynamic test with the lowest number of adverse scenarios J. This avoids labeling a static test as a dynamic one by staggering information disclosure across periods without real implications for risk-allocation.

Similar to static tests, we restrict attention to dynamic stress tests which are default free. Formally, $\mathcal{S}^d$ is default free if banks are able to meet their liabilities at $t = 2$ with certainty, i.e.

$$\tilde{m}_{i,J} + \tilde{q}_{i,J} \cdot \tilde{v} - d_i \geq 0, \quad \forall i \qquad P - a.s. \tag{DF$_d$}$$

where $\tilde{q}_{i,j}$ is the resulting quantity or the risky asset that bank i holds after j steps of the dynamic stress test

$$\tilde{q}_{i,j} = \tilde{q}_{i,j-1} - \frac{\tilde{m}_{i,j} - \tilde{m}_{i,j-1}}{\delta\left(h - \frac{1}{2}\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}_1, \ldots, \tilde{s}_j\right]\right)}, \qquad \forall i \quad \text{and} \quad j = 1, \ldots, J.$$

Dynamic solvency constraint (DF$_d$) ensures that no bank defaults regardless of the realization of $\tilde{v}$ and is similar to the static solvency constraint given by (DF).

Similar to the previous section, for every bank i define the unique triple (l_i^w, l_i^p, l_i^f) via

$$l_i^f = h - \frac{d_i - c_i}{n_i}, \qquad l_i^w = 2h - \frac{2}{\delta} \cdot \frac{d_i - c_i}{n_i}, \qquad \mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \geq l_i^p\right] = l_i^w.$$

By construction, this triple is increasing in i along with the bank's relative funding gap. Define $W_i(\cdot)$ in the same way as (3) for each bank i. To ensure that the optimal dynamic test satisfies a threshold rule we assume that

$$\frac{W_i(\underline{l})}{l_i^w - \underline{l}} \geq \frac{W_i(l_i^p)}{l_i^w - l_i^p} \quad \text{for every} \quad i = 1, 2, \ldots, I. \tag{7}$$

The following result shows that by stress testing the banks sequentially the regulator can achieve the indivually optimal allocation for every bank and, consequently, maximize expected welfare.

Proposition 2 (Dynamic Stress Test)**.** *Suppose condition* (7) *holds. The optimal dynamic stress test can be implemented with I adverse scenarios:*

$$\tilde{s}_j^* = \begin{cases} \tilde{l} & \text{if} \quad \tilde{l} < l_j^p, \\ l_j^w & \text{if} \quad \tilde{l} \geq l_j^p, \end{cases} \qquad\qquad j = 1, 2, \ldots, I.$$

After adverse scenario j the regulator imposes contingent capital requirements on bank i

$$m_{i,j}^*(s_j) = \begin{cases} c & \text{if} \quad j < i, \\ c + \left[\frac{d_i - c_i - n_i \cdot h + n_i \cdot s_i}{p_1(s_i) - (h - s_i)}\right]^+ \cdot p_1(s_i) & \text{if} \quad j \geq i. \end{cases}$$

Weak banks are recapitalized first, and, once their solvency is assured, regulator runs more informative adverse scenarios and recapitalizes stronger banks optimally.

Our main idea is that by recapitalizing banks sequentially, the regulator splits funding markets across time eliminating the information externality imposed by the weak banks on the rest of the financial system. Consider the case of two banks $I = 2$. The stress test discloses information about aggregate risk $\tilde{l}$ which affects asset prices and liquidity of both banks simultaneously. Because investors observe all available information, in a static test, the regulator must always keep $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}\right] \leq l_1^w$ to prevent the weak bank from going under. This imposes an information externality on the strong bank since under the individually optimal stress test for this bank $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}_2^*\right] > l_1^w$ with positive probability. By conducting stress tests sequentially and using the property that the optimal transparency is decreasing in the bank's relative funding gap, the regulator allows banks to recapitalize in markets split across time. This way she implements the optimal stress test for the weak bank first and then, after revealing additional information, recapitalize the strong bank. The second stress test scenario $\tilde{s}_2^*$ is transparent about higher levels of systemic risk, disclosing $\tilde{l}$ if it falls in $[l_1^p, l_2^p]$, which improves allocative efficiency for the strong bank's assets.

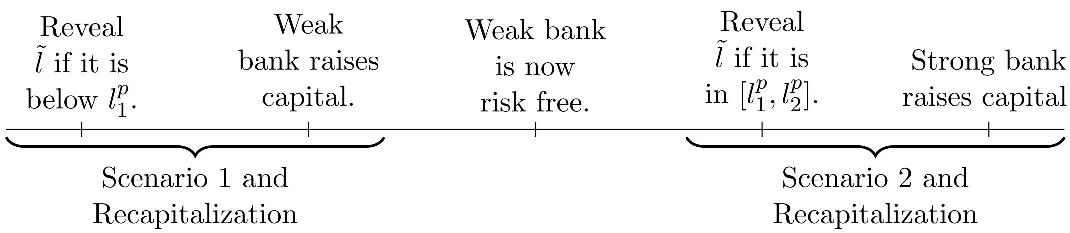

Figure 5: Timing of the dynamic stress test.

To highlight the increased efficiency of a dynamic stress test we also derive the optimal static stress test for multiple banks. Bank 1, the weakest bank with the largest relative funding gap $\frac{d_1 - c_1}{n_1}$, plays a special role in the design of the adverse scenario as it requires, by previous logic, that all posteriors satisfy $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}\right] \leq l_1^w$. Define function $\alpha(l)$ for each $l \in [l_1^w, \bar{l}]$ as the unique solution to the equation

$$\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \in [\alpha(l), l]\right] = l_1^w \quad \Rightarrow \quad \int_{l_1^w}^{l} (l_1^w - y) \cdot f(y)\, dy = \int_{\alpha(l)}^{l_1^w} (y - l_1^w) \cdot f(y)\, dy. \tag{8}$$

Intuitively, for each l choose $\alpha(l)$ such that $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \in \{\alpha(l), l\}\right] = l_1^w$. It is easy to see that function $\alpha(l)$ is decreasing in l over the defined interval and satisfies $\alpha(l_1^w) = l_1^w$ and $\alpha(\bar{l}) = l_1^p$. The following proposition characterizes the optimal static stress test for multiple banks which provides more granular information to the system than just the individually optimal test for the weak bank.

Proposition 3. *Suppose* $\frac{W_i(l_1^p)}{l_1^w - l_1^p} \leq \frac{W_i(\underline{l})}{l_1^w - \underline{l}}$ *for ever* $i = 1, 2, \ldots, I$. *The adverse scenario under the optimal static stress test* $\mathcal{S}^*$ *for* $I > 1$ *is determined by the weakest bank and is given by*

$$\hat{s}^* = \begin{cases} \tilde{l} & \textit{if} \quad l \in [\underline{l}, l_1^w], \\ \alpha(\tilde{l}) & \textit{if} \quad l \in (l_1^w, \bar{l}], \end{cases} \tag{9}$$

which takes values in $[0, l_1^p) \cup [l_1^w, \bar{l}]$*. Optimal contingent capital requirement for bank* i *is given by*

$$\hat{m}_i(s) = \begin{cases} c_i + \left[\frac{d_i - c_i - n_i h + n_i s}{p_1(s) - h + s}\right]^+ \cdot p_1(s) & \text{if} \quad \tilde{s} < l_1^p, \\ c_i + \left[\frac{d_i - c_i - n_i h + n_i \alpha^{-1}(s)}{p_1(l_1^w) - h + \alpha^{-1}(s)}\right]^+ \cdot p_1(l_1^w) & \text{if} \quad \tilde{s} \geq l_1^p. \end{cases} \tag{10}$$

Aggregate efficiency is strictly higher under the dynamic stress test whenever banks are heterogeneous in their relative funding gap: $\frac{d_1 - c_1}{n_1} > \frac{d_I - c_I}{n_I}$.

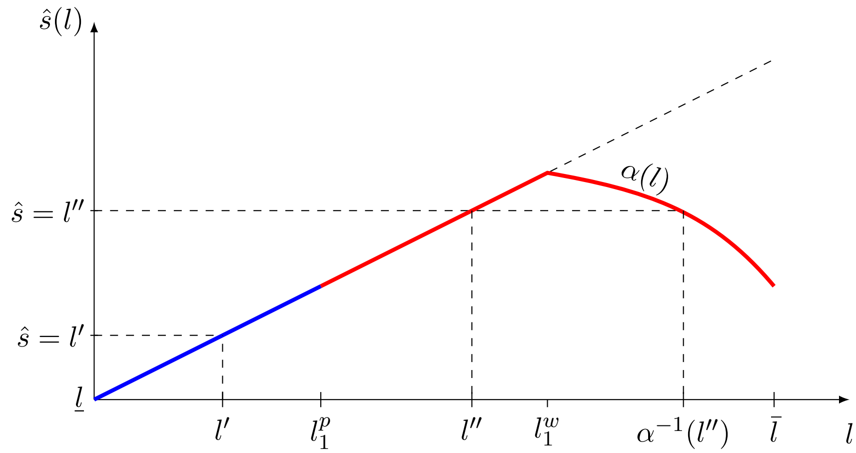

Figure 6: Signal realization $\tilde{s}$ as a function of $\tilde{l}$. For $\tilde{l} \geq l_1^p$ it pools signal realizations pairwise. For $\tilde{l} < l_1^p$ it simply reveals $\tilde{l}$.

Signal $\hat{s}^*$ defined in (9) generates the same posterior expectation of $\tilde{l}$ as $\tilde{s}^*$. On the other hand, it reveals more information about $\tilde{l}$ by spliting the states of nature into pairs $\{\alpha(l), l\}$ for each $l \in [l_1^w, \bar{l}]$. This is optimal for the regulator because by additionally disclosing that $\tilde{l} = l_1^w$, the regulator does not change the conditional average based on any of the signals, but improves the payoff of the stronger banks when $\tilde{l} = l_1^w$ by relaxing the worst-case scenario in the strong bank's solvency constraint (DF). The worst-case scenario does not matter when we design signal $\tilde{s}^*$ as in all of those states is sells the risky asset to the market completely thus completely eliminating exposure to it.

Lemma 1. *The welfare gain from performing the optimal dynamic stress test over the optimal static stress test is increasing in the funding gap of the weakest bank* $\frac{d_1 - c_1}{n_1}$.

Proof. The above result is a direct consequence that as the relative funding gap of the weakest bank increases, the strong bank's payoff under the optimal static test is decreasing since the set of adverse scenarios available to the regulator decreases. Under the optimal dynamic stress test the expected welfare of the strong bank is independent of the weak's bank relative funding gap. □

The optimal static stress test for multiple banks generates a Pareto improvement relative to pooling all states in $[l_1^p, \bar{l}]$ by generating a more informative signal about $\tilde{l}$ which allows the strong banks to allocate assets more efficiently. It does not, however, eliminate the externality on the strong bank completely as it is still forcing this bank to hold excess liquidity for $\tilde{l} \in [l_1^p, l_2^p]$. Lemma 1 states that this benefit is large when the difference in relative funding gaps is large.

3.3 Dividend Payments

During the financial crisis of 2007-2009 large bank holding companies paid in excess of $70 billion in dividends (see Acharya, Gujral, Kulkarni, and Shin (2011)). Acharya, Le, and Shin (2017a) argue that banks have incentives to free-ride on aggregate liquidity, while paying out dividends to their own shareholders. In this section we show that the optimal stress test might prohibit large dividend repayments in times when there is a lot of risk in the economy.

We can incorporate the incentive of the bank to pay out dividends by introducing the discount rate of the bank's shareholders. Specifically, if bank's shareholders discount future cash flows at rate β, their expected utility is given by

$$\mathrm{E}\left[(c - m(\tilde{s}))^+\right] + \beta \cdot \mathrm{E}\left[(m(\tilde{s}) + q(\tilde{s}) \cdot \tilde{v} - d)^+\right]$$

where the difference between starting cash and necessary capital requirements $c - m(\tilde{s})$ can be paid out as early dividends.[14] A dollar invested into the asset generates a discounted expected gain of $\beta \cdot \mathrm{E}\left[\frac{\tilde{v}}{\delta \cdot \mathrm{E}[\tilde{v}]}\right] = \frac{\beta}{\delta}$. We assume that $\frac{\beta}{\delta} > 1$ so that banks contribute enough to the economy to warrant existence.[15]

The adverse scenarios $\tilde{s}^*$ and $\{\tilde{s}_j^*\}_{j=1}^I$ derived in Propositions 1 and 2 respectively remain optimal. The capital requirements, however, allow the bank to pay out $c_i - d_i - n_i \cdot (h - \tilde{l})$ as dividends when $\tilde{l} \leq l_i^f$. By doing so, the regulator maximizes allocative efficiency of safe assets while ensuring the banks are safe. When $\tilde{l} > l_i^f$, the regulator forbids dividend repayment and requires bank i to raise liquid capital by selling risky assets as described in Proposition 2. The expected utility of bank i

[14] This formulation applies exactly to the static framework and is naturally extended to allow for the payoff under a dynamic stress test.

[15] Sections 2, 3.1 and 3.2 can be viewed as having $\beta = 1$ in which case there is no value of paying dividends early.

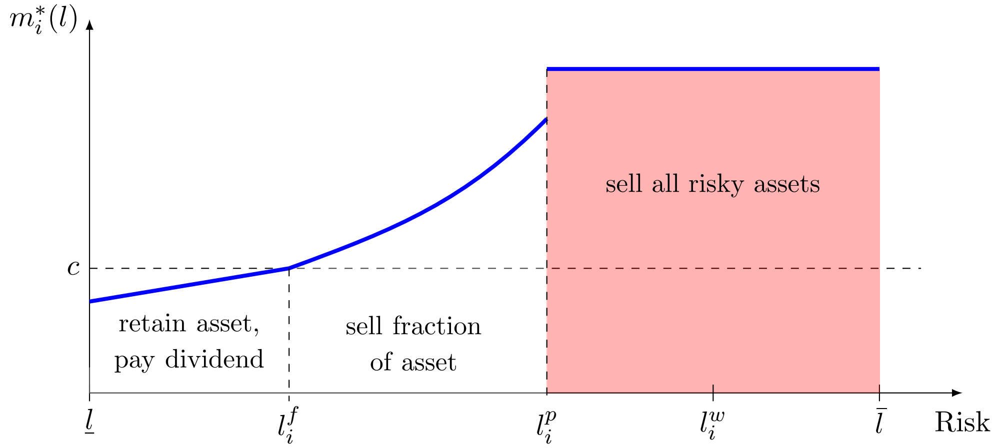

Figure 7: Optimal capital requirements as a function of the underlying state l when the bank can issue dividends.

under the optimal dynamic stress test can be written as

$$w_i^* = \underbrace{\beta \int_{\underline{l}}^{\bar{l}} \left(c_i + n_i\left(h - \frac{l}{2}\right) - d_i\right) f(l)\, dl}_{\text{undistorted value}} - \underbrace{\beta \frac{1-\delta}{\delta} \int_{l_i^f}^{\bar{l}} (m_i(l) - c_i) f(l)\, dl}_{\text{cost of capital requirements}} + \underbrace{(1-\beta) \int_{\underline{l}}^{l_i^f} (c_i - m_i(l)) f(l)\, dl}_{\text{benefit of early dividends}}.$$

The first term is the status-quo expected social value without asset reallocation costs and default deadweight losses. The second term is the social cost of selling assets to satisfy capital requirements. Third term is the benefit of paying dividends out early. It may be the case that bank's funding costs are lower than those of its shareholders if it benefits from the safe asset premium. Quantity $1-\beta$ captures this difference in discount rates between bank shareholders and outside markets. As long as the bank's dividend policy does not impact asset allocation, the regulator is accommodate it.

The bank may have incentives to pay out dividends before the stress test outcome is disclosed even when $\frac{\beta}{\delta} > 1$. The intuition is that capital requirements tax the asset holdings of the bank and reduce the expected marginal return below $\frac{1}{\delta}$ from holding them. If the regulator does not share the same relative value of dividends as the bank's equity holders, she would like to limit dividend payment prior to knowing the outcome of the stress test.

3.4 Interbank Trading of the Risky Asset

In this section we show that when a strong bank can provide additional liquidity to a weak bank in the interbank market for risky assets the total welfare and the profit of the strong bank under the optimal stress test go up. If the strong bank is sufficiently big, relative to the weak bank, the optimal stress test is equivalent to the sale of the weak bank to the strong one. The regulator then imposes capital requirements on the merged institution. A dynamic stress test is essential for the regulator to capture this benefit as it requires capital requirements to be set sequentially.

For ease of exposition, we restrict attention to two banks $I = 2$ and refer to bank 1 as the weak bank and bank 2 as the strong bank. When risky assets can be reallocated between banks, it is important to understand whether the acquiring bank has the same or higher valuation of the risky asset than the market. To derive the optimal stress test analytically, we assume that the banks have a common valuation for the risky asset. In this sense, the regulator cares about the total quantity of asset held in the banking system, rather than which bank holds which assets. Also, to demonstrate that interbank trade improvement does not simply rely on the weak bank getting a higher price for its assets, we assume that the buyer of the risky asset has all the bargaining power. This is consistent with the intuition that the bank who must sell assets to satisfy capital requirements is at a disadvantage.

Consider the aggregate bank which has a portfolio of $C = c_1 + c_2$ dollars in cash, $N = n_1 + n_2$ units of risky asset, and debt with face value $D = d_1 + d_2$. Similar to the static stress test for a single bank, define the triple (L^f, L^w, L^p) as

$$L^f = h - \frac{D - C}{N}, \qquad L^w = 2h - \frac{2}{\delta}\frac{D - C}{N}, \qquad \mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \geq L^p\right] = L^w.$$

Define W^* to be the expected welfare obtained by the regulator under the optimal stress test if there was just one large aggregate bank in the economy. Similarly, define w_i^* for $i \in \{1, 2\}$ to be the expected welfare of bank i's shareholders under the individually optimal stress test for bank i. If a stress test $\mathcal{S}^d$ is default free for the weak and strong banks, then it is default free for the aggregate bank. The converse is not necessarily true. This implies, generally, that when the regulator stress tests the aggregate bank, there is a broader range of policies available, weakly increasing her payoff:

$$w_1^* + w_2^* \leq W^*$$

with the inequality above being strict whenever the relative funding gaps of the banks are not equal $\frac{d_1-c_1}{n_1} > \frac{d_2-c_2}{n_2}$. The following proposition shows that when the strong bank has enough liquidity at $t=1$, we can achieve the expected welfare of the aggregate bank by relying on interbank trade.

Proposition 4. *If $c_2 \geq d_1 - c_1$, the optimal dynamic test achieves W^*. If $c_2 = 0$, the optimal dynamic stress test achieves $w_1^* + w_2^*$. Keeping the funding gaps fixed, the regulator's welfare is strictly increasing in c_2 over the range $(0, d_1 - c_1)$. The strong bank is always weakly better of than obtaining w_2^*.*

Sketch of Proof. If $c_2 \geq d_1 - c_1$, then the regulator can conduct scenario $\tilde{s}_1^*$. If $\tilde{s}_1^* < l_1^p$, then the weak bank sells some assets to the strong bank and the strong bank might later sell those to the market. In those states the aggregate allocation coincides with the outcome of the aggregate bank. If $\tilde{s}_1^* = l_1^w$, then the weak bank sells all of its assets to the strong bank at a price of $\delta\frac{2h-l_1^w}{2}$ and receives $d_1 - c_1$ units of cash in return. At this point the strong bank becomes equivalent to the aggregate bank since

$$\frac{d_2 - c_2 - d_1 + n_1}{n_1 + n_2} = \frac{D - C}{N}.$$

Thus, the optimal stress test is equivalent to that of an aggregate bank and, thus, the regulator reveals $\tilde{l}$ if it belongs to $[l_1^p, L^p]$ and requires the strong bank to sell assets to the market. Such a stress test achieves a payoff of W^*. Finally, for $c_2 \in (0, d_1 - c_1)$, even keeping all funding gaps and risky asset quantities constant, the set of feasible allocations strictly increases since the strong bank has liquidity available earlier to it. Thus, the expected value to the optimal stress test strictly increases. We do not explicitly characterize the optimal dynamic test when $c_2 \in (0, d_1 - c_1)$[16]

Corollary 2. *If $s_1^* = l_1^w$ the stress test outcome is equivalent to selling the weak bank to the strong bank at fair market value.*

Additional liquidity available to the strong bank benefits the system because it becomes possible to reallocate the asset within the system to approach the allocation of the aggregate bank. It may be the case, however, that the strong bank may prefer to pay out dividends rather than buy the assets of the weak bank. This is true if, for instance, the strong bank does not have perfect bargaining power when acquiring the asset. From the regulator's perspective the transaction price of the risky asset is just a transfer and she would prefer to force the strong bank into acquiring the

[16] Standard dynamic programing techniques are difficult to apply to our problem of sequential information design with a continuous distribution of uncertainty about $\tilde{l}$.

asset anyways. The regulator can provide incentives for the strong bank by restricting its dividend payouts and, thus, eliminating the opportunity cost of funds available to the strong bank.

3.5 Discussion of the Optimal Stress Test

Stress tests are aimed at maintaining the stability of the financial system against potential cash flow shocks. The regulator cannot reveal that there is a large quantity of aggregate risk held among SIFIs as it would hurt their ability to raise liquid funds in the capital markets. We show in Propositions 1 and 2 that as long as the regulator does not reveal the worst stress test outcomes to the markets, the safety of the system can be maintained by appropriately adjusting capital requirements to new information.

Our analysis is focused on the role of macro-prudential stress tests in which the regulator is concerned with systemic risk held by all banks. Our findings are complementary to those of Goldstein and Leitner (2015) and Williams (2017) who focus on the optimal way to restore secondary market liquidity when there is uncertainty about the quality of an individual bank. They show that when disclosing idiosyncratic information about bank portfolios, it is optimal to remove the bad banks from the system to restore efficiency of trade. Their main focus is on the setting in which financial system is already suffering from a credit freeze and the regulator unavoidably shuts some banks down. We, on the other hand, focus on the disclosure of macro-prudential information in a precautionary manner when the system is still able to recapitalize relying purely on public markets.[17]

Dynamic Stress Test. The main innovation in our paper is the introduction of dynamic stress tests in which the regulator runs several adverse scenarios and adjusts bank capital requirements sequentially. We show that this strictly improves expected welfare when testing multiple heterogeneous banks. A practical implementation is to stress test banks in the order from weakest to strongest. This minimizes the information spillover from strongest to weakest bank allowing the latter to recapitalize in a precautionary manner. In addition, it allows the regulator to efficiently manage the liquidity spillovers among banks.

Dynamic stress tests may come at a cost to the regulator. Running several adverse scenarios creates more work for the regulator and the bank, increasing compliance costs. In light of this, the regulator may choose to group banks into several scenarios, rather than run an individually optimal scenario

[17]In Section 4.2 we introduce the possibility of the regulator to infuse banks with cash in a precautionary way, showing that it is suboptimal when the cost of funds to the regulator is sufficiently high.

for each bank. This is best illustrated when the relative funding gaps between the banks are small and the incremental value obtained from a dynamic test is outweighed by the additional compliance costs. A dynamic stress test increases social value when the difference in the bank's relative funding gaps is sufficiently large. There may also be risks associated with delaying recapitalization of the banks when applying sequential stress test scenarios. Specifically, when there is rollover risk faced by the banks[18] the regulator may wish to recapitalize the banks as quickly as possible. As such, the regulator may, again group banks with close relative funding gaps and run a fewer number of adverse scenarios. Dynamic stress tests are valuable when there is large dispersion in the banks funding gaps.

Capital Requirements. Adverse scenarios inform capital requirements. As such, the stress test is well positioned in managing the gaps in other regulation and ensure the system is safe against major shocks. By allowing the regulator to implement the capital requirements as she sees fit, we are able to remain agnostic about the particular incentives of the bank's management and focus on the allocation that benefits the shareholders subject to bank's debt being risk-free. We implicitly rely on the leverage ratchet effect in which the banks prefer to raise capital by selling existing assets rather than diluting existing shareholders, as in Admati, DeMarzo, Hellwig, and Pfleiderer (2018). In Section 4.1 we discuss the incentives of the bank in response to the stress test. Then in Section 4.4 we discuss what changes in our model if the bank raises capital via equity issuance.

4 Extensions

4.1 Risky Asset Origination

We formulate a stylized pre-game in which the bank creates risky assets at a pecuniary cost. This can be naturally interpreted as originating credit-based securities and then holding them on its portfolio. Once these risky assets are originated, the regulator collects this information and conducts an optimal stress test. The bank chooses its portfolio knowing that the regulator will conduct a more stringent stress test in response to more portfolio risk. As such, we study the sub-game perfect equilibrium in a game where the bank first chooses its portfolio, then the regulator chooses the optimal stress test, and then the bank complies with capital requirements. We show

[18] See Gorton and Metrick (2012) for evidence of a run on the broker-dealers in the financial system and Wiggins and Metrick (2015) on the run of Lehman's broker-dealer clients.

that, under general conditions, there exists a default-free stress test and that the bank originates a socially efficient quantity of assets.

For ease of exposition we focus on one bank. The cost of creating n units of the risky asset is $k(n)$ which we assume is convex. The bank funds this cost by taking on debt that is to be repaid in the second period after the risky asset matures. The bank's portfolio, entering period 0, has no cash, n units of the risky asset, and debt with a face value of $k(n)$. The bank has no initial debt and the bank chooses its risky asset holdings out of the interest of its shareholders.

Define $\bar{n}$ to be the first best quantity of asset created which solves $k'(\bar{n}) = \mathrm{E}[\tilde{v}]$. The following lemma shows that the bank never wants to take on excessive leverage due to the incentive effects of the optimal stress test.

Lemma 2. *Suppose $k(\bar{n}) \leq \bar{n} \cdot \delta \mathrm{E}[\tilde{v}]$. There exists a default-free stress test in equilibrium. The bank creates an efficient quantity of the risky asset n^* which satisfies $k'(n^*) \in (\delta \mathrm{E}[\tilde{v}], \mathrm{E}[\tilde{v}])$.*

Proof. Result above is easiest derived using the expected welfare of the shareholders given by (3). The bank solves

$$\max_n \left[n\mathrm{E}[\tilde{v}] - k(n) - (1-\delta)\int_{l^f(n)}^{l^p(n)} \frac{k(n) - n(h-l)}{\delta(h-\frac{l}{2}) - (h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl - \frac{1-\delta}{\delta}\int_{l^p(n)}^{\bar{l}} k(n) f(l)\, dl \right]$$

where $l^f(n) = h - \frac{k(n)}{n}$ and $l^p(n)$ is the solution to $\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{l} \geq l^p(n)\right] = 2h - \frac{2}{\delta}\frac{k(n)}{n}$. The bank's asset choice is efficient since the stress test maximizes its welfare and the bank internalizes the effect of the portfolio choice on the stress test. The first order condition with respect to n, pinning down n^*, is given by

$$k'(n) = \mathrm{E}[\tilde{v}] - (1-\delta) DW'(n)$$

where the marginal dead-weight loss to the bank from capital requirements $DW(n)$ is given by

$$DW'(n) = \frac{k(n) - n(h-l^f)}{\delta(h-\frac{l}{2}) - (h-l)} \cdot \frac{l^f}{2} \cdot \frac{k'(n)n - k(n)}{n^2} + \int_{l^f}^{l^p} \frac{(k'(n) - h + l)}{\delta(h-\frac{l}{2}) - (h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl + \frac{k'(n)}{\delta}\Big(1 - F(l^p)\Big).$$

The marginal dead-weight loss $DW'(n)$ is the incremental regulatory cost for the bank of originating an additional unit of the risky asset. It consists of three components. The first term is the reduction in the probability that the bank will be able to retain all of its risky assets. The second term is the increase in the amount of asset sold when the stress test perfectly discloses the amount of risk. The third term is the increase in the quantity of the asset that needs to be sold when the amount of risk is high and the bank must sell all of its assets. Because the bank always has the option to sell the asset to the outside investors, $DW'(n^*) < (1-\delta)\mathrm{E}[v]$ implying the result of Lemma 2. $\square$

Multiple banks. When there are many banks, their heterogeneity may stem from the differential cost of issuing asset $k_i(\cdot)$. The optimal dynamic stress test implements the payoff of the individually optimal stress tests and, thus, the results of Lemma 2 carry over, i.e. there is a default-free stress test and banks invest efficiently. There is always a positive probability that the bank retains all of the risky assets.

4.2 Precautionary Infusions and Bailouts

In this section we discuss what happens when the regulator can subsidize banks directly. First, we consider a precautionary capital infusion performed by the regulator at $t = 1$ which is conducted by the regulator when she anticipates potential defaults in the second period. This is consistent with our narrative of the stress tests in which SIFIs are regulated ex-ante as to not cause market dysfunction if their solvency is ever questioned. Then we discuss what changes in our model if the regulator can conduct ex-post bailouts once the cash flows have been realized. While this approach is broadly consistent with our modeling framework, such ex-post interventions may not always be feasible as, during a financial panic, markets would be moving quickly and the regulator may not be able to act in time.

Precautionary capital infusions. Suppose the regulator can provide funds to the banks at a cost γ, i.e. every dollar given to the banks costs the regulator γ. The definition of the stress test can be easily amended to include the possibility of a monetary infusion at $t = 1$.

Lemma 3. *Suppose the cost of regulatory funds is sufficiently high $\gamma \geq \frac{l}{(2-\delta)\underline{l}-2h(1-\delta)}$. If $E\left[\tilde{l}\right] \leq l^w$, the optimal stress test does not involve a bailout. If $E\left[\tilde{l}\right] > l^w$, the regulator requires the bank to sell all of its risky assets and, only afterwards, she infuses the bank with $d - c - n\delta\left(h - \frac{1}{2}\mathrm{E}\left[\tilde{l}\right]\right)$ units of cash at $t = 1$.*

Lemma 3 states that when the cost of funds is sufficiently high, the regulator would rather recapitalize the banks in public markets, rather than infuse its own funds. This is an intuitive result that relies on the fact that the regulator recapitalizes the bank ex-ante.

Bailouts. If the regulator's cost of funds is γ ex-post, then if she discloses l and forces the bank to raise capital m, her payoff becomes

$$\hat{W}(l) = \max_m \mathrm{E}\left[c + n \cdot \tilde{v} - d - \frac{1-\delta}{\delta}(m-c) - \gamma \left(d - m - \left(n - \frac{m-c}{\delta \mathrm{E}\left[\tilde{v}\right]} \right) \tilde{v} \right)^+ \middle| \tilde{l} = l \right]$$

Under the optimal choice of m, the regulator intervenes with positive probability at $t = 2$ since the ex-ante likelihood of the bailout is sufficiently small. We can show that our results are qualitatively unchanged if either $\tilde{l}$ follows a binary distribution or the ex-post distribution of cash flows $\tilde{v}$ has an atom at $\tilde{l}$. Our results also remain unchanged if we limit the set of adverse scenarios to "monotone" signals in which only the adjacent realizations of $\tilde{l}$ can be pooled together. Such signals are a natural generalization of the optimal threshold scenarios derived in Propositions 1 and 2.

4.3 Decreasing Demand for the Risky Asset

The outside market may consist of heterogeneous investors who apply different discount rates to the assets offered by the bank when raising capital. Suppose potential investors have a discount rate of $\delta \in [0, \bar{\delta}]$ with $g(\delta)$ dollars available to investors of type δ. The bank must raise

$$G(\bar{\delta}) - G(\delta) = \frac{d - c - n(h-l)}{\delta\left(h - \frac{l}{2}\right) - (h-l)} \cdot \delta \left(h - \frac{l}{2} \right). \tag{11}$$

Define by $\delta(l)$ to be the level of discount that satisfies (11). In this setting not all investors will break even and the welfare of the bank is not the same as the welfare of the economy. The welfare of the bank and outside investors is given by

$$W(l) = c + n\left(h - \frac{l}{2}\right) - d - \int_{\delta(l)}^{1} (1-x) dG(x).$$

Such a formulation does not lend itself to an easy analytical characterization. If there is sufficient mass of wealth around $\bar{\delta}$ the objective of the regulator is close to the one analyzed in prior sections. As long as $\frac{W(l')}{l^w - l'} \geq \frac{W(l'')}{l^w - l''}$ for every $l' < l^p < l''$, then the static stress test derived in Proposition 1 remains optimal.

Solving for an optimal dynamic stress test becomes significantly more difficult in this context. The banks who first sell to the market get a better price. This implies that by requiring weak banks to raise capital first, the regulator implicitly subsidizes them at the expense of strong banks.

4.4 Equity Issuance

In this section we discuss what would happen if the bank could reduce its default risk by raising additional capital in the equity markets. In our main model the bank reduces its portfolio risk by selling assets to the market. This improves the ratio of bank's safe capital to risky capital and can be naturally viewed as an improvement in its capital ratio.

Suppose the bank were to raise capital by issuing new equity while keeping the asset side of its balance sheet constant. This would dilute existing shareholders, but, from the perspective of the regulator, is just a wealth transfer. We can assume that equity issuance is costly as the funds used in the issuance do not go to other productive uses. The regulator would prefer to minimize the amount of capital issued all else being equal. To become safe in the worst case scenario $\tilde{v} = \tilde{l}$, bank i must raise $[d_i - c_i - n_i \cdot (h - \tilde{l})]^+$. If $\tilde{l}$ is high, the shortfall that needs to be raised is greater. On the other hand, the new shareholders would be willing to pay at most $c_i + n_i \frac{2h - \tilde{l}}{2} - d_i$ for full ownership of the bank's assets. There is a critical level $\hat{l}_i^w$ at which bank i is barely solvent:

$$d_i - c_i - n_i \cdot \left(h - \hat{l}_i^w\right) = c_i + n_i \cdot \frac{2h - \hat{l}_i^w}{2} - d_i.$$

For $\tilde{l} > \hat{l}_i^w$ safe recapitalization is impossible and the bank defaults with a positive probability. The regulator needs to pool states of the world in $[\hat{l}_i^w, \bar{l}]$ with other outcomes in order to ensure the perceived level of risk in the economy is sufficiently low in order to support the value of bank's equity. The optimal stress test may not follow a threshold policy and features elements derived in Proposition 3 in which the regulator utilizes partial pooling. If we restrict attention to threshold stress tests, the optimal test looks qualitatively the same as in Proposition 1. If we focus on dynamic stress tests in which the bank raises capital only once, then we can show that dynamic stress tests strictly improve upon the optimal static policy.

5 Portfolio Correlations as Endogenous Downside Risk

Systemically important financial institutions impact asset prices. The nature of the downside risk analyzed in Sections 3 and 5 is likely endogenous to how well capitalized these banks are. A recent survey of the intermediary asset pricing literature by Benoit et al. (2016) points out that a common way to think of systemic risk is as correlation in bank portfolios. The cash flow shocks are then amplified as the banks are constrained simultaneously. The point of the stress test is, then, to uncover the degree of correlation among the portfolios of the banks. In this section we build a stylized model of intermediary asset pricing to show that the downside risk in our baseline setting may be an outcome of an aggregate comovement of bank portfolios. We demonstrate how, in this setting, the regulator is uniquely positioned in uncovering this information by being able to observe the cross-section of bank portfolios. As such, the regulator has both an informational advantage over the banks (and public markets) as well as the ability to certify this information to investors.

5.1 Augmented Setup

We extend our baseline model by an additional period $t = 3$ to incorporate the role of bank liquidity in asset prices at $t = 2$. In favor of tractability, namely to rely on the exact law of large numbers, we also assume that there is a continuum of identical banks $[0, 1]$. Each bank holds c units of cash, one unit of risky asset $\tilde{x}_i$ which pays at $t = 2$ and is non-tradable, n units of long-term tradable asset $\tilde{v}_i$ with expected payment b at $t = 3$. Each bank has outstanding debt with face value d that matures at $t = 2$. Risky cash flow $\tilde{x}_i$ at $t = 2$ which is given by

$$\tilde{x}_i = \begin{cases} \tilde{\xi} & \text{with probability} \quad \tilde{z}, \\ \tilde{\xi}_i & \text{with probability} \quad 1 - \tilde{z}, \end{cases}$$

where $\{\xi, \{\xi_i\}_{i \in [0,1]}\}$ are i.i.d. For simplicity, we assume that each ξ is either 0 or 1 with probability $1 - \mu$ and μ respectively.[19] Random variable $\tilde{z}$ is realized at $t = 1$ and captures how correlated the cash flows $\tilde{x}_i$ are between banks:

$$\mathrm{E}\left[\big(\tilde{x}_i - \mu\big)\big(\tilde{x}_j - \mu\big) \,\Big|\, \tilde{z} = z \right] = z^2 \cdot \mu(1 - \mu).$$

Banks raise liquid capital at $t = 1$ by selling their long-term assets to a competitive risk-neutral investors, who have an opportunity cost of δ between periods $t = 1$ and $t = 2$. These investors consume in period $t = 2$ and will need to sell the asset back to the bank. Banks are competitive, but, if they are cash constrained, they may only be able to offer investors a low price in the second period. The regulator seeks to maximize the expected welfare of the banks, which is equivalent to minimizing value of missed opportunities to investors while keeping the banks safe.

The regulator collects information from the banks about their portfolios $\tilde{v}_i$. The level of systemic

[19] The model is qualitatively unchanged if ξ's are not binary.

risk $\tilde{z}$ is realized at $t = 1$ which implies that by considering the cross-section of these portfolios the regulator can compute the cross-correlations between their portfolios

$$\int_0^1 \int_0^1 (\tilde{x}_i - \mu)(\tilde{x}_j - \mu)\, di\, dj = \tilde{z}^2 \cdot (1 - \mu)^2,$$

where the above equality holds by the exact law of large numbers. In this sense, the regulator has informational advantage by being able to exploit the cross-section of bank portfolios and obtain a forward looking estimate of $\tilde{z}$. The banks, on the other hand, are likely to be in a better position to learn $\tilde{z}$ from the time-series of their own portfolio, we assume that this is already incorporated in the prior $F(\cdot)$ about $\tilde{z}$.

5.2 Optimal Stress Test

Suppose the regulator reveals $\tilde{z}$ perfectly to the market in the first period. Denote by $p_1(z)$ the price of the long-term asset at time $t = 1$.[20] Also, denote by $p_2(z, \xi)$ the price of the long-term asset at $t = 2$ given the realization of the systemic component ξ. Since all banks need to be solvent with probability 1 at $t = 2$ the binding solvency constraint can be written through its worst case scenario as

$$m(z) + q(z) \cdot p_2(z, 0) = d, \qquad P - a.s. \tag{12}$$

where $m(z)$ is the capital requirement set by the regulator and $q(z) = n - \frac{m(z)-c}{p_1(z)}$ is the quantity of the long-term asset the bank is able to retain entering the second period. If b is sufficiently large, the cash in the market pricing constraint in the second period is given by

$$p_2(z, \xi) = m(z) + q(z) \cdot p_2(z, \xi) + \xi z + \mu(1 - z). \tag{13}$$

We assume that the cash in the market (13) is binding only when $\xi = 0$, then $p_2(z, 1) = b$ and $p_2(z, 0) = \frac{\mu}{n}(1 - z)$. This implies that, conditional on z, the price of the long-term asset at $t = 1$ is $p_1(z) = \delta \cdot [\mu b + (1 - \mu)p_2(z, 0)]$.

Given prices $p_1(z)$ and $p_2(z, \xi)$ it is easy to map them into the primitives of the model analyzed in Section 3. Downside risk $\tilde{l}$ is proportionate to $\tilde{z}$, and $p_1(z)$ is related to $p_1(l)$ through the expected asset pricing formula.

[20] We deliberately use the same notation for the price of the risky asset as in Section 3 as the two functions carry the same economic content and look very similar under the optimal capital requirements.

Define by z^f to be the level of systemic risk such that banks are solvent if they do not sell any of the long-term asset and z^w to be the level of risk at which the bank cannot remain safe even if they sell all of their long-term assets, i.e.:

$$c + n \cdot p_2(z^f, 0) = d, \qquad\qquad c + n \cdot p_1(z^w) = d.$$

The regulator seeks to maximize the welfare, which is equivalent to minimizing long-term assets sales. If $\tilde{z} = z$ is disclosed, she chooses the lowest $m(z)$ and, correspondingly, highest $q(z)$ to such that (12) is satisfied. Similarly to Section 3.1 the expected welfare of all banks is given by for $z \leq z^w$

$$\hat{W}(z) = c + \mu + n \cdot b - d - \frac{1-\delta}{\delta} \cdot (m(z) - c).$$

Function $\hat{W}(z)$, which is shown in in Figure 8, shares several qualitative features with $W(l)$ defined in Section 3.1: it is strictly convex in and decreasing when the risk is fairly high, $z \in (z^f, z^w)$, and it is flat when the risk is sufficiently small, $z < z^f$, because the banks are unconstrained in that region. This similarity underpins the following lemma, which is an analog of Proposition 1.

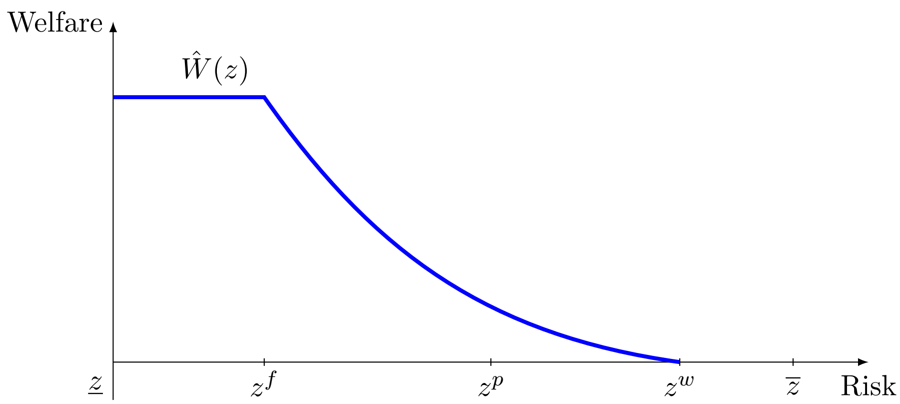

Figure 8: Expected Value to the Regulator if she discloses z and sets optimal capital requirements.

Lemma 4. *Define z^p to solve $\mathrm{E}[\tilde{z} \mid \tilde{z} \geq z^p] = z^w$ and suppose that $\frac{W(\underline{z})}{z^w - \underline{z}} \geq \frac{W(z^p)}{z^w - z^p}$. Then the optimal stress test is static with the optimal adverse scenario is given by*

$$\tilde{s}^* = \begin{cases} \tilde{z} & \text{if} \quad \tilde{z} < z^p, \\ z^w & \text{if} \quad \tilde{z} \geq z^p. \end{cases}$$

The optimal capital requirements are identical for all banks and given by

$$m^*(s) = c + \left[\frac{d - c - n \cdot p_2(s,0)}{p_1(s) - p_2(s,0)}\right]^+ \cdot p_1(s).$$

The intuition for the optimality of this stress test is analogous to that of Proposition 1: the optimal test must involve some pooling in order to keep the banks default free when $\tilde{z} > z^w$. When choosing which states to pool with $[z^w, \bar{z}]$ the regulator weighs the opportunity cost of pooling a particular state z $\hat{W}(z) - \hat{W}(z^w)$ against the benefit of increasing market belief $\mathrm{E}[\tilde{z}|\tilde{s}]$. The net cost of pooling a state z is equal to $[\hat{W}(z) - \hat{W}(z^w)]/(z^w - z)$, which is decreasing in z in the region (z^f, z^w), so pooling nearby states results in the highest bang for the buck.

Free-riding on Aggregate Liquidity. When there is a continuum of banks, an individual bank's asset choice does not impact prices $p_1(z)$ and $p_2(z,0)$. Specifically, each bank takes the outstanding asset quantity n^* as given and chooses how many assets to originate n without internalizing that this impacts $p_2(z,0)$. If all banks share the same origination cost $k(n)$, as in Section 4.1, each bank solves

$$\max_n \left[nb - k(n) - (1-\delta) \int_{z^f(n)}^{z^p(n)} \frac{k(n) - \frac{\mu}{n^*}(1-z)}{\delta(\mu b + \frac{\mu(1-\mu)}{n^*}(1-z)) - \frac{\mu}{n^*}(1-z)} f(z)\, dz - \frac{1-\delta}{\delta} \int_{z^p(n)}^{\bar{z}} k(n) f(z)\, dz \right]$$

Thus, if the regulator conducts the stress test in a subgame perfect way, there is inefficient overinvestment into the risky asset.

Heterogeneous Banks. The notion of a dynamic stress test can be naturally extended to a continuum of banks and uncertain correlation. For similar reasons as in Section 3.2 the stress test optimal for the weak banks is overly opaque for the strong banks. If the stronger banks have sufficient liquidity at $t = 1$, a dynamic stress test achieves the allocation of a representative bank, in all other cases dynamic test is welfare increasing relative to the static stress test derived in Lemma 4.

6 Conclusion

Bank regulation must be forward looking. A stress test is used to identify emerging risks and prepare the financial system for their eventuality. We show that in addition to being able to collect this information, it is crucial for the regulator to manage it correctly. The adverse scenario cannot be completely transparent as to not undermine the bank's ability to raise funds. Information policy and capital requirements work together in ensuring that banks are safe going forward at the lowest cost to the banking system and society. In this paper we analyze the optimal design of such tests. The optimal adverse scenario reveals low and intermediate levels of systemic risk, allowing for efficient asset allocation in those states, while not perfectly revealing the very risky states. When there is a lot of risk the regulator imposes strict capital requirements and banks raise large amounts of liquidity in the public markets. When banks are heterogeneous in their portfolios, dynamic disclosure of adverse scenario results is welfare improving. The weak bank is able to raise capital first and, after that, additional information gets disclosed leading to efficient recapitalization of the strong bank. In the presence of interbank trade whether a strong bank passes a stress test depends both on its own portfolio, but also on the portfolio of the weak bank. The regulator may need to limit the dividend policy of the banks to avoid capital leakage prior to the stress test, as well as channeling more banks to invest into the risky assets of their counter-parties. Dynamic information disclosure and capital requirements are essential for the regulator to achieve such flexibility.

Appendix

Proof of Proposition 1

This is a corollary of Proposition 3 when all the banks are identical.

Proof of Proposition 2

Lemma 5. *Suppose there is only one bank $I = 1$. The optimal dynamic stress test is static.*

Proof. We approach the problem by backward induction. We know if there is only 1 stress test period remaining, the optimal static stress test is characterized in Proposition 1. We then allow the regulator to require the bank sells a quantity of asset x prior to the information being revealed. We show that this is suboptimal. Thus, with 2 periods remaining it is efficient not to transact in the asset market. Thus running a second adverse scenario is unnecessary.

Since buying the asset back from the market is impossible, we can assume that $c = 0$. Suppose the bank first sells x at a pooling price $\delta\frac{2h-\mathrm{E}[\tilde{l}]}{2}$. Then reveal the level of risk if $\tilde{l} \leq l^p$ and sell y precautionary at the price $\delta \cdot \frac{2h-\mathrm{E}[\tilde{l}|\tilde{s}^*]}{2}$. Given the amount of asset x sold in the beginning, the ex-post budget constraint of the bank is

$$x \cdot \delta\frac{2h - \mathrm{E}[l]}{2} + y \cdot \delta\frac{2h - l}{2} + (n - x - y) \cdot (h - l) \geq d.$$

The above constraint is binding if $y(x) > 0$. Thus

$$y(x, l) = \left[\frac{d - x \cdot \delta\frac{2h-\mathrm{E}[l]}{2} - (n - x) \cdot (h - l)}{\delta\frac{2h-l}{2} - (h - l)}\right]^+.$$

Define $l^f(x)$ to be the threshold at which the bank retains $n - x$ units of the asset

$$\frac{d - x \cdot \delta\frac{2h-\mathrm{E}[\tilde{l}]}{2} - (n - x) \cdot (h - l^f(x))}{\delta\frac{2h-l^f(x)}{2} - (h - l^f(x))} = 0,$$

$$d - x \cdot \delta\frac{2h - \mathrm{E}\left[\tilde{l}\right]}{2} - (n - x) \cdot (h - l^f(x)) = 0.$$

This results in a threshold

$$l^f(x) = -\frac{d - x \cdot \delta \frac{2h - \mathrm{E}[\tilde{l}]}{2} - (n-x)h}{n-x} = h - \frac{d - x \cdot \delta \frac{2h - \mathrm{E}[\tilde{l}]}{2}}{n-x}.$$

If $l < l^f(x)$ then the quantity of the asset retained is $n - x$. If $l \geq l^f(x)$ the quantity of the asset retained is

$$\begin{aligned}
n - x - y(x, l) &= n - x - \frac{d - x \cdot \delta \frac{2h - \mathrm{E}[\tilde{l}]}{2} - (n-x) \cdot (h - l)}{\delta \frac{2h - l}{2} - (h - l)} \\
&= \frac{(n-x) \cdot \delta \frac{2h-l}{2} - d + x \cdot \delta \frac{2h - \mathrm{E}[\tilde{l}]}{2}}{\delta \frac{2h-l}{2} - (h-l)} \\
&= \frac{n \cdot \delta \frac{2h-l}{2} - d + x \cdot \delta \frac{l - \mathrm{E}[\tilde{l}]}{2}}{\delta \frac{2h-l}{2} - (h-l)}.
\end{aligned}$$

For a given l define threshold $l^w(x)$ as the solution (in l) to an equation

$$\begin{aligned}
n \cdot \delta \frac{2h - l^w(x)}{2} - d + x \cdot \delta \frac{l^w(x) - \mathrm{E}\left[\tilde{l}\right]}{2} &= 0, \\
l^w(x) \cdot \frac{\delta(x-n)}{2} + n \cdot \delta h - d - x \cdot \frac{\delta \mathrm{E}\left[\tilde{l}\right]}{2} &= 0, \\
l^w(x) \cdot (x - n) + 2nh - \frac{2}{\delta} d - x \cdot \mathrm{E}\left[\tilde{l}\right] &= 0.
\end{aligned}$$

This implies

$$l^w(x) = \frac{2nh - \frac{2}{\delta} d - x \mathrm{E}\left[\tilde{l}\right]}{n - x}.$$

Using this expression we can express the quantity of the asset retained by the bank as

$$\begin{aligned}
n - x - y(x) &= \frac{n \cdot \delta \frac{2h-l}{2} - d + x \cdot \delta \frac{l - \mathrm{E}[\tilde{l}]}{2}}{\delta \frac{2h-l}{2} - (h-l)}. \\
&= \frac{\frac{(n-x)\delta l^w(x)}{2} - \frac{(n-x)\delta l}{2}}{\delta \frac{2h-l}{2} - (h-l)} \\
&= \frac{\delta(n-x)(l^w(x) - l)}{\delta(2h - l) - 2(h-l)}
\end{aligned}$$

The derivative of $l^w(x)$ with respect to x is given by

$$\frac{d}{dx}l^w(x) = -\frac{\mathrm{E}\left[\tilde{l}\right]}{n-x} + \frac{l^w(x)}{n-x} = \frac{l^w(x) - \mathrm{E}\left[\tilde{l}\right]}{n-x} > 0.$$

Define $l^p(x)$ to be the cutoff level such that the conditional average above this cutoff is exactly equal to $l^w(x)$.

$$\frac{1}{1-F(l^p(x))} \cdot \int_{l^p(x)}^{\bar{l}} l f(l)\, dl = l^w(x) \qquad \Rightarrow \qquad \int_{l^p(x)}^{\bar{l}} l f(l)\, dl = l^w(x) \cdot \Big(1 - F(l^p(x))\Big). \tag{14}$$

Then the regulator's ex-ante value under the optimal test (dependent on x) is

$$\max_x \left[\int_{l^f(x)}^{l^p(x)} \frac{\delta(n-x)(l^w(x)-l)}{\delta(2h-l)-2(h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl + \int_{\underline{l}}^{l^f(x)} \left(c + x \cdot \delta \frac{2h - \mathrm{E}\left[l\right]}{2} - d + (n-x) \cdot \frac{2h-l}{2}\right) \cdot f(l)\, dl\right]$$

The derivative of the above expression respect to x is given by

$$\begin{aligned}
&\delta \int_{l^f(x)}^{l^p(x)} \frac{-l^w(x) + l + l^w(x) - \mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l)-2(h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl \\
&+ \int_{\underline{l}}^{l^f(x)} \left(\delta \frac{2h - \mathrm{E}\left[l\right]}{2} - \frac{2h-l}{2}\right) \cdot f(l)\, dl \\
&+ \delta \frac{(n-x)\cdot(l^w(x) - l^p(x))}{\delta(2h - l^p(x)) - 2(h - l^p(x))} \cdot \frac{l^p(x)}{2} \cdot f(l^p(x)) \cdot \frac{d}{dx} l^p(x)
\end{aligned}$$

Differentiating (14) with respect to x we obtain

$$\begin{aligned}
-l^p(x) \cdot f(l^p(x)) \cdot \frac{d}{dx} l^p(x) &= \Big(1 - F(l^p(x))\Big) \cdot \frac{d}{dx} l^w(x) - l^w(x) \cdot f(l^p(x)) \cdot \frac{d}{dx} l^p(x) \\
f(l^p(x)) \cdot \frac{d}{dx} l^p(x) &= \frac{l^w(x) - \mathrm{E}\left[\tilde{l}\right]}{n-x} \cdot \frac{1 - F(l^p(x))}{l^w(x) - l^p(x)}
\end{aligned}$$

Substituting $f(l^p(x)) \cdot \frac{d}{dx} l^p(x)$ into the first order welfare condition we obtain

$$\begin{aligned}
&\delta \int_{l^f(x)}^{l^p(x)} \frac{l - \mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l)-2(h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl + \int_{\underline{l}}^{l^f(x)} \left(\delta \frac{2h - \mathrm{E}\left[l\right]}{2} - \frac{2h-l}{2}\right) \cdot f(l)\, dl \\
&+ \delta \frac{(n-x)\cdot(l^w(x) - l^p(x))}{\delta(2h - l^p(x)) - 2(h - l^p(x))} \cdot \frac{l^p(x)}{2} \cdot \frac{l^w(x) - \mathrm{E}\left[\tilde{l}\right]}{n-x} \cdot \frac{1 - F(l^p(x))}{l^w(x) - l^p(x)}
\end{aligned}$$

We can simplify the last term

$$\delta\int_{l^f(x)}^{l^p(x)}\frac{l-\mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l)-2(h-l)}\cdot\frac{l}{2}\cdot f(l)\,dl+\int_{\underline{l}}^{l^f(x)}\left(\delta\frac{2h-\mathrm{E}\left[l\right]}{2}-\frac{2h-l}{2}\right)\cdot f(l)\,dl$$
$$+\frac{\delta}{2}\cdot\frac{l^p(x)\cdot\left(l^w(x)-\mathrm{E}\left[\tilde{l}\right]\right)(1-F(l^p(x)))}{\delta(2h-l^p(x))-2(h-l^p(x))}$$

We suppress explicit dependence of $l^p(x)$ and $l^w(x)$ for brevity and since we are looking at a first order condition. Note that

$$\begin{aligned}\mathrm{E}\left[\tilde{l}\right]&=\mathrm{E}\left[\tilde{l}\,|\,\tilde{l}\geq l^p\right](1-F(l^p))+\mathrm{E}\left[\tilde{l}\,|\,\tilde{l}<l^p\right]F(l^p)\\ \mathrm{E}\left[\tilde{l}\right](1-F(l^p))+\mathrm{E}\left[\tilde{l}\right]F(l^p)&=l^w(1-F(l^p))+\mathrm{E}\left[\tilde{l}\,|\,\tilde{l}<l^p\right]F(l^p)\\ \left(\mathrm{E}\left[\tilde{l}\right]-\mathrm{E}\left[\tilde{l}\,|\,\tilde{l}<l^p\right]\right)F(l^p)&=\left(l^w-\mathrm{E}\left[\tilde{l}\right]\right)(1-F(l^p))\end{aligned}$$

Thus, the derivative of welfare with respect to x can be written as

$$\delta\int_{l^f(x)}^{l^p(x)}\frac{l-\mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l)-2(h-l)}\cdot\frac{l}{2}\cdot f(l)\,dl+\int_{\underline{l}}^{l^f(x)}\left(\delta\frac{2h-\mathrm{E}\left[\tilde{l}\right]}{2}-\frac{2h-l}{2}\right)\cdot f(l)\,dl$$
$$+\frac{\delta}{2}\cdot\frac{l^p(x)\cdot\left(\mathrm{E}\left[\tilde{l}\right]-\mathrm{E}\left[\tilde{l}\,|\,\tilde{l}<l^p\right]\right)F(l^p)}{\delta(2h-l^p(x))-2(h-l^p(x))}$$

Consider the following sufficient condition for $l<l^f(x)$

$$\begin{aligned}\delta\cdot\frac{2h-\mathrm{E}\left[l\right]}{2}-\frac{2h-l}{2}&\leq\delta\cdot\frac{l-\mathrm{E}\left[l\right]}{\delta(2h-l)-2(h-l)}\frac{l}{2}\\ \left(\delta\cdot(2h-\mathrm{E}\left[l\right])-2h+l\right)\left(2h(\delta-1)+l(2-\delta)\right)&\leq\delta\cdot l(l-\mathrm{E}\left[l\right])\\ \left(2h(\delta-1)+l\right)\left(2h(\delta-1)+l(2-\delta)\right)&\leq\delta l^2+\delta\mathrm{E}\left[l\right]\left(-l+2h(\delta-1)+l(2-\delta)\right)\\ \left(2h(\delta-1)+l\right)\left(2h(\delta-1)+l(2-\delta)\right)&\leq\delta l^2+\delta\mathrm{E}\left[l\right]\left(2h(\delta-1)+l(1-\delta)\right)\\ \left(2h(\delta-1)+l\right)\left(2h(\delta-1)+l(2-\delta)\right)&\leq\delta l^2-\delta(1-\delta)\mathrm{E}\left[l\right]\left(2h-l\right)\\ 4h^2(1-\delta)^2+l^2(2-\delta)-2h(1-\delta)l(3-\delta)&\leq\delta l^2-\delta(1-\delta)\mathrm{E}\left[l\right]\left(2h-l\right)\\ 4h^2(1-\delta)^2+l^22(1-\delta)-2h(1-\delta)l(3-\delta)&\leq-\delta(1-\delta)\mathrm{E}\left[l\right]\left(2h-l\right)\\ 4h^2(1-\delta)+2l^2-2hl(3-\delta)&\leq-\delta\mathrm{E}\left[l\right]\left(2h-l\right)\end{aligned}$$

Sufficient condition holds for $l = h$

$$
\begin{aligned}
4(1-\delta) + 2 - 2(3-\delta) &\leq -\delta \\
-4\delta + 2\delta &\leq -\delta \\
-\delta &\leq 0
\end{aligned}
$$

This is satisfied. Thus for a sufficiently low range of l this holds.

If this holds, then

$$
\begin{aligned}
&\delta \int_{l^f(x)}^{l^p(x)} \frac{l - \mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l) - 2(h-l)} \cdot \frac{l}{2} \cdot f(l)\, dl + \int_{\underline{l}}^{l^f(x)} \left(\delta \frac{2h - \mathrm{E}\left[\tilde{l}\right]}{2} - \frac{2h-l}{2} \right) \cdot f(l)\, dl \\
&+ \frac{\delta}{2} \frac{l^p(x) \cdot \left(\mathrm{E}\left[\tilde{l}\right] - \mathrm{E}\left[\tilde{l} \,|\, \tilde{l} < l^p\right] \right) F(l^p)}{\delta(2h - l^p(x)) - 2(h - l^p(x))} \\
&\leq \delta \int_{\underline{l}}^{l^p(x)} \frac{l - \mathrm{E}\left[\tilde{l}\right]}{\delta(2h-l) - 2l} \cdot \frac{l}{2} \cdot f(l)\, dl + \frac{\delta}{2} \frac{l^p(x) \cdot \left(\mathrm{E}\left[\tilde{l}\right] - \mathrm{E}\left[\tilde{l} \,|\, \tilde{l} < l^p\right] \right) F(l^p)}{\delta(2h - l^p(x)) - 2(h - l^p(x))}
\end{aligned}
$$

The function under the integral is given by

$$
\begin{aligned}
\frac{l - \mathrm{E}[l]}{\delta(2h-l) - 2(h-l)} \cdot \frac{l}{2} &= \frac{l - \mathrm{E}[l]}{2h(\delta-1) + l(2-\delta)} \cdot \frac{l}{2} \\
&= \frac{l + h\frac{2(\delta-1)}{2-\delta} - h\frac{2(\delta-1)}{2-\delta} - \mathrm{E}[l]}{2h(\delta-1) + l(2-\delta)} \cdot \frac{l}{2} \\
&= \frac{l}{2(2-\delta)} + \frac{h\frac{2(1-\delta)}{2-\delta} - \mathrm{E}[l]}{2} \cdot \frac{l}{2h(\delta-1) + l(2-\delta)} \\
&= \frac{l}{2(2-\delta)} + \frac{h\frac{2(1-\delta)}{2-\delta} - \mathrm{E}[l]}{2} \cdot \frac{l + h\frac{2(\delta-1)}{2-\delta} - h\frac{2(\delta-1)}{2-\delta}}{2h(\delta-1) + l(2-\delta)} \\
&= \frac{l}{2(2-\delta)} + \frac{1}{2-\delta} \frac{h\frac{2(1-\delta)}{2-\delta} - \mathrm{E}[l]}{2} + \frac{h \cdot \left(h \cdot \frac{2(1-\delta)}{2-\delta} - \mathrm{E}[l] \right) \cdot \frac{1-\delta}{2-\delta}}{l(2-\delta) - 2h(1-\delta)}
\end{aligned}
$$

Then if $\mathrm{E}[l] \geq \underline{l} \geq h\frac{2(1-\delta)}{2-\delta}$ the above expression is concave. This condition is always satisfied when $\delta(h - \frac{l}{2}) > h - \underline{l}$ when it is profitable for the bank to sell assets in the market and suffer the discount, rather than obtaining the lowest realization of the signal. Applying Jensen's inequality to this function we obtain

$$\int_{\underline{l}}^{l^p} \frac{l - \mathrm{E}[l]}{\delta(2h-l)-2l} \cdot \frac{l}{2} \cdot f(l)\,dl + \frac{l^p}{2} \frac{\left(\mathrm{E}\left[\tilde{l}\right] - \mathrm{E}\left[\tilde{l}\,|\,\tilde{l} < l^p\right]\right) F(l^p)}{\delta(2h-l^p)-2(h-l^p)}$$

$$\leq \left[\frac{\mathrm{E}[l|l<l^p] - \mathrm{E}[l]}{\mathrm{E}[l|l<l^p](2-\delta) - 2h(1-\delta)} \cdot \frac{\mathrm{E}[l|l<l^p]}{2} + \frac{l^p}{2} \cdot \frac{\mathrm{E}\left[\tilde{l}\right] - \mathrm{E}\left[\tilde{l}\,|\,\tilde{l} < l^p\right]}{l^p(2-\delta)-2h(1-\delta)}\right] \cdot F(l^p)$$

$$= \frac{(\mathrm{E}[l] - \mathrm{E}[l|l<l^p]) \cdot F(l^p)}{2} \cdot \left(-\frac{\mathrm{E}[l|l<l^p]}{\mathrm{E}[l|l<l^p](2-\delta)-2h(1-\delta)} + \frac{l^p}{l^p(2-\delta)-2h(1-\delta)}\right) < 0$$

Suppose the optimal dynamic stress test for two banks features a sequence of J adverse scenarios $\{\tilde{s}_j\}_{j=1}^J$. Clearly, $\tilde{s}_J$ must be the static-optimal adverse scenario characterized by a threshold rule. The above implies that between $J-1$ and J the bank does not sell assets and, thus, it is without loss that information in the pair $\{\tilde{s}_{J-1}, \tilde{s}_J\}$ is revealed jointly. Since this corresponds to a static-test disclosure it follows that $\{\tilde{s}_{J-1}, \tilde{s}_J\}$ follows a static-optimal threshold disclosure rule. Thus, the dynamic stress test can be implemented with $J-1$ adverse scenarios. □

Based on Lemma 5 bank i achieves its optimum under a static stress test if such are available. By conducting these static stress tests sequentially from weakest to strongest we obtain the individual optimal welfare payoff for each bank i.

Proof of Proposition 3

Suppose that the optimal static stress test is $(\tilde{s}^*, q_i^*(s))$. Now we can characterize some properties of this test. Let $F(l|s) = \mathrm{P}\left(\tilde{l} \leq l | \tilde{s} = s\right)$ be a conditional cdf of $\tilde{l}$. Define the worst case scenario conditional on $\tilde{s}$ as

$$\bar{l}(\tilde{s}) = \sup\{l \,:\, F(l|\tilde{s}) < 1\}.$$

Lemma 6. *$\bar{l}(\tilde{s})$ is a random variable.*

Proof. We have

$$\{\omega : \bar{l}(\tilde{s}) < x\} = \left\{\omega : \mathrm{P}\left(\tilde{l} \geq x \,|\, \tilde{s}\right) = 1\right\} = \left\{\omega : F\left(x \,|\, \tilde{s}\right) = 1\right\} \in \mathcal{F},$$

since function $F(l|\tilde{s})$ is measurable. □

Recall that $q_i^*(\tilde{s})$ is given by

$$q_i^*(\tilde{s}) = \frac{c_i - d_i + n_i\delta \mathrm{E}\left[v|\tilde{s}\right]}{\delta \mathrm{E}\left[v|\tilde{s}\right] - (h - \bar{l}(\tilde{s}))} = \frac{c_i - d_i + n_i\delta\left(h - \frac{1}{2}\mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right)}{\delta\left(h - \frac{1}{2}\mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) - (h - \bar{l}(\tilde{s}))} = q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right).$$

The value of the asset given signal $\tilde{s}$ is given by

$$\mathrm{E}\left[\tilde{v}|\tilde{s}\right] = h - \frac{1}{2}\mathrm{E}\left[\tilde{l}|\tilde{s}\right].$$

Thus, the welfare conditional on $\tilde{s}$ generated by bank i can be written as

$$w_i\left(q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) = \begin{cases} q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right) \cdot \frac{\bar{l}(\tilde{s})}{2} & \text{if} \quad q_i^*(\tilde{s}) < n_i, \\ n_i \cdot \left(h - \frac{1}{2}\mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) + (1-\beta)\cdot(c_i + n_i\cdot(h - \bar{l}(\tilde{s})) - d_i) & \text{if} \quad q_i^*(\tilde{s}) \geq n_i. \end{cases}$$

Total welfare conditional on $\tilde{s}$ is

$$w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) = \sum_i w_i\left(q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right).$$

Define the pooling set of a signal $\tilde{s}$ to be

$$S_p = \left\{\omega : \mathrm{Var}\left(\tilde{l} \mid \tilde{s}\right) > 0\right\} \in \mathcal{F}.$$

Lemma 7. *For all pooling outcomes S_p^* of the optimal test $\tilde{s}^*$ the worst case scenario $\bar{l}(s)$ should be above l_1^w:*

$$\mathrm{P}\big(\bar{l}(\tilde{s}^*) \geq l_1^w \mid S_p^*\big) = 1.$$

Proof. From the contrary, suppose that $\mathrm{P}\big(\bar{l}(\tilde{s}^*) < l_1^w \mid \tilde{s}^* \in S_p^*\big) > 0$. Then we can improve on such test by disclosing $\tilde{l}$ perfectly in states of the world $G = \{\bar{l}(\tilde{s}^*) < l_1^w\} \cap S_p^*$. The original welfare function can be written as:

$$\begin{aligned} W &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right)\right] \\ &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right)\mathbb{1}\{G\}\right] + \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right)\mathbb{1}\{\Omega \setminus G\}\right] \end{aligned}$$

Similar to $\bar{l}(\tilde{s})$ define

$$\underline{l}(\tilde{s}) = \inf\{l : F(l|\tilde{s}) > 0\}.$$

Suppose that for bank i we have $h - \bar{l}(\tilde{s}) \leq \frac{d_i - m_i}{n_i}$. This means bank i needs to sell some assets under signal $\tilde{s}$. Since the welfare increases with the rise of the worst case scenario we have

$$w_i\left(q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) < w_i\left(q_i^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right).$$

for any informative signal $\tilde{\xi}$ shared by the regulator. Due to the convexity of $w(q^*(l,l),l)$ in $[\underline{l}(s), \bar{l}(s)]$ Jensen's inequality implies:

$$W < \mathrm{E}\left[w_i\left(q_i^*\left(\tilde{l}, \tilde{l}\right), \tilde{l}\right) \cdot \mathbb{1}\{G\}\right] + \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}\right], \bar{l}(\tilde{s})\right), \mathrm{E}\left[\tilde{l}|\tilde{s}\right]\right) \cdot \left(1 - \mathbb{1}\{G\}\right)\right]$$

for any informative signal $\tilde{s}$ disclosed by the regulator when event G occurs.

On the other hand, suppose that for bank i we have $h - \bar{l}(\tilde{s}) > \frac{d_i - m_i}{n_i}$ which implies that bank i could be paying out dividends. Conditional on S_p^* the regulator reveals an additional signal $\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} < \underline{l}(s) + \varepsilon\right\}$. This event occurs with probability $F(\underline{l}(s) + \varepsilon) > 0$ by definition of $\underline{l}(s)$. Then, for a sufficiently small ε the welfare from this additional signal is given by

$$\begin{aligned}&\Big(1 - F(\underline{l}(s) + \varepsilon)\Big) \cdot w\left(\frac{1}{1 - F(\underline{l}(s) + \varepsilon)} \int_{\underline{l}(s)+\varepsilon}^{\bar{l}(s)} y\, d\phi(y|s),\ \bar{l}(s)\right) \cdot \frac{1}{2(1 - F(\underline{l}(s) + \varepsilon))} \cdot \int_{\underline{l}(s)+\varepsilon}^{\bar{l}(s)} y\, d\phi(y|s) \\ &+ F(\underline{l}(s) + \varepsilon)\left[n_i \cdot \left(h - \frac{1}{2F(\underline{l} + \varepsilon)} \int_{\underline{l}(s)}^{\underline{l}(s)+\varepsilon} y\, d\phi(y|s)\right) + (1 - \beta)\left(c_i + n_i \cdot \left(h - \frac{1}{F(\underline{l}(s) + \varepsilon)} \int_{\underline{l}(s)}^{\underline{l}(s)+\varepsilon} y\, d\phi(y|s)\right) - d_i\right)\right]\end{aligned}$$

Define

$$x_0(\varepsilon) = \frac{1}{1 - F(\underline{l}(s) + \varepsilon)} \int_{\underline{l}(s)+\varepsilon}^{\bar{l}(s)} y\, dF(y|s).$$

The derivative with respect to ε at $\varepsilon = 0$ is given by

$$x_0'(0) = -\underline{l}(s)F'(\underline{l}(s)) + x_0(0)F'(\underline{l}(s)) = \Big(x_0(0) - \underline{l}(s)\Big)F'(\underline{l}(s)).$$

The derivative with respect to ε at $\varepsilon = 0$ is given by

$$\begin{aligned}&-dF(\underline{l}(s)) \cdot q_i^*(x_0, \bar{l}(s)) \cdot \frac{x_0(0)}{2} + \left(\partial_x q_i^*(x_0, \bar{l}(s))\frac{x_0}{2} + q_i^*(x_0, \bar{l}(s))\frac{1}{2}\right) \cdot F'(\underline{l}(s))(x_0(0) - \underline{l}(s)) \\ &+ dF(\underline{l}(s)) \cdot \left(n_i \frac{\underline{l}(s)}{2} + (1 - \beta)\left(c_i + n_i(h - \underline{l}(s)) - d_i\right)\right)\end{aligned}$$

I want the above term to be positive.

$$
\begin{aligned}
&-q_i^*(x_0,\bar{l}(s))\cdot\frac{x_0(0)}{2}+\left(\partial_x q_i^*(x_0,\bar{l}(s))\frac{x_0(0)}{2}+q_i^*(x_0,\bar{l}(s))\frac{1}{2}\right)\cdot(x_0(0)-\underline{l}(s))\\
&+n_i\frac{\underline{l}(s)}{2}+(1-\beta)\left(c_i+n_i(h-\underline{l}(s))-d_i\right)\vee 0
\end{aligned}
$$

Then

$$
\begin{aligned}
-q_i^*(x_0,\bar{l}(s))\cdot\frac{\underline{l}(s)}{2}+\partial_x q_i^*(x_0,\bar{l}(s))\frac{x_0(0)}{2}(x_0(0)-\underline{l}(s))+n_i\frac{\underline{l}(s)}{2}+(1-\beta)(c_i+n_i(h-\underline{l}(s))-d_i)\vee 0\\
(n_i-q_i^*(x_0,\bar{l}(s)))\frac{\underline{l}(s)}{2}-\partial_x q_i^*(x_0,\bar{l}(s))\frac{x_0(0)}{2}(\underline{l}(s)-x_0(0))+(1-\beta)(c_i+n_i(h-\underline{l}(s))-d_i)\vee 0
\end{aligned}
$$

Given that

$$
\begin{aligned}
q_i(x,l)&=\frac{c_i-d_i+n_i\delta(h-\frac{x}{2})}{\delta(h-\frac{x}{2})-(h-l)}=n_i-\frac{d_i-c_i-n_i(h-l)}{\delta(h-\frac{x}{2})-(h-l)}\\
\partial_x q_i(x,l)&=-\frac{\delta}{2}\cdot\frac{d_i-c_i-n_i(h-l)}{(\delta(h-\frac{x}{2})-(h-l))^2}
\end{aligned}
$$

Then the above becomes

$$
\frac{d_i-m_i-n_i(h-\bar{l}(s))}{\delta(h-\frac{x_0(0)}{2})-(h-\bar{l}(s))}\cdot\frac{\underline{l}(s)}{2}+\frac{\delta}{2}\cdot\frac{d_i-m_i-n_i(h-\bar{l}(s))}{(\delta(h-\frac{x_0}{2})-(h-\bar{l}(s))^2}\cdot\frac{x_0(0)}{2}\cdot(\underline{l}(s)-x_0)+\gamma(c_i+n_i(h-\underline{l}(s))-d_i)\vee 0
$$

A sufficient condition for the above term to be positive is

$$
\begin{aligned}
\frac{d_i-m_i-n_i(h-\bar{l}(s))}{\delta(h-\frac{x_0(0)}{2})-(h-\bar{l}(s))}\cdot\frac{\underline{l}(s)}{2}+\frac{\delta}{2}\cdot\frac{d_i-m_i-n_i(h-\bar{l}(s))}{(\delta(h-\frac{x_0}{2})-(h-\bar{l}(s))^2}\cdot\frac{x_0(0)}{2}\cdot(\underline{l}(s)-x_0)&\geq 0\\
\frac{\underline{l}(s)}{2}+\frac{\delta}{2}\frac{1}{\delta(h-\frac{x_0}{2})-(h-\bar{l}(s))}\frac{x_0}{2}(\underline{l}(s)-x_0)&\geq 0\\
\underline{l}(s)\left(\delta(h-\frac{x_0}{2})-(h-\bar{l}(s))\right)+\delta\frac{x_0}{2}(\underline{l}(s)-x_0)&\geq 0\\
\underline{l}(s)\left(\delta h-(h-\bar{l}(s))\right)-\delta\frac{x_0^2}{2}&\geq 0
\end{aligned}
$$

This expression is clearly decreasing in x_0. Take $x_0=\bar{l}(s)$

$$
\begin{aligned}
\underline{l}(s)(\delta h-h+\bar{l}(s))-\frac{\delta}{2}\bar{l}(s)^2&\geq 0\\
-(1-\delta)h\underline{l}(s)+\underline{l}(s)\bar{l}(s)-\frac{\delta}{2}\bar{l}(s)^2&\geq 0
\end{aligned}
$$

The above expression always holds at $\bar{l}(s) = \underline{s}$:

$$
\begin{aligned}
-(1-\delta)h + \underline{l}(s) - \frac{\delta}{2}\underline{l}(s) \geq 0 \\
\delta(h - \frac{1}{2}\underline{l}(s)) \geq h - \underline{l}(s)
\end{aligned}
$$

which holds by a parameter assumption. Following this, we need to verify that it holds at $\bar{l}(s) = \bar{l}$

$$
-(1-\delta)h\underline{l}(s) + \underline{l}(s)\bar{l} - \frac{\delta}{2}\bar{l}^2 \geq 0.
$$

This expression is increasing in $\underline{l}(s)$ and thus it must hold at $\underline{l}(s) = \underline{l}$:

$$
-(1-\delta)h\underline{l} + \underline{l}\bar{l} - \frac{\delta}{2}\bar{l}^2 \geq 0.
$$

□

Lemma 8. *Expected $\tilde{l}$ for any pooling realization of $\tilde{s}^*$ is always l_1^w, i.e.*

$$
\mathbb{1}\left\{S_p^*\right\} \cdot \left[\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}^*\right] - l_1^w\right] = 0 \qquad P - a.s.
$$

Proof. Clearly $\mathrm{P}\Big(S_p^*, \, \mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}^*\right] > l_1^w\Big) = 0$, because otherwise the weak bank fails with positive probability. Suppose that

$$
\mathrm{P}\Big(S_p^*, \, \mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}^*\right] < l_1^w\Big) > 0,
$$

then we can construct an improvement by disclosing the best realizations of $\tilde{l}$ conditional on $\tilde{s}$ since the pooling average will not fall above l_1^w. Formally define $\hat{l}(s), \alpha(s)$ as the unique solution of

$$
\int_{\hat{l}}^{\bar{l}(s)} l dF(l|s) + \alpha\hat{l}\left(F(\hat{l}|s) - F(\hat{l}-|s)\right) = l_1^w\left[1 - F(\hat{l}-|s) + \alpha\left(F(\hat{l}|s) - F(\hat{l}-|s)\right)\right],
$$

if $F(\hat{l}|s) = F(\hat{l}-|s)$ put $\alpha(s) = 0$. Then by construction above

$$
\mathbb{1}\left\{S_p^*\right\} \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,\middle|\, \tilde{s}^*, \, \mathbb{1}\left\{\tilde{l} > \hat{l}(\tilde{s}^*), \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} \leq \alpha(\tilde{s}^*)\right\}\right] = 0, \quad \mathrm{P} - a.s.
$$

where $\tilde{u} \sim U[0,1]$ is independent of $\tilde{l}, \tilde{v}, \tilde{s}^*$.

Since the residual set $\{\tilde{l} < \hat{l}(\tilde{s}^*)$, or $\tilde{l} = \hat{l}(\tilde{s}^*)$ and $\tilde{u} > \alpha(\tilde{s}^*)\} \cap S_p^*$ has a positive probability disclosing $\tilde{l}$ here creates an improvement due to the same argument as in Lemma 7 of $w(q^*(l,l),l)$.

This contradicts the optimality of $\tilde{s}^*$. $\square$

Lemma 9. *If $\tilde{s}^*$ is an optimal stress test, then the test*

$$\tilde{s}' = \begin{cases} \bar{l}(\tilde{s}^*), & \text{if} \quad S_p^*, \\ \tilde{l}, & \text{if} \quad \Omega \setminus S_p^* \end{cases}$$

is also optimal.

Proof. Write the welfare function under $\tilde{s}^*$ as

$$\begin{aligned} W &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}^*\right], \bar{l}(\tilde{s}^*)\right), \mathrm{E}\left[\tilde{l}|\tilde{s}^*\right]\right)\right] \\ &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}^*\right], \bar{l}(\tilde{s}^*)\right), \mathrm{E}\left[\tilde{l}|\tilde{s}^*\right]\right) \mathbb{1}\left\{S_p^*\right\}\right] + \mathrm{E}\left[w\left(q^*\left(\tilde{l}, \tilde{l}\right), \tilde{l}\right) \mathbb{1}\left\{\Omega \setminus S_p^*\right\}\right] \end{aligned}$$

Due to Lemma 8 we can replace $\mathrm{E}\left[\tilde{l} \mid \tilde{s}^*\right]$ with l_1^w in the first term

$$= \mathrm{E}\left[w\left(q^*\left(l_1^w, \bar{l}(\tilde{s}^*)\right), l_1^w\right) \mathbb{1}\left\{S_p^*\right\}\right] + \mathrm{E}\left[w\left(q^*\left(\tilde{l}, \tilde{l}\right), \tilde{l}\right) \mathbb{1}\left\{\Omega \setminus S_p^*\right\}\right]$$

next rewrite the events S_p^* and $\Omega \setminus S_p^*$ using the definition of $\tilde{s}'$

$$= \mathrm{E}\left[w\left(q^*\left(l_1^w, \bar{l}(\tilde{s}^*)\right), l_1^w\right) \mathbb{1}\left\{\tilde{s}' = \bar{l}(\tilde{s}^*)\right\}\right] + \mathrm{E}\left[w\left(q^*\left(\tilde{l}, \tilde{l}\right), \tilde{l}\right) \mathbb{1}\left\{\tilde{s}' = \tilde{l}\right\}\right]$$

next, Lemma 8 together with the law of iterated expectations imply that $\mathrm{E}\left[\tilde{l} \mid \tilde{s}'\right] = l_1^w$ when $\tilde{s}' = \bar{l}(\tilde{s}^*)$, hence

$$\begin{aligned} &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}'\right], \bar{l}(\tilde{s}')\right), \mathrm{E}\left[\tilde{l}|\tilde{s}'\right]\right) \mathbb{1}\left\{\tilde{s}' = \bar{l}(\tilde{s}^*)\right\}\right] + \mathrm{E}\left[w\left(q^*\left(\tilde{l}, \tilde{l}\right), \tilde{l}\right) \mathbb{1}\left\{\tilde{s}' = \tilde{l}\right\}\right] \\ &= \mathrm{E}\left[w\left(q^*\left(\mathrm{E}\left[\tilde{l}|\tilde{s}'\right], \bar{l}(\tilde{s}')\right), \mathrm{E}\left[\tilde{l}|\tilde{s}'\right]\right)\right]. \end{aligned}$$

This chain of equalities show that $\tilde{s}'$ achieves exactly the same ex-ante welfare W as does $\tilde{s}^*$. $\square$

Lemma 10. *The optimal test $\tilde{s}^*$ does not pool any of the two values $\tilde{l}$ in $[l_1^w, \bar{l}]$ together, i.e.*

$$\mathrm{P}\left(\tilde{l} = \bar{l}(\tilde{s}^*) \,\middle|\, \tilde{s}^*, \tilde{l} > l_1^w\right) = 1 \qquad \mathrm{P} - a.s.$$

Proof. Suppose the opposite, i.e. that there exists a subset $S' \subseteq S_p^*$ and a $\varepsilon > 0$ such that

$$\mathrm{P}\big(\tilde{s}^* \in S'\big) > 0 \quad \text{and} \quad \mathrm{P}\Big(\mathrm{E}\left[\tilde{l} \,\middle|\, \tilde{s}^*, \tilde{l} > l_1^w\right] < \bar{l}(\tilde{s}^*) - \varepsilon \,\Big|\, \tilde{s}^* \in S'\Big) = 1$$

For any $s \in S'$ the following inequalities are satisfied

$$0 < F(l_1^w - \,|\, s) < F(\bar{l}(s) - \varepsilon \,|\, s) < 1$$

the first one due to the definition of $\bar{l}(s)$, the second due to the definition of ε and the last one due to Lemma 8. Next $\hat{l}(s)$, $\alpha(s)$ as the unique solution of

$$\int_{\hat{l}}^{\bar{l}(s)-\varepsilon} l dF(l|s) + \alpha\hat{l}\left(F(\hat{l}|s) - F(\hat{l} - |s)\right) = l_1^w\left[F(\bar{l}(s) - \varepsilon) - F(\hat{l} - |s) + \alpha\left(F(\hat{l}|s) - F(\hat{l} - |s)\right)\right],$$

if $F(\hat{l}|s) = F(\hat{l} - |s)$ put $\alpha(s) = 0$, then by the construction above

$$0 < \mathrm{P}\Big(\tilde{l} \in [\hat{l}(\tilde{s}^*), \bar{l}(\tilde{s}^*) - \varepsilon], \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} < \alpha(\tilde{s}^*) \Big|\, \tilde{s}^* \in S'\Big) < 1$$

and

$$\begin{aligned} 0 &= \mathbb{1}\left\{\tilde{s}^* \in S'\right\} \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{s}^*, \tilde{l} \in [\hat{l}(\tilde{s}^*), \bar{l}(\tilde{s}^*) - \varepsilon], \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} < \alpha(\tilde{s}^*)\right] \\ &= \mathbb{1}\left\{\tilde{s}^* \in S'\right\} \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{s}^*, \tilde{l} \notin [\hat{l}(\tilde{s}^*), \bar{l}(\tilde{s}^*) - \varepsilon], \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} \geq \alpha(\tilde{s}^*)\right] \end{aligned}$$

Then refine the signal $\tilde{s}^* = s$ with the information of whether

$$\{\tilde{l} \in [\hat{l}(\tilde{s}^*), \bar{l}(\tilde{s}^*) - \varepsilon], \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} < \alpha(\tilde{s}^*)\}$$

whenever $\tilde{s}^* \in S'$ and call this more informative signal $\tilde{s}'$. Notice that capital requirements q^* for the optimal test $\tilde{s}^*$ are still feasible, i.e., lead to no defaults, under $\tilde{s}'$ because $\mathrm{E}\left[\tilde{l} \,|\, \tilde{s}'\right] = \mathrm{E}\left[\tilde{l} \,|\, \tilde{s}^*\right]$ and $\varepsilon > 0$ by construction.

Improvement in welfare can be achieved by relaxing liquidity constraint for the stronger banks on a set of positive probability $\{\tilde{s}^* \in S'\} \cap \{\tilde{l} \in [\hat{l}(\tilde{s}^*), \bar{l}(\tilde{s}^*) - \varepsilon], \text{ or } \tilde{l} = \hat{l}(\tilde{s}^*) \text{ and } \tilde{u} < \alpha(\tilde{s}^*)\}$ since the worst case scenario on this set is strictly less severe than $\underline{l}(\tilde{s}^*)$. $\square$

Lemma 11. *Suppose there are two signal realizations $\tilde{s} \in \{x_1, x_2\}$ such that $x_1 > l_1^w$, $\mathrm{P}\Big(\tilde{l} < l_1^p \,|\, \tilde{s} = x_1\Big) >$*

0, *and* $x_2 \in [l_1^p, l_1^w]$ *is fully revealing. There exists a signal* $\hat{s}$ *which is strictly better in the same set of outcomes.*

Proof. Ex-ante welfare can be written as

$$
\begin{aligned}
&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) + \mathrm{P}_{x_1,x_2}(\tilde{s} = x_2) \cdot w(q^*(x_2, x_2), x_2) \\
=&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) \cdot \left[\mathrm{P}\left(\tilde{l} = x_1 | \tilde{s} = x_1\right) + \mathrm{P}\left(\tilde{l} < l_1^w | \tilde{s} = x_1\right)\right] + \mathrm{P}_{x_1,x_2}(\tilde{s} = x_2) \cdot w(q^*(x_2, x_2), x_2)
\end{aligned}
$$

By rewriting the conditional probabilities this can be simplified to

$$
\begin{aligned}
&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) + \mathrm{P}_{x_1,x_2}(\tilde{s} = x_2) \cdot w(q^*(x_2, x_2), x_2) \\
=&p \cdot w(q^*(l_1^w, x_1), l_1^w) + (1 - p) \cdot w(q^*(x_2, x_2), x_2)
\end{aligned}
$$

where

$$p = \mathrm{P}_{x_1,x_2}(\tilde{s} = x_1)\,.$$

Consider a new signal $\hat{s}$ such that

$$
\hat{s}(\omega) = \begin{cases} x_1 & \text{if} \quad \omega \in \{\tilde{l} = x_1, \tilde{s} = x_1\} \cup \{\tilde{l} \in (x_2, l_1^w), \tilde{s} = x_1\} \cup \{\tilde{l} = x_2, \tilde{s} \in \{x_1, x_2\}, \tilde{u}_1 < \alpha(x_1, x_2)\}, \\ \tilde{l} & \text{if} \quad \omega \in \{\tilde{s} \in \{x_1, x_2\}\} \setminus \{\hat{s} = x_1\}. \end{cases}
$$

Choose $\alpha(x, z)$ such that

$$\mathrm{E}\left[\tilde{l} \,\middle|\, \hat{s} = x_1\right] = l_1^w.$$

The welfare generated by this signal is given by

$$
\begin{aligned}
&\mathrm{P}_{x_1,x_2}(\hat{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) + \mathrm{P}_{x_1,x_2}(\hat{s} < x_1) \cdot \mathrm{E}_{x_1,x_2}\left[w(q^*(\tilde{l}, \tilde{l}), \tilde{l}) \,\middle|\, \hat{s} < x_1\right] \\
=&q \cdot w(q^*(l_1^w, x_1), l_1^w) + (1 - q) \cdot \mathrm{E}\left[\tilde{w}(q^*(\tilde{l}, \tilde{l}), \tilde{l})\right]
\end{aligned}
$$

where

$$q = \mathrm{P}_{x_1,x_2}(\hat{s} < x_1)\,.$$

The difference between old and new welfare is given by

$$pw^* + (1 - p)w(x_2) - qw^* - (1 - q)\mathrm{E}\left[w(\tilde{l})\right] \vee 0$$

subject to the constraint that the expected value of $\tilde{l}$ must be equal under the two signals

$$p \cdot l_1^w + (1-p) \cdot x_2 = q \cdot l_1^w + (1-q) \cdot \mathrm{E}\left[\tilde{l} \,|\, \hat{s} < x_1\right].$$

Define $l = \mathrm{E}\left[\tilde{l} \,|\, \hat{s} < x_1\right]$. Then

$$q = \mathrm{P}_{x_1,x_2}(\hat{s} = x_1) = \frac{l - pl_1^w - (1-p)x_2}{l - l_1^w}.$$

Then

$$\begin{aligned}
&pw^* + (1-p)w(x_2) - qw^* - (1-q)\mathrm{E}\left[w(\tilde{l})\right] \\
\leq &pw^* + (1-p)w(x_2) - qw^* - (1-q)\mathrm{E}\left[w(x_2) + w'(x_2)(\tilde{l} - x_2)\right] \\
= &pw^* + (1-p)w(x_2) - qw^* - (1-q)\Big(w(x_2) + w'(x_2)(l - x_2)\Big) \\
= &(q-p)w(x_2) - (q-p)w^* - (1-q)w'(x_2)(l - x_2)
\end{aligned}$$

Note that

$$\begin{aligned}
q - p &= \frac{l - pl_1^w - (1-p)x_2}{l - l_1^w} - p = \frac{l - lp - (1-p)x_2}{l - l_1^w} = \frac{(1-p)(l - x_2)}{l - l_1^w} \\
1 - q &= 1 - \frac{l - pl_1^w - (1-p)x_2}{l - l_1^w} = \frac{(1-p)(x_2 - l_1^w)}{l - l_1^w}
\end{aligned}$$

Substuting in the above expression

$$\begin{aligned}
\frac{(1-p)(l-x_2)}{l - l_1^w}w(x_2) - \frac{(1-p)(l-x_2)}{l - l_1^w}w^* - \frac{(1-p)(x_2 - l_1^w)(l - x_2)}{l - l_1^w}w'(x_2) &\vee 0 \\
\frac{1}{l - l_1^w}w(x_2) - \frac{1}{l - l_1^w}w^* - \frac{(x_2 - l_1^w)}{l - l_1^w}w'(x_2) &\vee 0 \\
w(x_2) - w^* - (x_2 - l_1^w)w'(x_2) &\vee 0 \\
w(x_2) - (x_2 - l_1^w)w'(x_2) - w(q^*(l_1^w, x_1), l_1^w) &\vee 0
\end{aligned}$$

For the above expression to be negative it is sufficient to verify it for $x_1 = \bar{l}$. Thus

$$w(x_2) - (x_2 - l_1^w)w'(x_2) - w(q^*(l_1^w, \bar{l}), l_1^w) \leq 0.$$

□

Proposition 5. *The optimal signal always reveals $\tilde{l} \in [\underline{l}, l^p]$.*

Proof. According to the previous proposition it must be the case that every $\tilde{l} \in [l^p, l_1^w]$ is pooled with some other outcome. This implies that outcomes in $[l_p, \bar{l}]$ are pooled together. This, in turn, implies that only these outcomes need to be pooled. $\square$

Lemma 12. *Conditional on signal realizations $\tilde{s} \in \{x_1, x_2\}$ denote the two implied conditional distributions $\phi_i(y) = \mathrm{P}\Big(\tilde{l} \leq y \,|\, \tilde{s} = x_i, \, \tilde{l} < l_1^w\Big)$ for $i = 1, 2$, if $x_1 > x_2 > l_1^w$. Conditional on receiving signal $\tilde{s} \in \{x_1, x_2\}$ the welfare can be improved unless there exists $z^* \in (l^p, l_1^w)$ such that*

$$\phi_1(z^*) = 1 = 1 - \phi_2(z^*-),$$

the support of ϕ_2 is above the support of ϕ_1 and the two supports can only intersect at one point z^*.

Proof. Suppose the opposite, i.e. that the two supports have a non-trivial intersection, or that the support of ϕ_1 is fully above the support of ϕ_2.

Then conditional on $\tilde{s} = x_i$ the welfare is given by $w(q^*(l_1^w, x_i), l_1^w)$, hence we can write the welfare conditional on $\tilde{s} \in \{x_1, x_2\}$ as

$$\begin{aligned}
&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) + \mathrm{P}_{x_1,x_2}(\tilde{s} = x_2) \cdot w(q^*(l_1^w, x_2), l_1^w) \\
=&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_1) \cdot w(q^*(l_1^w, x_1), l_1^w) \cdot \Big[\mathrm{P}\Big(\tilde{l} = x_1 \,|\, \tilde{s} = x_1\Big) + \mathrm{P}\Big(\tilde{l} < l_1^w \,|\, \tilde{s} = x_1\Big)\Big] \\
+&\mathrm{P}_{x_1,x_2}(\tilde{s} = x_2) \cdot w(q^*(l_1^w, x_2), l_1^w) \cdot \Big[\mathrm{P}\Big(\tilde{l} = x_2 \,|\, \tilde{s} = x_2\Big) + \mathrm{P}\Big(\tilde{l} < l_1^w \,|\, \tilde{s} = x_2\Big)\Big],
\end{aligned}$$

where $\mathrm{P}_{x_1,x_2}(\,\cdot\,) = \mathrm{P}(\,\cdot\,|\, \tilde{s} \in \{x_1, x_2\})$.

By rewriting the conditional probabilities this can be simplified to

$$\begin{aligned}
&w(q^*(l_1^w, x_1), l_1^w) \cdot \Big[\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_1\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} < l_1^w, \tilde{s} = x_1\Big)\Big] \\
+&w(q^*(l_1^w, x_2), l_1^w) \cdot \Big[\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_2\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} < l_1^w, \tilde{s} = x_2\Big)\Big]
\end{aligned}$$

Consider a new signal $\hat{s} = \hat{s}(\tilde{l}, \tilde{s}, \tilde{u})$ such that

$$\begin{aligned}
&\{\hat{s} = x_1\} = \{\tilde{l} = x_1, \tilde{s} = x_1\} \cup \{\tilde{l} \in [l_p, \bar{y}), \tilde{s} \in \{x_1, x_2\}\} \cup \{\tilde{l} = \bar{y}, \tilde{s} \in \{x_1, x_2\}, \tilde{u} < \alpha(x_1, x_2)\},\\
&\{\hat{s} = x_2\} = \{\tilde{s} \in \{x_1, x_2\}\} \setminus \{\hat{s} = x_1\}
\end{aligned}$$

where $\tilde{u} \sim U[0,1]$ is a uniform random variable independent of $\tilde{l}, \tilde{v}, \tilde{s}$. Intuitively, the signal $\hat{s}$ pools x_1 with elements in $y \in [l_p, \bar{y})$ and x_2 with elements in $(\bar{y}, l_1^w)$, conditional on $\tilde{s} \in \{x_1, x_2\}$. Whenever $\tilde{l} = \bar{y}$ the signal $\hat{s}$ randomizes between pooling it with x_1 and x_2 with probabilities α and $1 - \alpha$ respectively.

We pick $\bar{y}$ and α such that

$$\mathrm{E}\left[\tilde{l} - l_1^w \mid \hat{s} = x_1\right] = \mathrm{E}\left[\tilde{l} - l_1^w \mid \hat{s} = x_2\right] = 0.$$

See previous lemmas for an analogous construction.

We can write welfare conditional on this new signal $\hat{s}$ as

$$\begin{aligned}
&w(q^*(l_1^w, x_1), l_1^w) \cdot \left[\mathrm{P}_{x_1,x_2}\left(\tilde{l} = x_1\right) + \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_1\right)\right]\\
+&w(q^*(l_1^w, x_2), l_1^w) \cdot \left[\mathrm{P}_{x_1,x_2}\left(\tilde{l} = x_2\right) + \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_2\right)\right]
\end{aligned}$$

The difference between the old and the new welfare is

$$\begin{aligned}
&w(q^*(l_1^w, x_1), l_1^w) \left[\mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \tilde{s} = x_1\right) - \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_1\right)\right]\\
+&w(q^*(l_1^w, x_2), l_1^w) \left[\mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \tilde{s} = x_2\right) - \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_2\right)\right]\\
=&\left[w(q^*(l_1^w, x_2), l_1^w) - w(q^*(l_1^w, x_1), l_1^w)\right] \cdot \left[\mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \tilde{s} = x_2\right) - \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_2\right)\right]
\end{aligned}$$

because $\{\hat{s} \in \{x_1, x_2\}\} = \{\tilde{s} \in \{x_1, x_2\}\}$.

Since $w(q^*(l_1^w, x_2), l_1^w) > w(q^*(l_1^w, x_1), l_1^w)$ the sign of the expression above depends on

$$\mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \tilde{s} = x_2\right) \quad \vee \quad \mathrm{P}_{x_1,x_2}\left(\tilde{l} < l_1^w, \hat{s} = x_2\right)$$

Recall that the average $\tilde{l}$ for any pooling signal is l_1^w, i.e.

$$
\begin{aligned}
l_1^w &= \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_2 | \tilde{s} = x_2\Big)\, x_2 + \int_{l_1^p}^{l_1^w} y \, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \mid \tilde{s} = x_2\Big) \\
0 &= \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_2, \tilde{s} = x_2\Big)\, (x_2 - l_1^w) + \int_{l_1^p}^{l_1^w} (y - l_1^w)\, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) \\
0 &= \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_2\Big)\, (x_2 - l_1^w) + \int_{l_1^p}^{l_1^w} (y - l_1^w)\, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) \qquad (*)
\end{aligned}
$$

where the last equality holds due to $\{\tilde{l} = x_2\} \subset \{\tilde{s} = x_2\}$. Similarly we get

$$
0 = \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = x_2\Big)\, (x_2 - l_1^w) + \int_{l_1^p}^{l_1^w} (y - l_1^w)\, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \hat{s} = x_2\Big) \qquad (**)
$$

from $\mathrm{E}\Big[\tilde{l} | \hat{s} = x_2\Big] = l_1^w$. Equating $(*)$ and $(**)$ we get

$$
\int_{l_1^p}^{l_1^w} (y - l_1^w)\, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) = \int_{l_1^p}^{l_1^w} (y - l_1^w)\, d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \hat{s} = x_2\Big)
$$

Using the definition of $\{\hat{s} = x_2\}$ we get

$$
\int_{l_1^p}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) = \int_{\bar{y}}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y\Big) + (\bar{y} - l_1^w)\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big)
$$

Splitting the r.h.s. of the above integral into $\tilde{s} = x_1$ and $\tilde{s} = x_2$ we get

$$
\begin{aligned}
\int_{l_1^p}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) &= \int_{\bar{y}}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_1\Big) \\
&\quad + \int_{\bar{y}}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) \\
&\quad + (\bar{y} - l_1^w)\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big)
\end{aligned}
$$

This implies

$$
\begin{aligned}
\int_{l_1^p}^{\bar{y}} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_2\Big) &= \int_{\bar{y}}^{l_1^w} (y - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \leq y, \tilde{s} = x_1\Big) \\
&\quad + (\bar{y} - l_1^w)\Big[\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big) - \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \tilde{s} = x_2\Big)\Big]
\end{aligned}
$$

where the last term is needed to account for the possibility that $\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \tilde{s} = x_2\Big) > 0$.

Increase y in the l.h.s. up to $\bar{y}$ and decrease it on the l.h.s down to $\bar{y}$ we get

$$\int_{l_1^p}^{\bar{y}} (\bar{y} - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \le y, \tilde{s} = x_2\Big) \ge \int_{\bar{y}}^{l_1^w} (\bar{y} - l_1^w) d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \le y, \tilde{s} = x_1\Big)$$
$$+(\bar{y} - l_1^w)\Big[\mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big) - \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \tilde{s} = x_2\Big)\Big]$$

Because $\bar{y} \le l_1^w$ dividing by $\bar{y} - l_1^w$ changes the sign

$$\int_{l_1^p}^{\bar{y}} d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \le y, \tilde{s} = x_2\Big) \le \int_{\bar{y}}^{l_1^w} d\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \le y, \tilde{s} = x_1\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big) - \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \tilde{s} = x_2\Big)$$
$$\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [l_1^p, \bar{y}], \tilde{s} = x_2\Big) \le \mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [\bar{y}, l_1^w], \tilde{s} = x_1\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big) - \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \tilde{s} = x_2\Big)$$
$$\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [l_1^p, l_1^w], \tilde{s} = x_2\Big) \le \mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [\bar{y}, l_1^w]\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big)$$

Finally notice that $\{\tilde{l} \in [\bar{y}, l_1^w]\} \cap \{\tilde{s} \in \{x_1, x_2\}\} \subseteq \{\hat{s} = x_2\} \cap \{\tilde{s} \in \{x_1, x_2\}\}$. Hence we can introduce $\hat{s} = x_2$ into the first term on the r.h.s.

$$\mathrm{P}_{x_1,x_2}(l_1^w \in [l_1^p, l_1^w], \tilde{s} = x_2) \le \mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [\bar{y}, l_1^w], \hat{s} = x_2\Big) + \mathrm{P}_{x_1,x_2}\Big(\tilde{l} = \bar{y}, \hat{s} = x_2\Big)$$
$$\mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [l_1^p, l_1^w], \tilde{s} = x_2\Big) \le \mathrm{P}_{x_1,x_2}\Big(\tilde{l} \in [l_1^p, l_1^w], \hat{s} = x_2\Big)$$

the inequality above is always strict unless both distributions $\phi_i(y)$ are fully concentrated on one point $\bar{y}$. Finally, if the inequality above is strict, then the signal $\hat{s}$ strictly improves welfare over the signal $\tilde{s}^*$ conditional on the event $\tilde{s} \in \{x_1, x_2\}$. □

Functions $g(l)$ and $G(l)$. Define function $g(l)$ on $[l_p, \bar{l}]$ in the following way. For $l \ge l_1^w$ define $g(l) = l$. For $l > l_1^w$ define it as the solution of

$$\mathrm{E}\Big[\tilde{l}\,|\,\tilde{l} \in [g(l), l]\Big] = l_1^w,$$
$$\int_{g(l)}^{l_1^w} x\, dF(x) + \int_{l_1^w}^{l} x\, dF(x) = l_1^w \cdot \Big[F(l) - F(g(l))\Big].$$

Define

$$G(l) = \begin{cases} l & \text{if} \quad l > l_1^w, \\ g^{-1}(l) & \text{if} \quad l \in [l^p, l_1^w]. \end{cases} \tag{15}$$

Lemma 13. *The optimal signal* $\tilde{s}^*$ *satisfies*

$$\mathrm{P}(\tilde{s} \leq z) \geq \mathrm{P}\Big(G(\tilde{l}) \leq z\Big)$$

for all any $z \in [l_1^w, \bar{l}]$.

Proof. Due to Lemma 8 the average $\tilde{l}$ for all pooling realizations of $\tilde{s}$ is l_1^w, i.e. $\mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{s}^* \geq z\right] = 0$. This implies

$$0 = \mathrm{P}\Big(\tilde{l} \geq z \,|\, \tilde{s}^* \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \geq z, \tilde{s}^* \geq z\right] + \mathrm{P}\Big(\tilde{l} < l_1^w \,|\, \tilde{s}^* \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} < l_1^w, \tilde{s}^* \geq z\right]$$
$$0 = \mathrm{P}\Big(\tilde{l} \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \geq z\right] + \mathrm{P}\Big(\tilde{l} < l_1^w, \tilde{s}^* \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} < l_1^w, \tilde{s}^* \geq z\right] \tag{*}$$

where in the last equation we used $\{\tilde{l} \geq z\} \subset \{\tilde{s} \geq z\}$.

Similarly, by construction of $G(l)$ we have

$$0 = \mathrm{P}\Big(\tilde{l} \geq z \,|\, G(\tilde{l}) \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \geq z, G(\tilde{l}) \geq z\right] + \mathrm{P}\Big(\tilde{l} < l_1^w \,|\, G(\tilde{l}) \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} < l_1^w, G(\tilde{l}) \geq z\right]$$
$$0 = \mathrm{P}\Big(\tilde{l} \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \geq z\right] + \mathrm{P}\Big(\tilde{l} < l_1^w, G(\tilde{l}) \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} < l_1^w, G(\tilde{l}) \geq z\right]$$
$$0 = \mathrm{P}\Big(\tilde{l} \geq z\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \geq z\right] + \mathrm{P}\Big(\tilde{l} < g(z)\Big) \cdot \mathrm{E}\left[\tilde{l} - l_1^w \,|\, \tilde{l} \leq g(z)\right] \tag{**}$$

where in the last equation we used $\{\tilde{l} < l_1^w, G(\tilde{l}) \geq z\} = \{\tilde{l} \leq g(z)\}$.

Subtracting $(*)$ from $(**)$ we get

$$\mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{\tilde{l} < l_1^w, \tilde{s}^* \geq z\right\}\right] = \mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{\tilde{l} \leq g(z)\right\}\right]$$

Splitting the r.h.s. according to $\{\tilde{s}^* \leq z\}$ and $\{\tilde{s}^* > z\}$ we get

$$\mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{\tilde{l} < l_1^w, \tilde{s}^* \geq z\right\}\right] = \mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{\tilde{l} \leq g(z)\right\} \Big(\mathbb{1}\left\{\tilde{s}^* \geq z\right\} + \mathbb{1}\left\{\tilde{s}^* < z\right\}\Big)\right]$$
$$\mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{l_1^w > \tilde{l} > g(z), \tilde{s}^* \geq z\right\}\right] = \mathrm{E}\left[\left(\tilde{l} - l_1^w\right) \mathbb{1}\left\{\tilde{l} \leq g(z), \tilde{s}^* < z\right\}\right]$$

Decreasing the $\tilde{l}$ down to $g(z)$ in the l.h.s. and increasing $\tilde{l}$ up to $g(z)$ in the r.h.s we get an inequality

$$
\begin{aligned}
\mathrm{E}\left[(g(z)-l_1^w)\,\mathbb{1}\left\{l_1^w > \tilde{l} > g(z), \tilde{s}^* \geq z\right\}\right] &\leq \mathrm{E}\left[(g(z)-l_1^w)\,\mathbb{1}\left\{\tilde{l} \leq g(z), \tilde{s}^* < z\right\}\right] \\
\mathrm{P}\Big(l_1^w > \tilde{l} > g(z), \tilde{s}^* \geq z\Big) &\leq \mathrm{P}\Big(\tilde{l} \leq g(z), \tilde{s}^* < z\Big) \\
\mathrm{P}\Big(l_1^w > \tilde{l}, \tilde{s}^* \geq z\Big) &\leq \mathrm{P}\Big(\tilde{l} \leq g(z)\Big) = \mathrm{P}\Big(l_1^w > \tilde{l}, G(\tilde{l}) \geq z\Big)
\end{aligned}
$$

Adding $\mathrm{P}\Big(\tilde{l} > z\Big) = \mathrm{P}\Big(\tilde{l} > z, \tilde{s}^* \geq z\Big) = \mathrm{P}\Big(\tilde{l} > z, G(\tilde{l}) \geq z\Big)$ to both sides of the above inequality

$$
\begin{aligned}
\mathrm{P}(\tilde{s}^* \geq z) &\leq \mathrm{P}\Big(G(\tilde{l}) \geq z\Big), \\
\mathrm{P}\Big(G(\tilde{l}) \leq z\Big) &\leq \mathrm{P}(\tilde{s}^* \leq z).
\end{aligned}
$$

□

Lemma 14. *Suppose* $\{\tilde{l} \leq g(z)\} \subseteq \{\tilde{s}^* \geq z\}$ *for a given* $z \in (l_1^p, l_1^w)$*. Then*

$$
\mathrm{P}\Big(\tilde{s}^* \geq z, \tilde{l} \in (g(z), l_1^w)\Big) = 0.
$$

Proof. Due to Lemma 8 we have

$$
l_1^w = \mathrm{E}\left[\tilde{l}\,|\,\tilde{s}^* \geq z\right] = \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{l} \geq z\right\}\,\Big|\,\tilde{s}^* \geq z\right] + \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{l} \leq l_1^w\right\}\,\Big|\,\tilde{s}^* \geq z\right]
$$

This is equivalent to

$$
\begin{aligned}
l_1^w\cdot\mathrm{P}(\tilde{s}^* \geq z) =& \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{l} \geq z\right\}\right] + \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{l} \leq l_1^w, \tilde{s}^* \geq z\right\}\right] \\
l_1^w\cdot\mathrm{P}\Big(\tilde{s}^* \geq z, \tilde{l} \geq g(z)\Big) + l_1^w\cdot\mathrm{P}\Big(\tilde{s}^* \geq z, \tilde{l} \leq g(z)\Big) =& \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{l} \geq z\right\}\right] + \mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{s}^* \geq z, \tilde{l} \leq g(z)\right\}\right] \\
&+\mathrm{E}\left[\tilde{l}\cdot\mathbb{1}\left\{\tilde{s}^* \geq z, \tilde{l} \in (g(z), l_1^w)\right\}\right]
\end{aligned}
$$

Using the assumption of the lemma, i.e., $\{\tilde{l} \le g(z)\} \subseteq \{\tilde{s}^* \ge z\}$ we obtain

$$
\begin{aligned}
l_1^w \cdot \mathrm{P}\Big(\tilde{s}^* \ge z, \tilde{l} \ge g(z)\Big) + l_1^w \cdot \mathrm{P}\Big(\tilde{l} \le g(z)\Big) =& \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} \ge z\right\}\right] + \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} \le g(z)\right\}\right] \\
&+ \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{s}^* \ge z,\, \tilde{l} \in (l_1^w, g(z))\right\}\right] \\
l_1^w \cdot \mathrm{P}\Big(\tilde{s}^* \le z, \tilde{l} \in (g(z), l_1^w)\Big) =& \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{s}^* \ge z,\, \tilde{l} \in (g(z), l_1^w)\right\}\right]
\end{aligned}
$$

However the above condition cannot hold unless both sides are equal to zero since $\tilde{l} \le l_1^w$ on the r.h.s. □

Lemma 15. *Suppose that for every $z \in [l_1^w, \bar{l}]$*

$$
\mathrm{P}\Big(\tilde{l} \le g(z) \,\Big|\, \tilde{s} \ge z,\, \tilde{l} \le l_1^w\Big) = 1 \tag{16}
$$

Then $\tilde{s} = G(\tilde{l})$ when $\tilde{l} \in [l_1^p, \bar{l}]$.

Proof. Equation (16) implies

$$
\mathrm{P}\Big(\tilde{l} \in (g(\tilde{s}),\, l_1^w] \,\Big|\, \tilde{s}\Big) = 0 \quad P - a.s. \tag{17}
$$

Note that for every $z \in [l_1^w, \bar{l})$

$$
\mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} < l_1^w\right\} \,\Big|\, \tilde{s} \ge z\right] + \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} > l_1^w\right\} \,\Big|\, \tilde{s} \ge z\right] = l_1^w.
$$

because of Lemma 8. Using equation (17)

$$
\mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} \le g(z)\right\} \,\Big|\, \tilde{s} \ge z\right] + \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} \ge z\right\} \,\Big|\, \tilde{s} \ge z\right] = l_1^w.
$$

This is equivalent to

$$
\begin{aligned}
l_1^w \cdot \mathrm{P}(\tilde{s} \ge z) &= \int_z^{\bar{l}} x f(x)\, dx + \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{s} \ge z, \tilde{l} \le g(z)\right\}\right] \\
&\le \int_z^{\bar{l}} x f(x)\, dx + \mathrm{E}\left[\tilde{l} \cdot \mathbb{1}\left\{\tilde{l} \le g(z)\right\}\right] \\
&= l_1^w \cdot \mathrm{P}\Big(G(\tilde{l}) \ge z\Big).
\end{aligned}
$$

According to Lemma 13, $\mathrm{P}(\tilde{s} \leq z) \geq \mathrm{P}\Big(G(\tilde{l}) \leq z\Big)$, thus, $\mathrm{P}(\tilde{s} \leq z) = \mathrm{P}\Big(G(\tilde{l}) \leq z\Big)$. As a result,

$$\mathbb{1}\left\{\tilde{s} \geq z, \tilde{l} \leq g(z)\right\} = \mathbb{1}\left\{\tilde{l} \geq g(z)\right\} \qquad\qquad P - a.s.$$

Due to Lemma 14 we have $\mathrm{P}\Big(\tilde{s} \geq z, \tilde{l} \in (g(z), l_1^w)\Big) = 0$, which implies that $\tilde{s} = G(\tilde{l})$. □

Lemma 16. *Suppose* (16) *does not hold for a given z. Then*

$$\mathrm{P}\Big(\tilde{l} \leq g(z) \,\Big|\, \tilde{s} < z,\, \tilde{l} \leq l_1^w\Big) > 0.$$

Proof. From the contrary, suppose that

$$0 = \mathrm{P}\Big(\tilde{l} \leq g(z), \tilde{s} < z, \tilde{l} \leq l_1^w\Big) \tag{18}$$

$$= \mathrm{P}\Big(\tilde{l} \leq g(z)\Big) - \mathrm{P}\Big(\tilde{l} \leq g(z), \tilde{s} \geq z\Big) \tag{19}$$

$$= \mathrm{P}\Big(\tilde{l} \leq g(z)\Big) \cdot \left(1 - \frac{\mathrm{P}\Big(\tilde{s} \geq z,\, \tilde{l} \leq g(z)\Big)}{\mathrm{P}\Big(\tilde{l} \leq g(z)\Big)}\right). \tag{20}$$

This implies that

$$\mathrm{P}\Big(\tilde{s} \geq z,\, \tilde{l} \leq g(z)\Big) = \mathrm{P}\Big(\tilde{l} \leq g(z)\Big). \tag{21}$$

According to Lemma 14 this implies that $\mathrm{P}\Big(\tilde{s} \geq z,\, \tilde{l} \in (g(z), l_1^w)\Big) = 0$. From here it follows that $\mathrm{P}\Big(\tilde{l} \in (g(z), l_1^w) \,|\, \tilde{s} \geq z, \tilde{l} \leq l_1^w\Big) = 0$. This contradicts the premise the premise that (16) does not hold. □

Proposition 6. *Signal $G(\tilde{l})$ is optimal.*

Proof. Suppose $\mathrm{P}\Big(\tilde{s}^* \neq G(\tilde{l}\,|\tilde{l} \in [l_p, \bar{l}]\Big) > 0$. Then Lemma 15 guarantees that there exists $z \in (l_1^w, \bar{l})$ such that

$$\mathrm{P}\Big(\tilde{l} \leq g(z) \,\Big|\, \tilde{s} \geq z, \tilde{l} < l_1^w\Big) < 1 \quad \Leftrightarrow \quad \mathrm{P}\Big(\tilde{l} > g(z) \,\Big|\, \tilde{s} \geq z, \tilde{l} < l_1^w\Big) > 0. \tag{22}$$

Then according to Lemma 16 it has also to be the case that

$$\mathrm{P}\Big(\tilde{l} \leq g(z) \,\Big|\, \tilde{s} < z, \tilde{l} > l_1^w\Big) > 0. \tag{23}$$

Since the distribution of $\tilde{s}$ is continuous in $[l_1^w, \bar{l}]$ ($\tilde{s}$ simply indexes the worst case $\tilde{l}$ in $[l_1^w, \bar{l}]$) there exists $\varepsilon > 0$ such that for sets A, B defined as

$$A = \{x \in (z, \bar{l}) : \mathrm{P}\Big(\tilde{l} > g(z) \,|\, \tilde{s} = x, \tilde{l} \le l_1^w\Big) > \varepsilon\}$$
$$B = \{x \in (l_1^w, z) : \mathrm{P}\Big(\tilde{l} \le g(z) \,|\, \tilde{s} = x, \tilde{l} \le l_1^w\Big) > \varepsilon\}$$

we have $\mathrm{P}(\tilde{s} \in A) > 0$ and $\mathrm{P}(\tilde{s} \in B) > 0$.

Define $q_A(x) = \mathrm{P}\Big(\tilde{l} \le x \,|\, \tilde{l} \in A\Big)$ and $q_B(x) = \mathrm{P}\Big(\tilde{l} \le y \,|\, \tilde{l} \in B\Big)$. Then $q_A(z) = 1$, while $q_B(l_1^w) = 1$. Functions $q_A(\cdot)$ and $q_B(\cdot)$ are increasing and continuous with values in $[0, 1]$. For $x \in [0, z]$ define

$$\kappa(x) = q_B^{-1}(q_A(x)).$$

Following Lemma 12 for each pair $(x, \kappa(x))$ and corresponding conditional right tail distributions $(\phi(y|x), \phi(y|\kappa(x)))$ there exists a welfare improving signal $\hat{s}(\tilde{l}, \tilde{s}^*, \tilde{u})$. By construction $\mathrm{P}(\tilde{s}^* \in A \cup B) > 0$, thus such modification improves welfare ex-ante as well. This contradicts optimality of $\tilde{s}^*$. □

Proof of Proposition 4

For a given l define $x(l)$ to be the amount the weak bank needs to sell to the strong bank at price $p(l)$ such that it has the same relative funding gap as the aggregate bank:

$$\frac{D - C}{N} = \frac{d_1 - c_1 - x(l)p_1(l)}{n_1 - x(l)}.$$
$$G(n_1 - x(l)) = n_1 g_1 - x(l)p_1(l)$$
$$n_1(G - g_1) = x(l)(G - p_1(l))$$
$$x(l) = \frac{n_1(G - g_1)}{G - p_1(l)}$$

where $G = \frac{D-C}{N}$ and $g_1 = \frac{d_1 - c_1}{n_1}$. The amount of revenut the weak bank needs to raise is given by

$$x(l)p_1(l) = \frac{n_1(G - g_1)}{G - p_1(l)} \cdot p(l) = n_1(G - g_1) \cdot \frac{p(l)}{G - p_1(l)}$$

Since $p_1(l) = \delta(h - \frac{l}{2})$ it is decreasing in l and thus the amount of revenue that needs to be raised is decreasing in l. This implies that the strong bank will be able to acquire the risky asset at the low price at $l = l^w$ at which point $x(l^w)p(l^w) = np(l^w) = d - c$ since the weak bank runs out of the asset to sell. Thus if $C_2 \ge d - c$, aggregation is possible.

Proof of Lemma 3

For any given l the regulator chooses whether to require the bank to sell assets or, simply, provide it with own funds. Denote by y the quantity of funds provided. Bank i's solvency constraint can be written as

$$c_i + (n_i - q)\delta\left(h - \frac{1}{2}l\right) + y + q(h-l) \geq d_i.$$

Clearly the aboce constraint must be binding in which case the regulator chooses $q \in [0, n]$ to maximize

$$\max_{q\geq 0}\left[q\frac{l}{2} - \gamma\cdot\left(d_i - c_i - (n_i - q)\delta\left(h - \frac{1}{2}l\right) - q(h-l)\right)\right]$$

The derivative with respect to residual quantity of the asset held by the bank is given by

$$\frac{1}{2}l - \gamma\cdot\left[\delta\left(h - \frac{1}{2}l\right) - (h-l)\right]$$

If this value is negative, then it implies that the regulator prefers the bank to sell assets rather than offer them a precautionary cash infusion:

$$\gamma \geq \frac{1}{2}\frac{l}{\delta\left(h - \frac{1}{2}l\right) - (h-l)}.$$

The right hand side is decreasing in l and thus a sufficient condition is given by

$$\gamma \geq \frac{\underline{l}}{\delta(2h - l) - 2(h-l)}.$$

Proof of Lemma 4

The solvency condition at $t = 2$ is given by

$$c + (n-q)p_2(z) + qp_2(z,0) \geq d.$$

It must be binding under the optimal capital requirements. Thus the asset pricing condition at $t = 2$ is given by

$$n\cdot p_2(z,0) = c + (n-q)\cdot p_1(z) + q\cdot p_2(z,0) - d + \mu(1-z)$$
$$p_2(z,0) = \frac{\mu(1-z)}{n}$$

The price of the asset at $t = 1$ is

$$p_1(z) = \delta \cdot \left(\mu b + \frac{\mu(1-\mu)}{n}(1-z) \right).$$

Define z^f, z^w as

$$p_2(z^f) = \frac{d-c}{n}, \qquad p_1(z^w) = \frac{d-c}{n}.$$

Then for for $z[\underline{z}, z^w]$ quantity of the asset retained by the banks is

$$q(z) = \min \left[\frac{d - c - np_2(z,0)}{p_1(z) - p_2(z,0)}, n \right].$$

The functional form, as well as the bank solvency constraint, is identical to the single bank analyzed in Section 3. The same proof as in Proposition 3 applies.

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