Frequent Monitoring in Dynamic Contracts

Dmitry Orlov*

First Draft: January 2, 2015

Current Draft: January 14, 2018

Abstract

This paper explores the agency costs of frequently monitoring a worker's performance. On one hand, monitoring the agent closely allows the principal to correct worker's mistakes as they arise. On the other hand, such actions inform the worker about his poor performance and endogenously increase the compensation costs. The optimal contract monitors poorly performing workers less to avoid demoralizing them. It keeps such workers in the dark about how well they are doing and reduces the need to inefficiently terminate them from the firm. At the same time the contract features elements of leniency following good performance: the supervisor corrects employee's mistakes without always punishing him for generating bad outcomes. Information design aspect of the optimal contract allows the principal to defer not only compensation, but also incentive provision to states in the world when it is cheapest.

Keywords: moral hazard, dynamic contracts, monitoring, information design, performance evaluations

1 Introduction

Well-informed employees are expensive to motivate. Fortunately, workers rarely observe the results of their labor directly and the supervisor has some control over how much information is available to them. In such environments, should a supervisor take corrective action when a worker makes a mistake? On one hand, information has productive value. On the other, such constructive criticisms

*A previous version of this paper was titled "Optimal Design of Internal Disclosure". I thank Andrzej Skrzypacz, Jeffrey Zwiebel, Peter DeMarzo, Peter Koudijs, Bob Wilson, Jonathan Levin, and Pavel Zryumov for insightful suggestions and comments on this paper. Email author at dmitry.orlov@simon.rochester.edu.

inform the agent of his poor performance and bring him closer to quitting. Future rewards need to be sufficiently high to keep the worker incentivized in this case. This is the first paper that studies the role of information design in settings where the principal can commit to any state contingent compensation policy satisfying only agent's limited liability. Even when transfers are contingent on performance, worker's limited liability still leads to incentive costs of information which, when optimally managed, leads to lack of monitoring of poorly performing workers.

Consider a professional services firm specializing in law or consulting. Typically, these long-term contracts have an up-or-out structure that provide strong incentives for associates to work hard. Information that makes an associate infer that he is unlikely to make it may motivate him to work even harder, but can also lead him to start looking for a new job and pay less attention to his current employer. Incentive misalignment creates an agency cost of information. Keeping tabs on the agent is valuable for the principal: if a worker is about to make a mistake, the supervisor can correct him and not suffer the consequences . How much ongoing monitoring should a junior employee receive in such circumstances? Instead of taking the up-or-out incentives for granted, in this paper I study the joint design of such careers and the monitoring policy that arises in the optimal contract.

I assume that realized output is seen only at agent's retirement while monitoring outcomes are public and contemporaneously observed. As such, they serve as a source of indirect feedback for the employee.[1] The decision of whether to monitor the agent or not is a solution to the following trade-off. Monitoring increases productive value since the principal catches some of agent's bad projects before they are invested into. However a less informed agent is cheaper to incentivize since fewer information nodes in his decision tree imply fewer incentive compatibility constraints that must bind. The optimal contract utilizes concurrent and delayed incentives. Concurrent incentives reflect how sensitive the agent's continuation value is with respect to principal's monitoring outcomes. Delayed incentives are determined by how sensitive the agent's continuation value is with respect to realized output. This model delivers three key results.

The first result is that instead of terminating the employee after bad performance, the optimal contract simply monitors the agent less. Incentive provision relies more on delayed incentives and concurrent incentives are low. Absent information about output, the agent's expectation over future rewards does not change and this mitigates the risk of demoralizing the agent and firing him. The

[1]This is a simplified way to think about feedback to the agent.

second result is that after an employee's mistake is uncovered by the principal prior to investment, there is less monitoring in the next instance – the worker is closer to his reservation utility and, thus, the supervisor cannot run the risk of criticizing him too much. Third, the pay for performance sensitivity with respect to contemporaneous output is non-monotone in the agent's continuation utility. For low continuation utilities it is increasing along with the monitoring probability, while for high continuation utilities it is decreasing since more and more incentives are delayed until the agent's retirement. The contract features elements of leniency towards good employees they are being monitored frequently and, when a mistake is discovered, they are not penalized for it.

It is important to note that the principal designs both the compensation scheme, i.e. career of the agent, as well as the monitoring frequency governing how informed the agent is about his past performance. A dynamic contract is, thus, a combination of compensation and monitoring policy.

Model. I start with a classic repeated moral-hazard problem in which performance pay is needed to motivate the agent to exert costly private effort. The quality of the agent is common knowledge and there is no learning about his ability. The principal and agent are risk-neutral and the agent is protected by limited liability. Every period the agent privately decides whether to work or shirk. His effort influences the distribution of projects. The principal can publicly verify the quality of the underlying projects. Monitoring is valuable to the principal since it reduces the number of bad projects that are undertaken.[2] The key trade-off is the conflict between the fundamental value of informed production and the ex-ante compensation needed to motivate the agent. If, for a given compensation plan, an informed agent finds it incentive compatible to work, an uninformed agent would also find it in his best interest to work. The intuition is best illustrated by comparing contracts in an environment where an agent observes his current performance in every period to one in which he observes nothing until the end of the contract. Under full information, incentive compatibility constraints have to be satisfied path by path for every performance history. If the agent does not observe his performance, he cannot distinguish the performance high and low paths, and so incentive compatibility constraints have to be satisfied only on average. This logic implies that agent's ex-ante compensation is lowest if there is no monitoring in the sense described above.

When the principal monitors the agent in period t, the results are public and contractable and the agent's beliefs over his future compensation change. If information indicating negative performance

[2]In the main Section 2 monitoring has no exogenous costs, unlike the monitoring technology studied in Piskorski and Westerfield (2016).

is disclosed, then the agent knows he has done poorly and a pay for performance scheme punishes him. Thus after bad performance is revealed it may become difficult to motivate the agent in the future. This role of information in the agent's decisions of exerting effort in subsequent periods alters the costs of incentive provision.

Theoretical insights. Joint characterization of optimal compensation and monitoring rule under repeated moral hazard is a difficult problem. An agent who is informed about his performance makes effort decisions sequentially. Absent information, this choice of the agent becomes a multi-tasking problem since the agent effectively chooses all of his actions in advance. In addition, each information structure induces a different set of optimally binding incentive constraints. In typical dynamic models the agent is informed about the performance measure of his effort at time t before making a $t+1$ effort decision. If the agent shirks at time t it will only have an effect on time t performance and subsequent performance will remain unaffected. This leads to single deviation incentive compatibility constraints being necessary and sufficient for global incentive compatibility. This is no longer true if the agent does not observe time t performance measure before making effort in period $t+1$. In this case if the agent has shirked at time t, his subsequent effort profile may include shirking again. This implies that single deviation incentive compatibility constraints may no longer be sufficient for global incentive compatibility. There are two difficulties. First, for a given information structure, determining the relevant global incentive compatibility constraint is not trivial. Second, for different information structures the optimally binding incentive compatibility constraints are different. This implies that the problem may be non-Markovian and makes it difficult to analyze using known methods based on dynamic programing.

To simplify this problem I allow the agent to observe an additional signal if she shirks. This signal either restores the agent to the equilibrium path, or reveals to him that he generated a bad project. Continuous time turns out to be more tractable as it is easier to verify sufficient conditions for optimality of the monitoring policy as well as use established methods in verifying the concavity of the Pareto frontier. Most interestingly, however, it allows me to prove equivalence between the main model and a version of the model in which the parties can renegotiate the delayed incentives, but the agent does not observe intermediate output signals. Continuous time also simplifies the closed form derivation of the optimal monitoring policy and verification of the concavity of the payoff frontier without relying on public randomization.

Empirical relevance. I frame the model as a problem of repeated capital allocation by the principal. A specific application of this work is in relation to internal allocation of resources in a large firm. The link between internal transparency and performance has been suggested in Hornstein and Zhao (2011), who relate transparency of multinational firms to the efficiency of their internal resource allocation. In my paper if a division is performing poorly, then the principal monitors it less and back-loads incentives into the future more. This preserves incentives going forward, but skews the capital allocations in favor of the poorly performing division. This way I address the questions raised in Scharfstein and Stein (2000) and Rajan, Servaes, and Zingales (2000). They point out that it is strange that divisions would be compensated with private benefits of capital allocations, rather than performance sensitive contracts. I show that these ex-post inefficient distortions may occur as a result of an ex-ante optimal mechanism which commits to reducing monitoring efficiency after the division performed poorly.

1.1 Related Literature

Incentive contracts have been widely studied in economics. In this paper the technological efficiency of informed production is traded off against endogenous agency cost of compensating an informed agent. The importance of agent's information environment for his decisions has been highlighted in classic papers. Harris and Raviv (1978) show that the amount of information the agent observes about the performance measure before making the private decision determines how much rent he receives under the optimal contract. They show that if the agent observes the noise realization before making his decision, he receives higher rents from the principal. Lizzeri, Meyer, and Persico (2002) show in a two period model that when compensation is optimally chosen jointly with the information-sharing rule, not providing the agent with intermediate performance evaluations reduces the agent's expected compensation for exerting effort. If, however, being informed increases the value of agent's effort, timely information sharing may be optimal. The economic tension outlined in this paper is that the agent can extract more rents from the principal when he observes more information about the contractible performance measure. The principal is willing to share this information only when the exogenous value of the agent knowing this information is sufficiently large. Laux (2001) uses the same economic trade off to show that for incentive reasons it is optimal to let the manager operate multiple projects simultaneously, rather than individual projects in sequence. When the agent works on projects at the same time, this is the same as him not observing intermediate output of individual projects before making subsequent decisions.

I explore these ideas in a rich dynamic environment showing the optimal way of pooling incentives across dynamic histories, while simultaneously quantifying the agency costs of information.

Piskorski and Westerfield (2016) and Chen, Sun, and Xiao (2017) consider costly monitoring in dynamic contracts. The principal spends resources in acquiring additional signals about agent's effort and thus may avoid the need for inefficient termination of the agent. As such, more monitoring by the principal substitutes for the pay for performance sensitivity that is needed to motivate the agent to work. There are several differences between these and present paper. First, I model monitoring as the decision to observe the agent's performance, rather than agent's effort. It is not about "what" performance information to acquire, but rather "when" to acquire it. Second, in most of the paper, monitoring is costless for the principal. It highlight the endogenous agency cost channel stemming from incentive provision to the agent. I show that if the status-quo of observing performance is with a delay, which is a realistic assumption in many settings, frequent monitoring is expensive exactly because of agency considerations.

This work is also related to a large literature on dynamic contracts which focus on the implications of optimal contract design on optimal firm management and dynamics. The closest paper in terms of technical analysis is Biais, Mariotti, Rochet, and Villeneuve (2010a), while other papers include DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), Sannikov (2008), Zhu (2013), and Varas (2013). While most of these papers derive implications about optimal effort levels and project termination, I explore a new question in firm design the optimal level of internal communication between a supervisor and an employee.

A key difference between the environment analyzed in this paper and a the literature above is that I assume the agent observes interim performance information only when the principal engages in concurrent monitoring. Otherwise, performance information is observed only at retirement. At a first glance, it may seem counterintuitive that the agent would know less about his own performance compared to the principal. However, the agent does perfectly observe his own effort and we must remember that a performance measure is a noisy signal of it. The agent is still more informed about the relevant variable the principal imperfectly estimates. For instance, Baker (1992) points out that the agent's output may not be contractible and the principal may be better informed about the relevant performance signals. Results of principal's monitoring are public and contractable, and this differentiates this work from papers on subjective evaluations such as Baker, Gibbons, and Murphy (1994), MacLeod (2003), and Fuchs (2007).

Benefits of information disclosure for technological purposes have been considered in papers such as Lizzeri et al. (2002), Ray (2007), and Manso (2011). In this present paper, I assume that the principal improves investment efficiency from monitoring, however the results of this monitoring are public. This allows to explicitly compute the agency costs of sharing information in a dynamic principal-agent setting with a stationary technology.

Feedback and Performance Evaluations. The problem of joint design of compensation and performance evaluations has been pointed out in Chapter 12 of Murphy and Cleveland (1995). They argue that it is important to understand how performance evaluations impact incentives in a performance sensitive environment beyond their psychological aspects. A good review of the psychology literature on performance evaluations can be found in Benabou and Tirole (2002) and Benabou and Tirole (2003). Smolin (2017) analyzes the design of performance feedback to an agent of uncertain ability and shows that the principal benefits from keeping the agent optimistic about his ability. This paper is different since the agent's type is public information and agency costs arise due to the need for performance sensitivity in the agent's contract.

2 Main Model

I begin with an informal description of the model. The principal (she) employs the agent (worker, he) who is tasked with finding good projects to invest in. There is no uncertainty about the agent's overall ability, but the likelihood of coming up with a good project each period depends on his private effort. The principal contributes investment capital and monitors the incoming projects for quality prior to investment. Monitoring results and subsequent investment decisions are publicly observable by the worker who does not directly see project quality otherwise.[3] Performance of undertaken projects is also observable, but only at agent's retirement.[4] Monitoring outcomes and performance results are contractable. The principal designs a contract specifying a monitoring policy and a compensation plan contingent on the monitoring outcomes and eventual performance of the projects. The optimal contract determines the optimal tradeoff between investment efficiency and the agency costs of compensation. In what follows all random variables are defined on a probability space $(\Omega, \mathbb{F}, \mathrm{P})$.

[3] Monitoring outcomes serve a feedback role informing the agent about his performance. I can also assume that the agent observes monitoring results with some probability. This models the risk of information leakage within a firm. A simple example of such risk is a legal associate who learns that he is not going to make partner.

[4] This is without loss if performance is observed privately by the principal and she ex-ante commits to a disclosure rule. In this case it would be optimal to disclose results only at agent's retirement.

Players. Time is continuous and there are two long-lived players: the principal and the agent. Both are risk-neutral and discount the future at a common rate r. The agent has limited liability and retires at an exogenous non-contractable time τ exponentially distributed with parameter γ.[5] The agent generates a flow of investment projects, but does not have the funds to pursue them. The principal provides the necessary investment capital and can also monitor the quality of incoming projects. This captures the idea that, in addition to the principal being wealthy, she also has valuable knowledge and experience incremental to the agent's effort. Principal has outside option R when the agent retires and I normalize the agent's outside option to 0.

Production technology. Each period the agent finds a new project for the principal to invest in. Investment into projects every period requires exactly one unit of capital and is not scalable. Each project can be one of two types. The high quality (good) project generates a return of $\mu\, dt$ if invested. The low quality (bad) project generates a return of -1 if invested.[6] Good projects occur frequently and generate a modest return while bad projects are rare, but lead to large losses. Define process $X = \{X_t\}_{t\geq 0}$ to be the number of bad projects recommended by the agent up to time t. The total (undiscounted) return to the principal if she invests into every project is

$$\mu t - X_t.$$

Once invested, the performance of the project is observed only at the agent's retirement time. This assumption is equivalent to assuming that post-investment the principal decides when to disclose performance results to the agent. It is then optimal for the principal to disclose these results only at agent's retirement as it expands the set of contractable performance signals that can be used for compensation purposes and keep the agent in the dark prior to retirement.

Moral hazard problem. There is no uncertainty about the agent's ability to generate good projects, but he affects the likelihood of good projects by exerting costly private effort. A project is low quality either because of bad luck or due to agent's shirking. I formalize this by decomposing the number of bad projects X_t into the sum of X_t^N and X_t^A:

$$X_t = X_t^N + X_t^A.$$

[5]This assumption is analogous to assuming the contracting game has a finite, but long, horizon while preserving the analytical tractability of a single-dimensional dynamic problem.

[6]I can equivalently assume the return is $-l$ from investing into a bad project.

Process $X^N = \{X_t^N\}_{t\geq 0}$ describes the nature's draw of bad luck. I model it as a Poisson process with a constant intensity λ. Process $X^A = \{X_t^A\}_{t\geq 0}$ is determined by the agent's effort. If the agent works in period t ($a_t = 1$), the intensity of arrival of X^A is equal 0, while if the agent shirks in period t ($a_t = 0$) this intensity is equal to Δ.

For tractability, I assume the agent privately process process X^A. Under this assumption the optimal incentive compatibility conditions can be characterized in closed-form and along the equilibrium path of high effort agent's information coincides with public information. It also generates a sufficient incentive compatibility condition for the case when the agent does not observe X^A. In Section 5 I provide parametric conditions under which the optimal renegotiation-proof contract in which the agent does not observe X^A is identical to one in which the agent privately observes it.

Monitoring technology. The principal publicly monitors projects recommended by the agent. She chooses a fraction f_t projects to monitor up to $\bar{f} \leq 1$. This way she avoids investing into a fraction f_t of bad projects in period t. I assume that monitoring results are public. This assumption is motivated by the fact that if the agent sees the decision of the principal he can infer bad projects from the times when the principal has forgone the investment. It is a natural assumption since investment likely requires the participation of the party who came up with the idea for the project.[7] One can also interpret the monitoring results being public if they stem from information leakage within the organization. The benefit of monitoring is the improved investment efficiency. The downside is the agent learns that he recommended a bad project.

It is useful to decompose the number of bad projects recommended by the agent X_t into the sum of Y_t bad projects that were screened out as a result of principal's monitoring prior to investment and of Z_t bad projects that were invested into:

$$X_t = Y_t + Z_t. \tag{1}$$

Because the monitoring results are public, process $Y = \{Y_t\}_{t\geq 0}$ is observable by both players in real time. For a given effort of the agent a_t and monitoring frequency f_t the intensity of discovering a bad project is $(\lambda + \Delta(1 - a_t)) \cdot f_t$. Process $Z_t = X_t - Y_t$ is latent and is observed by the agent

[7] As long as the principal cannot commit to invest into projects she knows are bad quality, the observability of investment is equivalent to monitoring results being public.

only at his retirement. The arrival rate of this latent process is given by $(\lambda + \Delta(1 - a_t)) \cdot (1 - f_t)$.[8]

The realized returns from the undertaken projects are observed with a delay. To keep the model simple, I assume they are observable only at agent's retirement. There are two arguments in favor of this assumption. First, if the agent does not continue working on a given project post investment, it is up to the principal to communicate the results. This is a reasonable assumption in this context since in a big firm workers rarely observe realized performance and it is up to the employer (principal) to report this information back to them. It is easy to show that the principal would do it only at agent's retirement. Second, if the agent observes the results with a delay, but prior to retirement, this leads to a difficult incentive provision problem as shown in Sannikov (2014) and Varas (2013). The focus of this paper is on the agency costs of observing output immediately, or with a delay, and so I simplify the problem and move my analysis away from known theories of repeated moral hazard under persistent private information.

Private and public histories. Following Orlov, Skrzypacz, and Zryumov (2018), I identify the history of the continuous time contract with the corresponding filtration on the probability space $(\Omega, \mathcal{F}, \mathrm{P})$. There are three types of histories that are relevant for the principal-agent relationship:

- Public filtration $\mathbb{F}^Y = \{\mathcal{F}_t^Y\}_{t \geq 0}$ where $\mathcal{F}_t^Y = \sigma(\{Y_s\}_{s \leq t \wedge \tau})$ is the information available to both players up to time $\min(t, \tau)$ generated by the public monitoring process Y. Performance information Z is not observed until the agent's retirement and, thus, is not part of the public history.

- Agent's private filtration $\mathbb{F}^A = \{\mathcal{F}_t^A\}_{t \geq 0}$, where $\mathcal{F}_t^A = \sigma(\{Y_s, X_s^A\}_{s \leq t \wedge \tau})$ – information available to the agent up to time $\min(t, \tau)$ generated by public monitoring process Y and the agent's private output process X^A. Agent's past effort process a is also part of his private history, but it is sunk and its long-term implications by time t are contained in $\mathcal{F}_t^A$.

- All available information $\mathbb{F} = \{\mathcal{F}_t\}_{t \geq 0}$ where $\mathcal{F}_t = \sigma(\{Y_s, Z_s\}_{s \leq \tau})$ – all of the performance information that can, in principle, be observed and contracted on. It includes both monitoring results Y and performance results Z. It is without loss that $\mathcal{F}_t$ captures the entire path of process Z, however the optimal contract only relies on Z_τ.

[8] Agent's effort a and principal's monitoring f are substitutes. I focus on contracts which implement high effort $a_t = 1$. This turns out to be optimal if $\bar{f} < 1$ and Δ is sufficiently large. In the case of $\bar{f} = 1$ and costless monitoring, the principal can achieve first best by allowing the agent to shirk and setting $f_t \equiv 1$. This is a special case, and, once a linear monitoring cost $k > 0$ is introduced, implementing high effort is optimal under reasonable parameters.

My approach of modeling information is rather simple – the agent either observes his performance immediately after taking an action, or at retirement. All available information $\mathbb{F}$ exceeds public information, since it contains information about process Z. On the other hand, it is not contained in the private information of the agent since the agent also does not observe process Z. Along the equilibrium path in which $X^A \equiv 0$ and agent's information is equivalent to public information.

2.1 Optimal contracting problem

Principal offers the agent a long-term contract. It specifies both the monitoring frequency and compensation to the agent. Because both players are risk-neutral, equally patient, and the principal has commitment power, any compensation profile is payoff equivalent to a single payment made to the agent at his retirement time τ. Thus, a compensation plan is a single random variable C_τ measurable with respect to $\mathcal{F}_\tau$. The monitoring frequency is a process $F = \{f_t\}_{t\geq 0}$ predictable with respect to public history $\mathbb{F}^Y$.[9] Monitoring policy F is predictable with respect to $\mathbb{F}^Y$ and it is easy to show that there is no benefit of conditioning monitoring frequency f_t on the realizations of the latent process Z.

The principal chooses contract $\mathcal{C} = (C, F)$ to maximize her expected payoff

$$\sup_{\mathcal{C}} \mathrm{E}_F \left[\int_0^\tau e^{-rt}(\mu\, dt - dX_t + dY_t) - e^{-r\tau} C_\tau \right]$$

subject to high effort process $\bar{a} = \{\bar{a}_t\}_{t\geq 0}$ in which $\bar{a}_t \equiv 1$ being a best response of the agent

$$\bar{a} \in \arg \max_{\hat{a}\in\mathcal{A}(\mathbb{F}^A)} \mathrm{E}_{\hat{a}} \left[e^{-r\tau} C_\tau - \int_0^\tau e^{-rt} \hat{a}_t h \cdot dt \right].$$

The argmax$(\cdot)$ is taken among all effort processes $\mathcal{A}(\mathbb{F}^A)$ which are predictable with respect to the agent's private history $\mathbb{F}^A$.[10] Along the path of high effort, the information set of the agent is identical to that of the principal, however if the agent shirks, he has additional information that the project led to a low payout. This implies that if the compensation process is sensitive to the realized profit of the project, the agent may have private information about his expected compensation. The set of effort levels to which the agent can deviate is greater than the set of effort profiles predictable with respect to the public history.

[9] And, possibly, a public randomization device $\mathcal{F}^R$.

[10] Note that agent's private history is endogenous to principal's monitoring rule F. This creates the endogenous cost of monitoring for the principal.

3 Optimal Contract

The optimal compensation plan and monitoring policy are interconnected and must be solved for jointly. First, I characterize the optimal incentive compatibility constraint in terms of monitoring frequency, agent's expected compensation, and the sensitivity of agent's expected compensation to monitoring outcomes. Second, I provide conditions under which the optimal monitoring frequency is a closed-form increasing function of the agent's continuation value in the contract. Third, I characterize the Pareto frontier and show that it is attained without the use of public randomization across continuation utilities. Finally, I provide conditions under which high effort until agent's retirement is optimal for the principal.

3.1 Incentive Compatibility under the Optimal Contract

The first step in deriving the optimal contract is to show that the optimal compensation does not reward the agent at retirement τ if there was a bad project that was not screened out by the principal. This property allows characterization of the necessary and sufficient incentive compatibility conditions for a given monitoring frequency in that period. The optimal incentive scheme penalizes the agent if a bad project is uncovered prior to investment, or if an undertaken project leads to a loss. The timing of when these penalties accrue is crucial for the optimal contract.

Define the agent's continuation value along the path of high effort

$$w_t = \mathrm{E}_{\bar{a}} \left[e^{-r(\tau - t)} C_\tau - \int_t^\tau e^{-r(s-t)} h \, ds \,\middle|\, \mathcal{F}_t^Y \right] \cdot \mathbb{1}\{\tau \geq t\}. \tag{2}$$

For a given monitoring policy F define the probability that no bad projects where invested into up to time t:

$$p_\tau = \mathrm{P}\big(Z_\tau = 0 \,|\, \mathcal{F}_\tau^Y\big) = e^{-\int_0^\tau \lambda(1-f_t)\, dt}. \tag{3}$$

Incentive compatibility conditions under the optimal contract rely on the composition of delayed incentives based on performance Z, and concurrent incentives given by the sensitivity of agent's expected pay to monitoring results Y. Lemma 1 tackles optimal sensitivity of the agent's compensation with respect to investment results Z. Lemma 2 characterizes the sensitivity of agent's continuation value with respect to monitoring results Y. Proposition 1 builds on these observations and characterizes the incentive compatibility conditions for the optimal contract and constitutes the first result of the paper.

Agent's retirement time τ is not contractable. And so when he chooses to leave, he requests his promised utility from the principal. This implies that in expectation, prior to retirement, the agent is promised a value of w_τ. However, the principal can reveal information about performance results Z_τ and further condition this payment on realized performance. The following lemma shows that the principal should only reward the agent if there were no bad projects that were invested into.

Lemma 1. *There exists an optimal contract such that* $C_\tau = \frac{w_\tau}{p_\tau} \cdot \mathbb{1}\{Z_\tau = 0\}$.

Sketch of proof. Outcome $Z_\tau = 0$ is most informative about high effort. For any incentive compatible contract $\mathcal{C}$ there exists the associated continuation value process $w = \{w_t\}_{t \geq 0}$ defined by (2) and probability process $p = \{p_t\}_{t\geq 0}$ defined by (3). Define the new contract $\hat{\mathcal{C}} = (\hat{C}, F)$ in which $\hat{C}_\tau = \frac{w_\tau}{p_\tau}\mathbb{1}\{Z_\tau = 0\}$. Such a contract delivers the same expected compensation to the agent if he works, but penalizes him more if he shirks. Contract $\hat{\mathcal{C}}$ is incentive compatible, delivers the agent the same continuation utility along the equilibrium path, and implements the same monitoring policy F. Thus the principal's payoff would be unaffected were she to switch from $\mathcal{C}$ to $\hat{\mathcal{C}}$. Thus, it is without loss to search for an optimal contract which satisfies $C_\tau = 0$ if $Z_\tau > 0$.

Lemma 2. *For a contract* $\mathcal{C}$ *and the associated continuation value process* $w = \{w_t\}_{t\geq 0}$ *there exists a predictable process* $\beta = \{\beta_t\}_{t\geq 0}$ *with respect to filtration* $\mathbb{F}^Y$ *such that for any* $t < \tau$

$$dw_t = rw_t\,dt + h\,dt + \beta_t \cdot (\lambda f_t\,dt - dY_t). \tag{4}$$

The proof of Lemma 2 is quite standard within the dynamic contracting literature.[11] It is notable that in this setting both β_t and f_t govern the dynamics of the continuation value process w. In any given period a lower sensitivity β_t implies that the agent is less sensitive to the contemporaneous project quality Y. I refer to β_t as the contemporaneous performance sensitivity. A lower monitoring frequency f_t implies that process Y_t has a lower arrival intensity. By reducing f_t the principal is able to lower the variance of the agent's continuation utility and, thus, limit inefficiencies that may be incurred if it becomes too small and the agent would need to be terminated.

The incentive compatibility condition derived below highlights the composition of concurrent incentives captured by β_t and optimal delayed incentives captured by w_t.

[11] See DeMarzo and Sannikov (2006), Sannikov (2008), Biais, Mariotti, Rochet, and Villeneuve (2010b), Varas (2013) among others.

Proposition 1 (Optimal Incentive Compatibility)**.** *There exists an optimal contract under which effort is incentive compatible if and only if*

$$h \leq \Delta \cdot \Big(f_t \cdot \beta_t + (1 - f_t) \cdot w_t \Big). \tag{5}$$

Sketch of proof. If the agent shirks in period t, he saves the private cost of effort h. The agent may get away with shirking if $dX_t^A = 0$. However if $dX_t^A > 0$, which happens with intensity Δ, the project is bad and negatively affects the agent's compensation. With probability f_t, the principal screens out this bad project, in which case the agent's continuation value drops by β_t. Thus, in expectation the penalty from being monitored is equal to $f_t \cdot \beta_t$. If the principal does not monitor, which occurs with probability $1 - f_t$, the principal invests into the project, but the agent now knows the compensation at τ is 0 since $Z_\tau \geq X_\tau^A > 0$ in this case. Thus, the agent loses all of his continuation value in the firm. The expected loss from losing this compensation is given by $(1 - f_t) \cdot w_t$. The expected cost to the agent of proposing a bad project is thus given by $f_t\beta_t + (1 - f_t)w_t$. In order for the agent to find effort incentive compatible, the cost of effort must be lower than the probability of generating a bad project (Δ) multiplied by the sum of contemporaneous and delayed incentives. Thus, we obtain (5). Incentives are provided in two ways – contemporaneously when the project is being monitored and with delay at retirement when all of the agent's compensation is at stake.

In order to ensure the agent can be incentivized to exert effort until retirement, he must always have some stake in the firm. Because of limited liability of the agent it must be the case that $w_t - \beta_t \geq 0$. Thus, the lowest continuation value at which the principal can still induce effort can be obtained if we consider $f_t \equiv 0$.

Corollary 1. *Effort can be incentive compatible after every public history if and only if the agent's continuation utility never drops below* $\frac{h}{\Delta}$.

Equation (4) together with Corollary 1 implies that after a bad project is identified the agent's continuation value cannot drop below $\frac{h}{\Delta}$:

$$w_t - \beta_t \geq \frac{h}{\Delta}.$$

Substituting this into the incentive compatibility condition (5) it follows that

$$\frac{h}{\Delta} \leq f_t \cdot \beta_t + (1 - f_t) \cdot w_t \leq w_t - f_t \cdot \frac{h}{\Delta} \qquad \Rightarrow \qquad f_t \leq \frac{w_t - \frac{h}{\Delta}}{\frac{h}{\Delta}}. \tag{6}$$

The upper bound on monitoring frequency imposed by inequality (6) is a consequence of the incentive compatibility condition and the requirement that after a bad shock the agent must still have enough continuation value in the firm to exert effort in subsequent periods. Constraint (6) is relevant only if $w_t < (1 + \bar{f})\frac{h}{\Delta}$. It captures the intuition that frequent negative feedback can demoralize the agent when he has a low stake in the firm.

3.2 Optimal Compensation and Monitoring Policy

The optimal monitoring policy F is chosen by the principal to maximize ex-ante profits. I show that if the players are sufficiently impatient, the optimal contract specifies a "greedy" policy – the principal monitors the agent the maximum amount subject to the agent always having enough of a stake in the firm to continue working in future periods. Taken together with the optimal incentive compatibility condition (5), it also pins down the optimal contemporaneous pay for performance sensitivity $\beta = \{\beta_t\}$.

Going forward I assume that players are impatient:

$$r \geq \max\left[\gamma, \quad \lambda + h - \gamma + \max\left(0, \frac{(\gamma - \Delta)\lambda}{rh}\right)\right]. \tag{7}$$

Because agent's incentives only depend on his promised utility, it is natural to take w as the state variable of the optimal contract. Assumption (7) is a sufficient condition that the principal finds it optimal to implement as much monitoring as possible subject to keeping the agent's continuation value $w_t \geq \frac{h}{\Delta}$ with probability 1. It results in the principal preferring instantaneous investment efficiency over one that is spread out over future periods.

Proposition 2 (Optimal Monitoring)**.** *Under condition* (7) *the optimal monitoring policy* $f(\cdot)$ *and contemporaneous pay for performance sensitivity* $\beta(\cdot)$ *are given by functions of agent's continuation* *value*

$$f(w) = \min\left[\frac{w - \frac{h}{\Delta}}{\frac{h}{\Delta}},\ \bar{f}\right], \tag{8}$$

$$\beta(w) = \begin{cases} w - \frac{h}{\Delta} & \textit{if} \quad w \in \left[\frac{h}{\Delta},\ \frac{(1+\bar{f})h}{\Delta}\right], \\ \max\left[w + \frac{\frac{h}{\Delta} - w}{\bar{f}},\ 0\right] & \textit{if} \quad w > \frac{(1+\bar{f})h}{\Delta}. \end{cases} \tag{9}$$

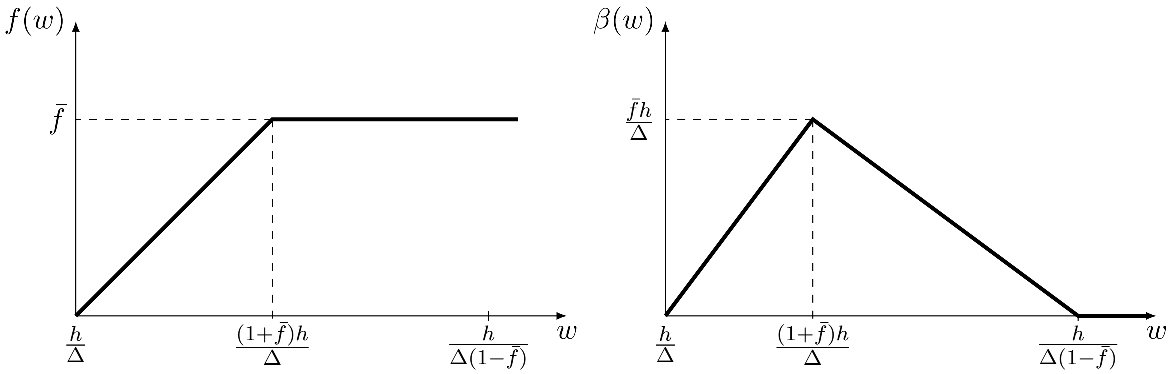

Figure 1: Optimal monitoring rule and contemporaneous pay for performance sensitivity as functions of agent's promised utility w.

Figure 1 highlights two features of the optimal monitoring and incentives in Proposition 2. First, when the agent's continuation value is low, it implies that the contemporaneous punishment for a bad project $\beta(w)$ must be low as to keep $w - \beta(w) \geq \frac{h}{\Delta}$. To satisfy both this requirement and maintain sufficient incentives for the agent to exert effort, the optimal incentive compatibility condition (5) keeps $(1 - f(w)) \cdot w$ large by keeping the probability of monitoring $f(w)$ small. As a result, monitoring frequency is small for low continuation values. In other words, when $\beta(w)$ must be small as to keep the agent incentivized in the future, the principal has no choice but to backload more incentives to retirement by monitoring the agent less now.

The second result of Proposition 2 refers to high continuation value states w. If $\bar{f} < 1$, then $(1-\bar{f})\cdot w$ is so large that the principal can all but eliminate $\beta(w)$ in (5) and incentives are maintained. The agent is willing to exert effort since he has so much tied up in the firm that he does not want to realize he lost it all when he retires. As a result, contemporaneous incentives are low when delayed incentives are very powerful. A corollary of this is that the largest contemporaneous pay for performance sensitivity $\beta(w)$ is observed for managers with intermediate performance driven purely by the information management aspect of principal's monitoring.

The optimal contract requires keeping track of two state variables: agent's continuation value w_t and the probability process p_t defined in (3). Given Lemma 1, the optimal compensation at agent's retirement is then $\frac{w_\tau}{p_\tau} \cdot \mathbb{1}\{Z_\tau = 0\}$. Because of risk-neutrality however, prior to agent's retirement the probability p_t is not payoff relevant as the expected payment to the agent is just w_τ along the equilibrium path. As a result, the second state variable p is irrelevant for ex-ante payoffs along the path of agent's high effort. Thus, optimal controls $\{\beta(w), f(w)\}_{t\geq 0}$ determining the dynamics of the optimal contract depend only on w.

3.3 Pareto Frontier and Principal's Value Function

I close out the formal analysis by characterizing the Pareto dominant payoffs of the players. I do so by fixing the agent's expected value w and solve for the maximum expected value $v(w)$ that can be achieved by the principal. First, I show that if $w \geq \frac{h}{\Delta(1-\bar{f})}$, the principal can achieve first best surplus. This stems from the fact that if $\bar{f} < 1$ and w is large, the principal can set $\beta(w) = 0$ and $f(w) = \bar{f}$ which keeps (5) satisfied. Absent any variance in the agent's continuation value, the regions for which $f(w) < \bar{f}$ are no longer reached.

For $w < \frac{h}{\Delta(1-\bar{f})}$ the principal's value as a function of agent's continuation utility satisfies a delay differential equation.[12] There exists a unique solution to this differential equation subject to the boundary condition that when $w > \frac{h}{\Delta(1-\bar{f})}$ the optimal contract implements a first best outcome: high effort and maximum monitoring $\bar{f}$ until the agent's retirement. This solution is concave. This implies that the optimal contract does not rely on public randomization across the agent's promised utilities.

Define the principal's value function under the optimal contract to be

$$v(w_0) = \sup_{\mathcal{C}} \mathrm{E}\left[\int_t^\tau e^{-r(s-t)}(\mu\, dt - dZ_t) + e^{-r(\tau-t)} \cdot (R - C_\tau)\right]$$

where the sup is taken across all incentive compatibility contracts delivering the agent an initial expected value of

$$w_0 = \mathrm{E}\left[e^{-r(\tau-t)} \cdot C_\tau - \int_t^\tau e^{-r(s-t)} h\, ds\right].$$

The sum of $v(w_0) + w_0$ corresponds to the social surplus generated by the optimal contract. Since the principal aims to maximize her value holding the promise of w_0 to the agent fixed, her problem

[12] Similar to prior contracting literature with Poisson shocks, e.g. Biais et al. (2010b), Varas (2013).

is equivalent to that of the social planner.

The next lemma shows that if w_0 is sufficiently large, there exists an optimal contract that leads to a first best social surplus.[13]

Lemma 3. *Suppose $\bar{f} < 1$. The optimal contract delivering the agent an initial continuation value $w_0 \geq \frac{h}{\Delta(1-\bar{f})}$ results in a first best social surplus:*

$$v(w_0) + w_0 = \frac{\mu + \lambda(\bar{f} - 1) - h + \gamma R}{r + \gamma} \qquad \text{for} \qquad w \geq \frac{h}{\Delta(1 - \bar{f})}. \tag{10}$$

Sketch of proof. If $w \geq \frac{h}{\Delta(1-\bar{f})}$ then the principal can set $f_t = \bar{f}$, $\beta = 0$ and the incentive compatibility condition $h \leq \Delta \cdot (1 - \bar{f}) \cdot w_0$ is satisfied. According to Lemma 2, if $\beta = 0$, then w_t is monotonically increasing and thus the incentive compatibility is satisfied going forward for the first best level of monitoring $\bar{f}$. The social surplus, denoted by M going forward, is given by

$$M = \frac{\mu + \lambda(\bar{f} - 1) - h + \gamma R}{r + \gamma}.$$

It is the expected discounted value of agent's effort under maximum monitoring $\bar{f}$ until his retirement. It also accounts for the principal's outside option when the agent retires and the effort cost of the agent up until his retirement.

It is worth noting that the discount rate r does not impact the threshold at which first best social surplus is reached at $\frac{h}{\Delta(1-\bar{f})}$ and the agent need not have residual ownership of the project at that point yet. The optimal contract does not have contemporaneous performance sensitivity ($\beta = 0$) only because all of the incentive provision is done through back-loaded incentives. I use the result of Lemma 3 to specify the boundary condition for the contract which promises the agent $w_0 < \frac{h}{\Delta(1-\bar{f})}$.

Proposition 3. *The optimal contract does not involve public randomization. Principal's value function $v(w)$ is the unique solution to the delay differential equation*

$$\begin{aligned}(r + \gamma)v(w) = \mu + \lambda(f(w) - 1) + \gamma(R - w) + v'(w) \cdot \big(rw + h + \lambda f(w)\beta(w)\big) \\ + \lambda f(w) \cdot \big(v(w - \beta(w)) - v(w)\big)\end{aligned} \tag{11}$$

for every $w < \frac{h}{\Delta(1-\bar{f})}$ and satisfying boundary condition (10) at $w = \frac{h}{\Delta(1-\bar{f})}$. Functions $f(w)$

[13] It is worthwhile to note that Proposition 2 implies Lemma 3. In the formal proofs in the Appendix the sequence of proofs is reversed as I first establish Lemma 3 (which is straightforward), and then use it to derive the sufficient conditions for the optimal monitoring rules.

and $\beta(w)$ are specified in (8) *and* (9). *Principal's value function $v(w)$ is strictly concave for $w \in \left[\frac{h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right]$.*[14]

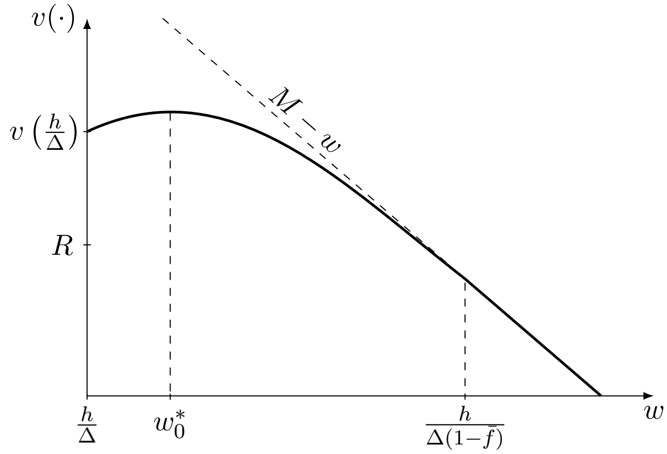

Figure 2: Principal's value function $v(\cdot)$ is the solid line. The dashed line is the payoff to the principal under the first-best social surplus, where $M = \frac{\mu+\lambda(\bar{f}-1)-h+\gamma R}{r+\gamma}$.

Proposition 3 verifies that the optimal contract does not require public randomization beyond the implicit randomization of embedded in monitoring outcome process Y. The proof of Proposition 3 is somewhat technical, but is standard in the literature. First, I show existence of the solution to the delay differential equation (11) relying on its linearity. The case of $\bar{f} = 1$ turns out to be special and I show that it can be obtained as the increasing limit to solutions corresponding to $\bar{f}_n \to 1$. Then, I prove that the solution is concave for $w \in \left[\frac{h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right]$ following similar arguments to Biais et al. (2010b) adapted to the particular delay differential equation (11).

Initial continuation value w_0^* is chosen to maximize the principal's continuation utility, leading to a first order condition $v(w_0^*) = 0$ pinning down the ex-ante value the agent obtains in the optimal contract. Subsequent monitoring and compensation rules are then given by (8) and (9), while the continuation value dynamics are governed by (4). This closes out deriving the optimal level of monitoring by the principal and the associated optimal contract. In the next section I provide conditions under which high effort until agent's retirement is optimal from the perspective of the principal and shows that termination is suboptimal.

[14]When $\bar{f} = 1$ define $\bar{w} = \infty$. Boundary condition (10) turns into a transversality condition $\lim_{w\to\infty}(v(w) + w) = M$. The solution to (11) characterizing principal's value function is unique and strictly concave for all w.

3.4 Optimality of High Effort and Monitoring Costs

In this section I provide sufficient parametric conditions under which high effort until retirement is globally optimal. Instead of firing the agent or employing him with low effort, the principal simply reduces the monitoring frequency and reduces how informed the agent is about his performance.

If the agent does not exert effort, then he recommends bad projects with intensity $\lambda + \Delta$. The principal can screen out at most $\bar{f}$ of them leaving $(\lambda + \Delta)(1 - \bar{f})$ of bad projects that the principal loses money on. If $\bar{f} < 1$ and Δ is large then this loss is big and agent's effort is valuable. In the extreme case of $\bar{f} = 1$, however, the principal is so good at monitoring the agent that, even if the latter shirks and generates bad projects with intensity $\lambda + \Delta$, the principal can substitute agent's costly effort with free monitoring and achieve efficiency. If $\bar{f} = 1$ implementing high effort can be optimal for the principal only if monitoring the agent is costly.

Suppose it costs the principal $k \cdot f$ to monitor the agent with frequency f in any given period. Agent's effort is preferable to maximal monitoring of a shirking agent if the monitoring cost is large. Specifically, I assume that

$$k > \lambda + h. \tag{12}$$

The following Lemma provides conditions under which the optimal contract always implements high effort from the agent. It implies that the optimal contract promises the agent an expected utility above $\frac{h}{\Delta}$ and it stays above $\frac{h}{\Delta}$ after every history prior to agent's retirement.

Lemma 4. *Suppose r, μ, and Δ are sufficiently large. The optimal contract starts with $w_0 \geq \frac{h}{\Delta}$ and implements high effort $\bar{a}$ until agent's exogenous retirement time τ.*

Sketch of proof. For simplicity, I assume that μ is sufficiently large so that the principal prefers to implement no effort from the agent, rather than firing him and claiming her outside option R. I show that if $w < \frac{h}{\Delta}$, then it is optimal for the principal to randomize between continuation values of 0 and $\frac{h}{\Delta}$, rather than to wait for w_t to increase over time. As a result, the value function of the principal $v(\cdot)$ is linear over $\left[0, \frac{h}{\Delta}\right]$ and has a downward kink at $w = \frac{h}{\Delta}$. The size of this kink is large if Δ is large because the principal ends up investing into more bad projects. Thus the principal optimally chooses a lower monitoring frequency f if w is close to $\frac{h}{\Delta}$ to avoid the continuation value to drop below $\frac{h}{\Delta}$ and prevent the agent from shirking.

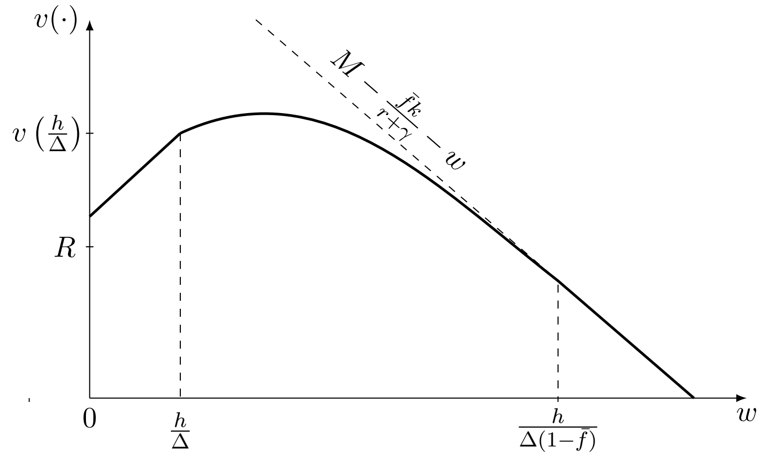

Figure 3: Principal's value function $v(\cdot)$ is the solid line. In the region $w \in [0, \frac{h}{\Delta}]$ the principal randomizes between the agent's continuation utilities of 0 and $\frac{h}{\Delta}$. For $w \geq \frac{h}{\Delta}$ continuation value process follows (4).

4 Economics behind the Optimal Contract

Value of delayed incentives. In the large literature on dynamic contracts the principal backloads payments to states of the world in which her shadow cost of monetary transfers is lowest, i.e. when the residual agency friction is sufficiently small. I show that by influencing how informed the agent is about his performance, the principal can also backload incentives to states of the world in which her shadow cost of continuation value volatility is low, e.g. when the principal no longer cares about demoralizing the agent. This is why at τ the agent gets nothing unless the highest likelihood path corresponding to $Z_\tau = 0$ is realized. This also allows the principal to "reuse" incentives as was shown in Fuchs (2007) in the context of subjective evaluations. This is most easily seen from (5). If we set $f_t = 0$ across periods, then sufficient incentive compatibility becomes

$$\frac{h}{\Delta} \leq w_0 < w_t \quad \text{for} \quad t > 0.$$

A single continuation value $w_0 \geq \frac{h}{\Delta}$ becomes sufficient to incentive effort from the agent in all subsequent periods.

The agent observes his contractable performance with a delay and this introduces the possibility of delaying performance sensitivity. Dynamic contracting papers such as Sannikov (2014) have this channel in play since the agent's performance sensitivity is spread across many periods. In this paper the principal can costlessly monitor projects recommended by the agent and yet under the optimal monitoring policy the monitoring frequency is below first best for $w < \frac{(1+\bar{f})h}{\Delta}$ leading the principal to invest into some bad quality projects. The assumption driving this result is that monitoring results are public and the principal wishes to implement high effort going forward even if the agent has recommended a bad project. This economic force is stronger when the agent is closer to his outside option and, thus, monitoring is limited following bad performance.

Interraction of monitoring and compensation. It is important that the compensation profile is designed jointly with the monitoring policy of the principal. The structure of my model is mostly are applicable to performance contracts within organizations as they require the supervisor being better informed than the worker. This is different from a large part of the literature which focused on CEO and CFO compensation because of data constraints. Because of this, my predictions on monitoring and pay for performance complement the monitoring structure analyzed in Piskorski and Westerfield (2016).

The first prediction is that the amount of optimal monitoring $f(\cdot)$ is monotone in performance. As long as no bad projects are screened out, agent's continuation value is increasing and so is the amount of monitoring he receives. This is quite intuitive – a well informed agent requires a large amount of expected compensation to exert effort. Thus when the promised utility is already high, the principal might as well publicly monitor performance. When the promised utility is low, the agency problem becomes binding and the only way to provide incentives is by keeping the agent in the dark.

The second prediction is that employees working for the firm for a long-time obtain the most monitoring/feedback on their effort, however this information has no implications on their future compensation. In the optimal contract this can be seen as $\beta(w) = 0$ and $f(w) = \bar{f}$ for $w \geq \frac{h}{\Delta(1-\bar{f})}$. Because agent's continuation value is so large, delayed incentives alone are sufficient to incentivize effort. Thus the agent is not punished for recommending bad projects that the principal succeeds at screening out. A consequence of this more broadly is that the agent is rewarded for being monitored if a bad project has been uncovered prior to retirement. A consequence of this is that the employees with intermediate performance, as proxied by the continuation value w close to $\frac{(1+\bar{f})h}{\Delta}$, face the highest contemporaneous pay for performance sensitivity.

Finally, if the agent's effort is sufficiently valuable, the optimal contract implements high effort and does not involve termination. In my model, instead of firing the agent, the principal can reduce monitoring and keep the agent in the dark about his performance. The continuation value, which is essentially the expected compensation, is then steadily increasing away from termination/low effort region.

Possible Implementations. The optimal contract derived in Section 3 requires the principal to condition agent's compensation at retirement on the entire performance path of the agent. As such, this paper does not carry a strong positive theory of existing contracts, but rather a more normative view of how contracts should be designed. Nevertheless, a possible implementation can be a dynamic budget which grows at a rate of r and the principal contributes h dollars to it each period. For a given investment project recommended by the agent, he commits $\beta(w)$ of his own funds, while the principal contributes the residual $1-\beta(w)$. The agent gets a riskless return r on good projects, while the principal claims the residual return $\mu - r$. At agent's retirement τ, the principal takes the residual budget of the agent and either multiplies it by $\frac{1}{p_\tau}$ if $Z_\tau = 0$, i.e. pays a bonus, or takes it all away.

Effect of risk-aversion. I solve for the optimal contract in the case of the principal and the agent being both risk-neutral. This follows a large literature on dynamic contracts DeMarzo and Sannikov (2006), Biais et al. (2010b), Varas (2013) among others. Risk-aversion and learning are important considerations in agency theory. If the agent were risk-averse, then the principal may find it costly to backload all of the incentives until the agent's retirement and the optimal contract would likely smooth out compensation across states of the world in which $Z_\tau > 0$. The economic force of placing more value in states of the world at which $Z_\tau = 0$ is still present following the informativeness principle of Holmstrom (1979), however it becomes very difficult to obtain a sufficient statistic for total delayed incentives. In a risk-neutral world this is quite simple since all of expected payment is made in the most informative node. This requires that the output process has an upper bound on the likelihood ratio of high effort, similar to the setting of Hoffmann, Inderst, and Opp (2017).

5 Renegotiation of Delayed Incentives

So far I've assumed that the agent can observe an intermediate signal about output X^A. If the agent shirks and does not observe an arrival of X^A, this returns him to equilibrium path. Otherwise, he knows there was a bad project and, by Lemma 1 he knows that he will get 0 at τ if the bad project was not screened out. This assumption led to a simple and tractable incentive compatibility constraint (5) which was key in deriving the optimal contract. As an unfortunate side-effect, however, it exogenously increases the agent's information rent since he has access to additional performance information even though it happens out of equilibrium. In this section I show that private observability of X^A is irrelevant in a renegotiation-proof contract. This happens because the principal may have an incentive to renegotiate the ex-post inefficient delayed punishments that might have been prescribed by the original contract.

I adopt the following notion of renegotiation-proofness from DeMarzo and Sannikov (2006) where the contract must be renegotiation-proof all following public histories of the game.

Definition 1. Contract $\mathcal{C}$ is renegotiation-proof if there is no history after which there exists an alternative contract $\hat{\mathcal{C}}$ which results in a weakly higher payoff for the agent and a strictly higher payoff for the principal.

The notion that dynamic contracts need not be renegotiation-proof has been highlighted in DeMarzo and Sannikov (2006). If the principal's value function is ever increasing in the agent's continuation value after some history, then she would benefit from offering the agent a new contract with a higher continuation value. Such promises break ex-ante promise keeping and potentially create incentives for bad performance of the agent. By this logic, if $v'\left(\frac{h}{\Delta}\right) \leq 0$, then the contract derived in Section 3 is renegotiation-proof.

Corollary 2. *Suppose $v'\left(\frac{h}{\Delta}\right) \leq 0$. Contract $\mathcal{C}^*$ is the optimal renegotiation-proof contract. Condition $v'\left(\frac{h}{\Delta}\right) \leq 0$ is satisfied if the players are sufficiently impatient $r \geq \frac{\lambda\Delta}{h}$.*

Corollary 2 states that in the setting of Section 2 a contract fails to be renegotiation-proof only if the principal benefits from offering a higher continuation value to the agent. Thus, renegotiation-proofness does not add anything beyond existing literature if the agent privately observes X^A. If the agent does not observe X^A, however, the players could benefit from restarting the contract and eliminate future punishments prescribed by prior incentive compatibility requirements.

Suppose the agent does not observe X^A and his information after every history coincides with public information $\mathbb{F}^Y$. Incentive compatibility (5) remains sufficient since the agent understands that even if he observed X^A, then he would have exerted effort regardless. However condition (5) is no longer necessary and the principal could do better as she is now contracting with an agent who observes less information about his performance. The analysis of the optimal contract in this

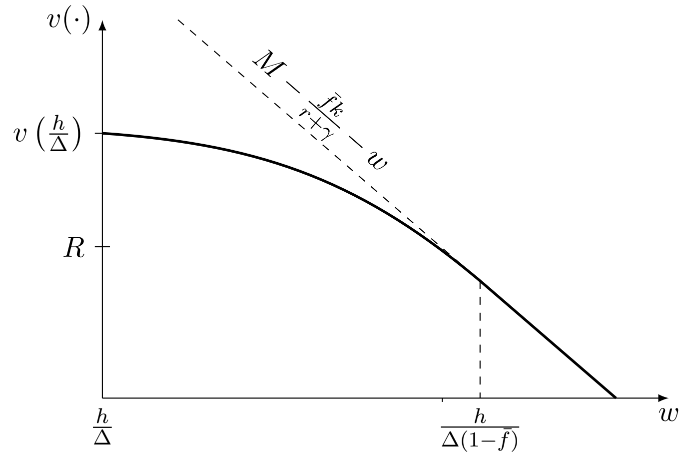

Figure 4: Principal's value function $v(\cdot)$ is downward sloping for $w \geq \frac{h}{\Delta}$ if r is sufficiently large. The optimal contract never enters region $w \in \left[0, \frac{h}{\Delta}\right)$ and so it is not plotted.

setting is quite complicated and is beyond the scope of this paper. However I am able to show that the optimal renegotiation-proof contract coincides with one obtained earlier if the resulting payoff frontier is strictly concave in w, which holds in the special case of $\bar{f} = 1$.

Proposition 4. *Assume the principal is sufficiently impatient $r \geq \frac{\lambda\Delta}{h}$, private effort cost h is small, and $\bar{f} = 1$. Suppose the agent does not observe process X^A and observes only the public filtration $\mathbb{F}^Y$. The optimal renegotiation-proof contract coincides with the one obtained in Proposition 2.*

Sketch of proof. The first step is to derive the necessary incentive compatibility conditions in this setting where the agent does not observe X^A. Because the principal and the agent do not observe process Z, the optimal contract still pays 0 to the agent at retirement if $Z_\tau > 0$. This implies that the agent's continuation value is given by

$$\begin{aligned}\mathrm{E}_{\hat{a}}\left[e^{-r\tau} \cdot C_\tau - \int_0^\tau e^{-rt}\hat{a}_t h\, dt\right] &= \mathrm{E}_{\hat{a}}\left[e^{-r\tau}\frac{w_\tau}{p_\tau}\mathbb{1}\{Z_\tau = 0\} - \int_0^\tau e^{-rt}\hat{a}_t h\, dt\right] \\ &= \mathrm{E}_{\hat{a}}\left[e^{-r\tau} \cdot w_\tau \cdot \frac{\hat{p}_\tau}{p_\tau} - \int_0^\tau e^{-rt} \cdot \hat{a}_t h\, dt\right]\end{aligned}$$

where $\hat{p}_t = \mathrm{P}_{\hat{a}}(Z_t = 0)$. If the agent shirks at time t, he affects his private belief $\hat{p}_t$ and, thus, reduces the expected reward from the contract. This leads him to, potentially, exert less effort in subsequent periods. The partial derivative with respect to $\hat{p}_0$ at $\hat{p}_0 = p_0 = 1$ of the above expression is given by $\mathrm{E}_{\hat{a}}\left[e^{-r\tau}w_\tau\right]$. Because the agent picks out-of-equilibrium effort optimally, this expression is evaluated with respect to the lowest (in terms of expected cost) effort $\hat{a}$ such that

$$\mathrm{E}_{\hat{a}}\left[e^{-r\tau}C_\tau - \int_0^\tau e^{-rt}\hat{a}_t h\,dt\right] = \mathrm{E}_{\bar{a}}\left[e^{-r\tau}C_\tau - \int_0^\tau e^{-rt}h\,dt\right] = w \tag{13}$$

For each history of the game define by A_t to be the smallest expected discounted cost of an effort profile which keeps the agent ex-ante indifferent between it and exerting high effort:

$$A_t = \inf_{\hat{a}\in\mathbb{F}^Y} \mathrm{E}\left[\int_t^\tau e^{-r(s-t)}\hat{a}_s\,ds \mid \mathcal{F}_t^Y\right]$$

subject to effort process $\hat{a}$ satisfying (13). High effort is incentive compatible going forward if

$$\frac{h}{\Delta} \leq f_t \cdot \beta_t + (1-f_t)\cdot(w_t + A_t). \tag{14}$$

The second step is to show that the payoff frontier needs to be concave in agent's promised utility. If it was not, then the principal can either increase f_t, or the first-best is attained which is infeasible for $\bar{f} = 1$. This is the only step that requires $\bar{f} = 1$. It implies that the agent's incentive compatibility condition (14) must be satisfied with equality, since the principal could otherwise reduce β_t and renegotiate in the future.

The final step is to show that it is necessarily the case that $A_t = 0$ along the path of the renegotiation-proof contract. If this were not the case, then one can show, by considering the relaxed problem of the agent, that he prefers to exert 0 effort after every history. If $A_t = 0$, then (14) is equivalent to (5).

Proposition 4 shows that the optimal renegotiation-proof contract does not depend on whether or not the agent can observe X^A if $\bar{f} = 1$. This provides partial justification for analyzing the original problem in which X^A is observed as it results in a clean optimal contract characterization and maps into the more difficult problem under some parameters.

6 Conclusion

In this paper I show that by keeping the agent uninformed about his output the principal can optimally delay incentive provision until the agent's retirement. This is a new idea in repeated moral hazard environments since it highlights the benefits of backloading not only pay, but also the sensitivity of pay with respect to current output. Opaqueness of the agent's information environment makes this possible. These results are hinged on the novel observation that rewards and information structures need to be understood jointly and that optimal information management is important even when the principal can commit to a wide array of compensation profiles to mitigate inefficiencies. As a result, poorly performing agent is not terminated, but receives less monitoring. On the other hand, after good performance, the agent is not punished when he recommends bad projects, but the principal screens out those projects. The steepest contemporaneous incentives are observed for intermediate levels of performance.

7 Appendix

Proof of Lemma 1 (optimal delayed incentives)

Proof. Any contract can be characterized by a payment process $\{C_t\}$. The agent retires at time τ and the principal rewards him with C_τ. The payment depends on information that the agent does not directly observe and, thus, there could be residual uncertainty right before retirement. As before, denote the continuation value of the agent if he exerts high effort to be

$$\begin{aligned} w_t &= \mathrm{E}\left[e^{-r(\tau-t)}C_\tau - \int_t^\tau e^{-r(s-t)}h\,ds \,\Big|\, \mathcal{F}_t^A\right] = \mathrm{E}\left[e^{-r(\tau-t)}C_\tau - \left(1-e^{-r(\tau-t)}\right)\frac{h}{r} \,\Big|\, \mathcal{F}_t^A\right] \\ &= \mathrm{E}\left[e^{-r(\tau-t)}\left(C_\tau + \frac{h}{r}\right) \,\Big|\, \mathcal{F}_t^A\right] - \frac{h}{r} = \mathrm{E}\left[e^{-r(\tau-t)}C_\tau \,\Big|\, \mathcal{F}_t^A\right] - \frac{h}{r+\gamma} \\ &= \mathrm{E}\left[e^{-r(\tau-t)}\mathrm{E}\left[C_\tau \,\Big|\, \mathcal{F}_\tau^A\right] \mid \mathcal{F}_t^A\right] - \frac{h}{r+\gamma} = \mathrm{E}\left[e^{-r(\tau-t)}w_\tau \,\Big|\, \mathcal{F}_t^Y\right] - \frac{h}{r+\gamma}. \end{aligned}$$

where the last equality holds since, when the agent exerts effort, his private information is equal to public information $\mathcal{F}_t^A = \mathcal{F}_t^A$.

Define process p_t to be the probability that the manager has not generated an bad project which was not disclosed if he exerted high effort:

$$p_t = \mathrm{P}\big(Z_t = 0 \,|\, \mathcal{F}_t^A\big) = \mathrm{P}\big(Z_t = 0 \,|\, \mathcal{F}_t^Y\big)$$

For a given monitoring policy F and corresponding process Y, consider an alternative compensation scheme

$$\hat{C}_t = \frac{w_t}{p_t} \cdot \mathbb{1}\{Z_t = 0\}.$$

If the agent exerts effort, then $\mathcal{F}_t^A = \mathcal{F}_t^Y$. This implies that for every $t < \tau$

$$\begin{aligned} \hat{w}_t &= \mathrm{E}\left[e^{-r(\tau-t)}\hat{C}_\tau \,\Big|\, \mathcal{F}_t^Y\right] - \frac{h}{r+\gamma} = \mathrm{E}\left[e^{-r(\tau-t)}\frac{w_\tau}{p_\tau}\mathbb{1}\{Z_\tau = 0\} \,\Big|\, \mathcal{F}_t^Y\right] - \frac{h}{r+\gamma} \\ &= \mathrm{E}\left[e^{-r(\tau-t)}\mathrm{E}\left[\frac{w_\tau}{p_\tau}\mathbb{1}\{Z_\tau = 0\} \,\Big|\, \mathcal{F}_\tau^Y\right] \,\Big|\, \mathcal{F}_t^Y\right] - \frac{h}{r+\gamma} = \mathrm{E}\left[e^{-r(\tau-t)}w_\tau \mid \mathcal{F}_t^Y\right] - \frac{h}{r+\gamma} = w_t \end{aligned}$$

If the agent exerts high effort, his payoff is exactly the same under new contract $(\hat{C}, F)$ as it was under the original contract (C, F).

Verification of incentive compatibility. Suppose the agent deviates to an alternative effort profile $\hat{a} = \{\hat{a}_t\}_{t\geq 0}$. The original contract was incentive compatible and we need to show that such a deviation is weakly suboptimal under the new contract. Define $\hat{p}_t$ to be

$$\hat{p}_t = \mathrm{P}\big(Z_t = 0 \,|\, \mathcal{F}_t^A\big) \begin{cases} p_t & \text{if} \quad X_t^A = 0 \\ 0 & \text{if} \quad X_t^A > 0 \end{cases}$$

Also, define

$$\hat{q}_t = \mathrm{P}_{\hat{a}}\big(X_t^A = 0 \,|\, \mathcal{F}_t^Y\big) = e^{-\int_0^t \Delta(1-\hat{a}_t)\, dt}.$$

Given process $\hat{p}_t$ I need to show the expected payoff under contract $(\hat{C}, F)$ is weakly lower than the payoff under contract (C, F). For the same effort level $\hat{a}$ the information sets of the agent are identical under (C, F) and $(\hat{C}, F)$ and thus the effort paths are identical.

Thus we need to only rank expected compensation:

$$\begin{aligned}
\mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} C_\tau\right] &= \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} \mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^A\right]\right] \\
&= \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} q_\tau \mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^A, X_\tau^A = 0\right] + (1-q_\tau)\mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^A, X_t^A > 0\right]\right] \\
&= \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} q_\tau \mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^Y\right] + (1-q_\tau)\mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^A, X_\tau^A > 0\right]\right] \\
&= \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} q_\tau w_\tau + (1-q_\tau)\mathrm{E}\left[C_\tau \,\middle|\, \mathcal{F}_\tau^A, X_\tau^A > 0\right]\right] \\
&\overset{(1)}{\geq} \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} q_\tau w_\tau\right] = \mathrm{E}_{0,\hat{a}}\left[e^{-r(\tau-t)} q_\tau \hat{C}_\tau\right]
\end{aligned}$$

where inequality (1) is strict unless $\mathrm{E}\left[C_\tau \,|\, \mathcal{F}_\tau^A, X_\tau^A > 0\right] = 0$. Thus, if contract (C, F) was incentive compatible, then contract $(\hat{C}, F)$ is also incentive compatible. □

Proof of Lemma 2 (application of martingale representation theorem)

Proof. Agent's ex-ante expected value conditional on $\tau \geq t$ is given by

$$\begin{aligned}
W_t &= \mathrm{E}\left[e^{-r\tau} w_\tau - \int_0^\tau e^{-rs} h\, ds \,\middle|\, \mathcal{F}_t^Y\right] \\
&= e^{-rt} \cdot \mathrm{E}\left[e^{-r(\tau-t)} w_\tau - \int_t^\tau e^{-r(s-t)} h\, ds \,\middle|\, \mathcal{F}_t^Y\right] - \int_0^t e^{-rs} h\, ds
\end{aligned}$$

Process $W = \{W_t\}$ is a Levy martingale. By Theorems T9 and T17 from Brémaud (1981) there exists a predictable process $\beta = \{\beta_t\}$ with respect to the filtration $\mathbb{F}^Y$ such that

$$dW_t = e^{-rt}\beta_t(\lambda f_t\, dt - dY_t)$$

On the other hand, applying Ito's lemma to W_t obtain:

$$dW_t = -re^{-rt}w_t\, dt + e^{-rt}dw_t - e^{-rt}h$$

Equating these two expressions

$$dw_t = rw_t\, dt + h\, dt + \beta_t(\lambda f_t\, dt - dY_t)$$

This proves the decomposition of the dynamics of agent's continuation value into promise keeping $(rw_t\, dt)$ and the pay for performance sensitivity term (β_t). □

Proof of Proposition 1 (incentive compatibility)

Proof. According to Lemma 1 there exists an optimal contract such that $C_\tau = 0$ if $Z_\tau = 0$. Output process X_t is given by

$$X_t = X_t^N + X_t^A \tag{15}$$

in which the agent privately observes process X^A. If the agent exerts high effort at time t, then $dX_t^A \equiv 0$. If the agent deviates, then dX_t^A could be positive. Consider an alternative effort process of the agent given by $\hat{a} = \{\hat{a}_t\}_{t\geq 0}$. For a given monitoring policy F and implied definitions of processes Y and Z we have

$$X_t = Y_t + Z_t.$$

Define an auxiliary process $G = \{G_t\}_{t\geq 0}$ as a stochastic integral

$$G_t = \int_0^t (X_s^A - X_{s-}^A - Y_s + Y_{s-})\, dX_s^A.$$

Process G is positive only if $dX_t^A > 0$, but at the same time $dY_t = 0$. This implies that the agent knows there was a bad project that the principal has invested in. This implies, according to Lemma 1, that the agent's compensation at retirement will be 0 regardless of what he does afterwards. Process G is a counting process with intensity $\Delta \cdot (1-\hat{a}_t)\cdot(1-f_t)$. Define the continuation value of the agent conditional on time t information from following effort $\hat{a}$ until time t and then following high effort:

$$\begin{aligned}\hat{W}_t &= e^{-rt}\mathrm{E}_t\left[e^{-r(\tau-t)}C_\tau - \int_t^\tau e^{-r(s-t)}h\,ds\right]\mathbb{1}\{\tau\geq t, G_t=0\} + e^{-r\tau}C_\tau\mathbb{1}\{\tau\leq t\} - \int_0^{t\wedge\tau}\hat{a}_s h\,ds\\ &= e^{-rt}w_t\cdot\mathbb{1}\{\tau\geq t, G_t=0\} + e^{-r\tau}w_\tau\mathbb{1}\{\tau\leq t\} - \int_0^{t\wedge\tau}\hat{a}_s h\,ds.\end{aligned}$$

Applying Ito's formula

$$e^{rt}d\hat{W}_t = -rw_t\,dt + dw_t - w_t\,dG_t - \hat{a}_t h\,dt.$$

The drift of the above expression is equal to

$$\begin{aligned}\mathrm{E}_t\left[e^{rt}d\hat{W}_t\right] &= -rw_t\,dt + rw_t\,dt + h\,dt - \beta_t\Delta(1-\hat{a}_t)f_t\,dt - w_t\Delta(1-\hat{a}_t)(1-f_t)\,dt - \hat{a}_t h\,dt\\ &= (1-\hat{a}_t)\Big(h - \beta_t\Delta f_t - w_t\Delta(1-f_t)\Big)\,dt\\ &= \Delta(1-\hat{a}_t)\left(\frac{h}{\Delta} - f_t\beta_t - (1-f_t)w_t\right)\,dt\end{aligned}$$

This implies that if there is a positive measure of points at which the incentive compatibility condition is not satisfied, there is a profitable deviation. This implies that incentive compatibility condition (5) is necessary for high effort to be incentive compatible. □

Proof of Proposition 2 (optimal monitoring)

The following sequence of results proves that the principal's value function is concave in w under the optimal control $\beta(w), f(w)$. In what follows $\mathrm{E}_t[\cdot] = \mathrm{E}\left[\cdot \mid \mathcal{F}_t^Y\right]$.

Proof. Assume there exists an optimal contract for every agent's continuation value w. This is an assumption that can be verified ex-post by verifying the optimality of the constructed contract. Define the principal's value function under the optimal contract to be

$$\begin{aligned}v(w) &= \max_{\mathcal{C}}\mathrm{E}_t\left[\int_t^\tau e^{-r(s-t)}(\mu\,ds - dX_s + dY_s) + e^{-r(\tau-t)}(R-C_\tau)\right]\\ &= \max_{\mathcal{C}}\mathrm{E}_t\left[\int_t^\tau e^{-r(s-t)}\mathrm{E}_s\left[(\mu\,ds - dX_s + dY_s)\right] + e^{-r(\tau-t)}(R-C_\tau)\right]\end{aligned}$$

subject to

$$w = \mathrm{E}\left[e^{-r(\tau-t)}C_\tau - \int_t^\tau e^{-r(s-t)} h\, ds\right].$$

Principal's value function $v(w)$ is weakly concave since the principal can use a public lottery across the agent's continuation values. Thus $v(\alpha w_1 + (1-\alpha)w_2) \geq \alpha v(w_1) + (1-\alpha)v(w_2)$.

Lemma 5. *Under contract terms $(\beta(w), f(w))$ the principal's value function satisfies the delay differential equation*

$$(r+\gamma)v(w) = \mu + \lambda(f(w)-1) + \gamma(R-w) + v'(w)(rw + h + \lambda f(w)\beta(w)) + \lambda f(w)(v(w-\beta(w)) - v(w)). \quad (16)$$

Proof. The ex-ante payoff to the principal conditional on the information available at time t is

$$V_t = \int_0^{\tau\wedge t} (\mu\, dt - dX_t + dY_t) - e^{-r\tau} C_\tau \mathbb{1}\{t > \tau\} + e^{-rt} v(w_t)\mathbb{1}\{t \leq \tau\}$$

Principal's objective is a martingale and thus for $t < \tau$ we can apply Ito's formula to $v(\cdot)$.

$$\begin{aligned} e^{rt} \cdot dV_t &= (\mu\, dt - dX_t + dY_t) + (R - C_t - v(w_t)) \cdot \mathbb{1}\{\tau \leq t\} - rv(w_t)\, dt \\ &\quad + v'(w_t)\Big(rw_t + h + \lambda f(w_t)\beta(w_t)\Big)\, dt + (v(w_t - \beta(w_t)) - v(w_t))\, dY_t \end{aligned}$$

Because V_t is a martingale, it implies that $\mathrm{E}_{t-}[dV_t] = 0$. Thus we can substitute the differentials dX_t and dY_t with their arrival intensities $\lambda\, dt$ and $\lambda f_t\, dt$ respectively. This implies

$$\begin{aligned} 0 &= \mu - \lambda + \lambda f(w_t) + \gamma(R - w_t - v(w_t)) - rv(w_t) \\ &\quad + v'(w_t)\Big(rw_t + h + \lambda f(w_t)\beta(w_t)\Big) + \lambda f(w_t)(v(w_t - \beta(w_t)) - v(w_t)) \end{aligned}$$

This implies that the value function of the principal must satisfy

$$(r+\gamma)v(w) = \mu + \lambda(f(w)-1) + \gamma(R-w) + v'(w)(rw + h + \lambda f(w)\beta(w)) + \lambda f(w)(v(w-\beta(w)) - v(w))$$

which proves the lemma. □

Lemma 6. *Incentives and monitoring $(\beta(w), f(w))$ are optimal only if they are the solutions to*

$$(r+\gamma)v(w) = \max_{f,\beta}\left[\mu + \lambda(f-1) + \gamma(R-w) + v'(w)\Big(rw + h + \lambda f\beta\Big) + \lambda f(v(w-\beta) - v(w))\right] \quad (17)$$

where the max is taken subject to the constraint $\frac{h}{\Delta} \leq f(w) \cdot \beta(w) + (1 - f(w)) \cdot w$.

Proof. Equation Suppose there exists a positive measure of w for which

$$(r+\gamma)v(w) \leq \max_{f,\beta}\left[\mu + \lambda(f-1) + \gamma(R-w) + v'(w)\Big(rw + h + \lambda f\beta\Big) + \lambda f(v(w-\beta) - v(w))\right]. \quad (18)$$

and the inequality is strict for a positive measure of continuation values w. Denote by $f^*(w), \beta^*(w)$ to be the solution to the above argmax for every w. Define the ex-ante value to the principal from following f^*, β^* up to time t, and then switching to the original contract $\mathcal{C}$:

$$\hat{V}_t = \int_0^{\tau\wedge t} (\mu\, dt - dX_t + dY_t) - e^{-r\tau} C_\tau \mathbb{1}\{t > \tau\} + e^{-rt} v(w_t) \mathbb{1}\{t \leq \tau\}$$

Using Ito's formula we have

$$\begin{aligned} e^{rt} d\hat{V}_t = (\mu\, dt - dX_t + dY_t) + (R - C_t - v(w_t)) \cdot \mathbb{1}\{\tau \leq t\} - rv(w_t)\, dt \\ + v'(w_t)\Big(rw_t + h + \lambda f^*(w_t)\beta^*(w_t)\Big)\, dt + (v(w_t - \beta^*(w_t)) - v(w_t))\, dY_t \end{aligned}$$

If (18) holds, then the above expression implies that $\hat{V}_t$ is a submartingale. According to Doob's optional sampling theorem it implies that $\mathrm{E}\left[\hat{V}_\tau\right] > \hat{V}_0$. This implies that the original contract is suboptimal. □

According to Lemma 6 and the fact that principal's objective is concave, it follows that the necessary and sufficient incentive compatibility condition must be binding under the optimal contract:

$$\frac{h}{\Delta} = f(w) \cdot \beta(w) + (1 - f(w)) \cdot w.$$

This implies that

$$\beta = \frac{h/\Delta - (1-f)w}{f} = \frac{h/\Delta - w}{f} + w \qquad \Rightarrow \qquad w - \beta = \frac{w - h/\Delta}{f}.$$

The first order condition of (17) with respect to f is

$$\lambda + \lambda v'(w)w + \lambda\left[v\left(\frac{w - h/\Delta}{f}\right) - v(w)\right] - \lambda\frac{w - h/\Delta}{f} \cdot v'\left(\frac{w - h/\Delta}{f}\right) \geq 0.$$

Dividing both sides by λ obtain

$$1 + v'(w)w - v(w) + v\left(\frac{w - h/\Delta}{f}\right) - \frac{w - h/\Delta}{f} \cdot v'\left(\frac{w - h/\Delta}{f}\right) \geq 0.$$

The maximum amount of feedback for $w \geq \frac{h}{\Delta}$ is given by

$$f(w) = \min\left(\frac{w - h/\Delta}{h/\Delta}, \bar{f}\right).$$

We need to check that at the maximum amount of feedback $f(w)$ and the implied $\beta(w)$ we have:

$$1 + v'(w)w - v(w) + v\left(\frac{w - h/\Delta}{f(w)}\right) - \frac{w - h/\Delta}{f(w)} \cdot v'\left(\frac{w - h/\Delta}{f(w)}\right) \geq 0.$$

Consider the two cases:

- Case 1. Suppose $w \leq (1 + \bar{f}) \cdot \frac{h}{\Delta}$. Thus $f(w) = \frac{w - h/\Delta}{h/\Delta}$. The maximum monitoring is optimal if and only if

  $$1 + wv'(w) + v(h/\Delta) - v(w) - \frac{h}{\Delta} \cdot v'(h/\Delta) \geq 0.$$

  The derivative of the above expression with respect to w is given by

  $$w \cdot v''(w) \leq 0.$$

- Case 2. Suppose $w > (1 + \bar{f}) \cdot \frac{h}{\Delta}$. Thus $f(w) = \bar{f}$ and the maximum monitoring is optimal if and only if

  $$1 + v'(w)w - v(w) + v\left(\frac{w - h/\Delta}{\bar{f}}\right) - \frac{w - h/\Delta}{\bar{f}} \cdot v'\left(\frac{w - h/\Delta}{\bar{f}}\right) \geq 0.$$

  The derivative of the above expression with respect to $\bar{f}$ is

  $$\frac{(w - h/\Delta)^2}{\bar{f}^3} \cdot v''\left(\frac{w - h/\Delta}{\bar{f}}\right) \leq 0$$

  Thus if the necessary inequality holds at $\tilde{f} = \frac{w - h/\Delta}{h/\Delta}$, maximum feedback $f(w)$ is sufficient.

This implies that the sufficient condition for $f(w)$ to be optimal is

$$1 + wv'(w) - v(w) + v\left(\frac{h}{\Delta}\right) - \frac{h}{\Delta} \cdot v'\left(\frac{h}{\Delta}\right) \geq 0$$

for all $w \leq \bar{w}$. Define the auxiliary function

$$g(w) = w \cdot v'(w) - v(w).$$

The condition for $f(w)$ to be optimal can be written as

$$1 + g(w) - g\left(\frac{h}{\Delta}\right) \geq 0.$$

Function $g(\cdot)$ is decreasing since $g'(w) = wv''(w) \leq 0$. Thus, if it holds at $w = \bar{w}$, it holds for all $w \in \left[\frac{h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right]$. At the upper threshold we have

$$\begin{aligned} g(\bar{w}) &= \bar{w} \cdot v'(\bar{w}) - v(\bar{w}) \\ &= -\bar{w} - \frac{\mu - \lambda(1-\bar{f}) + \gamma R}{r+\gamma} + \bar{w} \\ &= -\frac{\mu - \lambda(1-\bar{f}) + \gamma R}{r+\gamma}. \end{aligned}$$

Similarly

$$g\left(\frac{h}{\Delta}\right) = \frac{h}{\Delta} \cdot v'\left(\frac{h}{\Delta}\right) - v\left(\frac{h}{\Delta}\right).$$

Note that at $w = \frac{h}{\Delta}$ the objective of the agent satisfies the differential equation

$$\begin{aligned} (r+\gamma)v\left(\frac{h}{\Delta}\right) &= \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'\left(\frac{h}{\Delta}\right)\left(r\frac{h}{\Delta} + h\right) \\ (r+\gamma)v\left(\frac{h}{\Delta}\right) - (r+\gamma)\frac{h}{\Delta}v'\left(\frac{h}{\Delta}\right) &= \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + \left(1 - \frac{\gamma}{\Delta}\right)h \cdot v'\left(\frac{h}{\Delta}\right) \\ -(r+\gamma)g\left(\frac{h}{\Delta}\right) &= \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + \left(1 - \frac{\gamma}{\Delta}\right)h \cdot v'\left(\frac{h}{\Delta}\right) \end{aligned}$$

Then the sufficient condition for maximum feedback to be optimal is

$$\begin{aligned} 1 + g(\bar{w}) - g(h/\Delta) &= 1 - \frac{\mu - \lambda(1-\bar{f}) + \gamma R}{r+\gamma} + \frac{\mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + (1 - \frac{\gamma}{\Delta})h \cdot v'\left(\frac{h}{\Delta}\right)}{r+\gamma} \\ &= 1 - \frac{\lambda\bar{f}}{r+\gamma} + \frac{-\gamma\frac{h}{\Delta} + \left(1 - \frac{\gamma}{\Delta}\right)h \cdot v'\left(\frac{h}{\Delta}\right)}{r+\gamma} \\ &= \frac{r + \gamma - \lambda\bar{f} - h + \left(1 - \frac{\gamma}{\Delta}\right)h\left(v'\left(\frac{h}{\Delta}\right) + 1\right)}{r+\gamma} \end{aligned}$$

A sufficient condition for $(\beta(w), f(w))$ is optimal is given by

$$r + \gamma - \lambda \bar{f} - h + \left(1 - \frac{\gamma}{\Delta}\right) h \left(v'\left(\frac{h}{\Delta}\right) + 1\right) \geq 0 \tag{19}$$

This condition is satisfied if r is sufficiently large – i.e. the principal is sufficiently impatient. This follows from the fact that

$$v'\left(\frac{h}{\Delta}\right) = \frac{(r+\gamma)v(h/\Delta) - \mu + \lambda + \gamma(h/\Delta - R)}{rh/\Delta + h}$$

The derivative $v'\left(\frac{h}{\Delta}\right)$ is thus bounded above by

$$\begin{aligned} v'(h/\Delta) &= \frac{(r+\gamma)v(h/\Delta) - \mu + \lambda + \gamma(h/\Delta - R)}{rh/\Delta + h} \\ &\leq \frac{\mu + \lambda(\bar{f} - 1) + \gamma R - (r+\gamma)h/\Delta - \mu + \lambda + \gamma(h/\Delta - R)}{rh/\Delta + h} \\ &= \frac{\lambda \bar{f} - rh/\Delta}{rh/\Delta + h}. \end{aligned}$$

The same derivative is also bounded from below by

$$\begin{aligned} v'(h/\Delta) &= \frac{(r+\gamma)v(h/\Delta) - \mu + \lambda + \gamma(h/\Delta - R)}{rh/\Delta + h} \\ &\geq \frac{\mu + \lambda + \gamma R - (r+\gamma)h/\Delta - \mu + \lambda + \gamma(h/\Delta - R)}{rh/\Delta + h} \\ &= -\frac{rh/\Delta}{rh/\Delta + h}. \end{aligned}$$

Thus

$$v'(h/\Delta) \in \left[-\frac{r\frac{h}{\Delta}}{r\frac{h}{\Delta} + h}, \frac{\lambda \bar{f} - r\frac{h}{\Delta}}{r\frac{h}{\Delta} + h}\right] \subseteq \left[-1, -1 + \frac{\lambda \bar{f} \Delta}{rh}\right]$$

This implies that the sufficient condition (19) is satisfied if r is sufficiently large since $v'(h/\Delta)$ is bounded from both sides. The exact condition is

$$r + \gamma - \lambda \bar{f} - h + \min\left[0, \left(1 - \frac{\gamma}{\Delta}\right) \cdot \frac{\lambda \bar{f} \Delta}{rh}\right] \geq 0.$$

The condition above is a sufficient condition of optimality of $f(w)$. □

Proof of Lemma 3 (first best contract)

Proof. Need to show that for $w \geq \frac{h}{\Delta(1-\bar{f})}$ the optimal contract results in first best output

$$v(w) + w = \frac{\mu - \lambda(1-\bar{f}) - h + \gamma R}{r+\gamma}. \tag{20}$$

For these continuation values there exists an optimal contract that leads to a first best social surplus and delivers the agent exactly rent w_0. This implies that such a contract is optimal for the principal. If $w_0 \geq \bar{w}$ it implies that $\beta(w_0) = 0$. By Lemma 2 this implies that $\frac{dw_t}{dt} \geq 0$. Thus for $w \geq \bar{w} = \frac{h}{\Delta(1-\bar{f})}$

$$v(w) = \frac{\mu + \lambda(\bar{f}-1) - h + \gamma R}{r+\gamma} - w.$$

□

Proof of Proposition 3 (concavity of principal's value function)

The following sequence of results proves that the principal's value function is concave in w under the optimal control $\beta(w), f(w)$. In what follows $\mathrm{E}_t[\cdot] = \mathrm{E}\left[\cdot \mid \mathcal{F}_t^Y\right]$.

Lemma 7. *There exists a unique solution to* (16) *satisfying* (20).

Proof. First consider the case $\bar{f} < 1$.

- Step 1. There exists a unique solution to (16) satisfying an initial condition $v(\underline{w}) =$. In the region $w \in \left[\frac{h}{\Delta}, (1+\bar{f})\frac{h}{\Delta}\right)$ the delayed differential equation (16) is a first order differential equation. Thus there exists a unique solution satisfying the initial boundary condition. The rest of the solution can be constructed iteratively in each intervals $[w_n, w_{n+1})$ where $w_{n+1} = \bar{f} \cdot w_n + \frac{h}{\Delta}$.

- Step 2. By the same argument there exists a unique solution to the differential equation

$$(r+\gamma)v_G(w) = v_G'(w)(rw + h + \lambda f(w)\beta(w)) + \lambda f(w)(v_G(w-\beta(w)) - v_G(w))$$

  subject to the boundary condition $v_G(\underline{w}) = 1$. Function $v_G(\cdot)$ is strictly increasing. Since $f(\underline{w}) = 0$ and thus

$$v_G'(h/\Delta) = \frac{(r+\gamma)v_G(h/\Delta)}{rh/\Delta + h} > 0$$

Let $\tilde{w} = \inf\{w : v'_G(\tilde{w}) \leq 0\}$.

$$v'_G(\tilde{w})(r\tilde{w} + h + \lambda f(\tilde{w})\beta(\tilde{w})) = (r+\gamma)v_G(\tilde{w}) + \lambda f(\tilde{w}) \cdot \Big(v_G(\tilde{w}) - v_G(\tilde{w} - \beta(\tilde{w}))\Big) > 0$$

since $v_G(\tilde{w}) - v_G(\tilde{w} - \beta(\tilde{w})) \geq 0$ and $v_G(\tilde{w}) \geq v_G(h/\Delta) > 0$.

– Step 3. By the argument in Step 1 there exists a unique solution to the differential equation

$$\begin{aligned}(r+\gamma)v_P(w) = \mu + \lambda(f(w)-1) + \gamma(R-w) + v'_P(w) \cdot (rw + h + \lambda f(w)\beta(w)) \\ + \lambda f(w) \cdot (v_P(w - \beta(w)) - v_P(w))\end{aligned}$$

subject to the boundary condition $v_P(h/\Delta) = \frac{\mu - \lambda + \gamma R}{r+\gamma} - h/\Delta$.

– Step 4. Any solution to (16) satisfying an initial condition $v(h/\Delta) = A$ is given by $v(w) = v_P(w) + A \cdot v_G(w)$ for some constant parameter A. Because (16) is linear, it implies that $v_P(w) + A \cdot v_G(w)$ satisfies (16). At $w = h/\Delta$

$$v_P(h/\Delta) + A \cdot v_G(w/\Delta) = 0 + A \cdot 1 = A.$$

Because of Step 1, this implies that $v(w) = v_P(w) + A \cdot v_G(w)$ for $w \geq h/\Delta$.

– Step 5. To satisfy the boundary condition at $w = \bar{w}$ define A via an equation

$$v_P(\bar{w}) + Av_G(\bar{w}) = \frac{\mu - h + \gamma R}{r+\gamma} - \bar{w}$$

Thus function

$$v(w) = v_P(w) + \frac{\frac{\mu - h + \gamma R}{r+\gamma} - \bar{w} - v_P(\bar{w})}{v_G(\bar{w})} \cdot v_G(w)$$

is the unique solution to (16) satisfying the boundary condition (20).

Now consider the case $\bar{f} = 1$.

– Step 1. Index $v(w, \bar{f})$ is the value function of the principal if $f(w) \leq \bar{f}$. The previous step identified that $v(w, \bar{f})$ is the expected integral of value under the contract defined by initial continuation value w_0 and functions $\beta(w), f(w)$. By the optimality of $f(w)$ proven in the previous proposition it follows that the expected value of the contract can only increase if $\bar{f}$ was higher. Thus $\frac{\partial v(w,\bar{f})}{\partial \bar{f}} \geq 0$.

– Step 2. There exists a unique monotone limit $v(w,1) = \lim_{\bar{f}\to 1} v(w,\bar{f})$. Functions $v(w,\bar{f}_n)$ converge monotonically for any $\bar{f}_n \to 1$. Because each $v(w,\bar{f}_n)$ satisfies (16), this implies the convergence is uniform in any compact interval $C \in [h/\Delta, +\infty)$. Moreover, because $v(w,\bar{f}_n)$ satisfy (16), it also implies that $\partial_w v(w,\bar{f}_n)$ converge uniformly to some function $g(w)$. It follows that $\partial_w v(w,1) = g(w)$ and, thus, $v(w,1)$ satisfies (16).

– Step 3. It remains to show that

$$\lim_{w\to\infty}(v(w,1)+w) \to \frac{\mu - h + \gamma R}{r+\gamma}$$

The social surplus and thus the original optimal control is still feasible. Thus $v(w,\bar{f}) + w$ is a monotone function in w with the upper bound of $\frac{\mu+\lambda(\bar{f}-1)-h+\gamma R}{r+\gamma}$.

This concludes the proof that we can compute principal's value function by solving for the ordinary differential equation (16) subject to (20). □

Lemma 8. *Function $v(\cdot)$ defined by* (16) *satisfying boundary condition* (20) *is strictly concave at $w = \frac{h}{\Delta}$ if $r \geq \gamma\bar{f}$.*

Proof. Differentiating (16) with respect to w at $w = \frac{h}{\Delta}$

$$(r+\gamma)v'\left(\frac{h}{\Delta}\right) = \frac{\lambda\Delta}{h} - \gamma + v''\left(\frac{h}{\Delta}\right)\left(r\frac{h}{\Delta} + h\right) + rv'\left(\frac{h}{\Delta}\right)$$
$$\gamma v'\left(\frac{h}{\Delta}\right) = \frac{\lambda\Delta}{h} - \gamma + v''\left(\frac{h}{\Delta}\right)\left(r\frac{h}{\Delta} + h\right)$$

In order for $v(w)$ to be concave at $w = \frac{h}{\Delta}$ we must have

$$\gamma\left(v'\left(\frac{h}{\Delta}+1\right)\right) - \frac{\lambda\Delta}{h} \leq 0$$

A sufficient condition is

$$\gamma \cdot \frac{\lambda\bar{f}\Delta}{rh} \leq \frac{\lambda\Delta}{h}$$
$$\gamma \cdot \frac{\bar{f}}{r} \leq 1$$
$$\gamma\bar{f} \leq r$$

Thus, if $r \geq \gamma\bar{f}$, it implies that $v(w)$ is weakly concave at $w = \frac{h}{\Delta}$. □

Lemma 9. *Function $v(\cdot)$ defined by* (16) *satisfying boundary condition* (20) *satisfies $v''(\bar{w}) = 0$.*

Proof. In the neighborhood of $w = \bar{w}$ (16) is given by

$$(r+\gamma)v(w) = \mu+\lambda(\bar{f}-1)+\gamma(R-w)+v'(w)\left(rw+h+\lambda\left(\frac{h}{\Delta}+(\bar{f}-1)w\right)\right)+\lambda\bar{f}\left(v\left(\frac{w-h/\Delta}{\bar{f}}\right)-v(w)\right)$$

Differentiating both sides with respect to w in the vicinity of $w = \bar{w}$

$$(r+\gamma)v'(w) = -\gamma + v'(w)\left(r+\lambda(\bar{f}-1)\right) + v''(w)\left(rw+h+\lambda\left(\frac{h}{\Delta}+(\bar{f}-1)w\right)\right)$$
$$+\lambda\bar{f}\left(v'\left(\frac{w-h/\Delta}{\bar{f}}\right)\frac{1}{\bar{f}} - v'(w)\right)$$

Simplifying the terms this becomes

$$(\gamma+\lambda)v'(w) = -\gamma + v''(w)\left(rw+h+\lambda\left(\frac{h}{\Delta}+(\bar{f}-1)w\right)\right) + \lambda v'\left(\frac{w-h/\Delta}{\bar{f}}\right)$$

Substituting $\bar{w} = \frac{h}{\Delta(1-\bar{f})}$ we obtain

$$\gamma(v'(\bar{w})+1) = v''(\bar{w})\cdot(r\bar{w}+h)$$

At $w = \bar{w}$ this results in $v''(\bar{w}) = 0$. □

Lemma 10. *Function $v''(w)$ is continuous over $\left[\frac{h}{\Delta}, \frac{(1+\bar{f})h}{\Delta}\right)$ and over $\left[\frac{(1+\bar{f})h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right]$. The first derivative $v'(w)$ is continuous in w.*

Proof. Function $v(w)$ is continuous. This implies that function $v'(w)$ backed out of (16) is also continuous.

The value function satisfies the differential equation

$$(r+\gamma)v(w) = \mu - \lambda(1-f(w)) + \gamma(R-w) + v'(w)(rw+h-\lambda f(w)\beta(w))$$
$$+\lambda f(w)(v(w-\beta(w)) - v(w))$$

There are three regions within which $f(w), \beta(w)$ are continuously differentiable given by $\left(\frac{h}{\Delta}, \frac{(1+\bar{f})h}{\Delta}\right)$ and $\left(\frac{(1+\bar{f})h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right)$ and $\left(\frac{h}{\Delta(1-\bar{f})}, +\infty\right)$. Within each set the second derivative is continuous. □

Lemma 11. *The solution to* (16) *satisfying* (20) *is strictly concave for $w \in \left[\frac{h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right)$.*

Proof. Define

$$w^* = \inf\left\{w \geq \frac{h}{\Delta} \,\middle|\, v''(w) \geq 0\right\}.$$

According to Lemma 8 it follows that $w^* > \underline{w}$. Because $v''(\cdot)$ is upper-semi continuous according to Lemma 10 it follows that $v''(w^*) \geq 0$ and $v''(w) < 0$ for all $w < w^*$.

Case 1. Suppose $w^* \in \left(\frac{(1+\bar{f})h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right)$. Then $f(w^*) = \bar{f}$ and $\beta(w^*) = w^* + \frac{\frac{h}{\Delta} - w^*}{\bar{f}}$. Equation (16) becomes

$$(r+\gamma)v(w) = \mu + \lambda(\bar{f}-1) + \gamma(R-w) + v'(w)(rw + h + \lambda\bar{f}(\beta(w))) + \lambda\bar{f}(v(w-\beta(w)) - v(w)).$$

Differentiating both sides with respect to w in the vicinity of w^*

$$\gamma v'(w) = -\gamma + v''(w)(rw + h + \lambda\bar{f}\beta(w)) + v'(w)\lambda\bar{f}\beta'(w) + \lambda\bar{f}(v'(w-\beta(w))(1-\beta'(w)) - v'(w))$$
$$\gamma v'(w) = -\gamma + v''(w)(rw + h + \lambda\bar{f}\beta(w)) + v'(w)\lambda(\bar{f}-1) + \lambda\bar{f}\left(v'(w-\beta(w))\frac{1}{\bar{f}} - v'(w)\right)$$

It is useful to write it out as

$$(\gamma+\lambda)v'(w) = -\gamma + v''(w)(rw + h + \lambda\bar{f}\beta(w)) + \lambda v'\left(\frac{w - \frac{h}{\Delta}}{\bar{f}}\right). \tag{21}$$

Differentiating again with respect to w

$$(\gamma+\lambda)v''(w) = v'''(w)(rw + h + \lambda\bar{f}(\beta(w))) + \lambda v''\left(\frac{w - \frac{h}{\Delta}}{\bar{f}}\right) \tag{22}$$

Then at $w = w^*$

$$v'''(w^*) = \frac{(\gamma+\lambda)v''(w^*) - \lambda v''\left(\frac{w^* - \frac{h}{\Delta}}{\bar{f}}\right)}{rw^* + h + \lambda\bar{f}(\beta(w^*))} > 0$$

since $v''\left(\frac{w^* - \frac{h}{\Delta}}{\bar{f}}\right) < 0$ and $v''(w^*) \geq 0$. This implies that $v''(w)$ is strictly positive in some right neighborhood of w^* given by $(w^*, w^* + \varepsilon)$.

Define

$$\tilde{w} = \inf\left\{w > w^* \,\middle|\, v''(w^*) \leq 0\right\}.$$

According to Lemma 9 $v''(\bar{w}) = 0$ and it follows that $\tilde{w} \leq \bar{w}$. Because $v''(w)$ is continuous in

$\left\{w : w \geq w^* \geq \frac{(1+\bar{f})h}{\Delta}\right\}$ it follows that $v''(\tilde{w}) = 0$.

Suppose $\tilde{w} - \beta(\tilde{w}) < w^*$. Then consider equation (22)

$$(\gamma + \lambda)v''(\tilde{w}) = v'''(\tilde{w})(r\tilde{w} + h + \lambda\bar{f}\beta(\tilde{w})) + \lambda v'' \left(\frac{\tilde{w} - \frac{h}{\Delta}}{\bar{f}}\right).$$

Since $v''(\tilde{w}) = 0$ this implies that $v'''(\tilde{w}) > 0$ since $v''\left(\frac{\tilde{w}-\frac{h}{\Delta}}{\bar{f}}\right) < 0$. Thus it cannot be the case that $v''(\tilde{w}) = 0$ and $\tilde{w}$ is the first point where it crosses 0 after w^*. This contradicts the fact that $\tilde{w} - \beta(\tilde{w}) < w^*$ and implies that $\tilde{w} - \beta(\tilde{w}) \geq w^*$.

At $\tilde{w}$ equation (21) is

$$(\gamma + \lambda)v'(\tilde{w}) = -\gamma + v''(\tilde{w})(r\tilde{w} + h + \lambda\bar{f}\beta(\tilde{w})) + \lambda v' \left(\frac{\tilde{w} - \frac{h}{\Delta}}{\bar{f}}\right)$$

$$(\gamma + \lambda)v'(\tilde{w}) + \gamma = \lambda v' \left(\frac{\tilde{w} - \frac{h}{\Delta}}{\bar{f}}\right).$$

This implies

$$v'\left(\frac{\tilde{w} - \frac{h}{\Delta}}{\bar{f}}\right) = v'(\tilde{w}) + \frac{\gamma}{\lambda}(v'(\tilde{w}) + 1) \geq v'(\tilde{w})$$

which contradicts the fact that $v(w)$ is strictly convex in the region $(w^*, \tilde{w})$. This implies that if there exists a $w^* < \bar{w}$ satisfying $v''(w^*) \geq 0$, then it cannot be greater than $\frac{(1+\bar{f})h}{\Delta}$.

Case 2. Consider the alternative: $w^* \in \left[\frac{h}{\Delta}, \frac{(1+\bar{f})h}{\Delta}\right)$. Then for ever $w \in \left[\frac{h}{\Delta}, w^*\right]$ we have $\beta(w) = w - \frac{h}{\Delta}$ and $f(w) = \frac{w-h/\Delta}{h/\Delta}$. Equation (16) becomes

$$(r+\gamma)v(w) = \mu + \lambda(f(w) - 1) + \gamma(R - w) + v'(w)\left(rw + h + \lambda f(w) \cdot \beta(w)\right) + \lambda f(w)\left(v\left(\frac{h}{\Delta}\right) - v(w)\right).$$

Differentiating this ODE with respect to w

$$(r+\gamma)v'(w) = \lambda f'(w) - \gamma + v''(w)\left(rw + h + \lambda\frac{\left(w - \frac{h}{\Delta}\right)^2}{\frac{h}{\Delta}}\right) + v'(w)\left(r + 2\lambda\frac{w - \frac{h}{\Delta}}{\frac{h}{\Delta}}\right)$$
$$+ \frac{\lambda\Delta}{h}\left(v\left(\frac{h}{\Delta}\right) - v(w)\right) - \lambda\frac{w - \frac{h}{\Delta}}{\frac{h}{\Delta}}v'(w)$$

$$\gamma(v'(w) + 1) = \frac{\lambda\Delta}{h} + v''(w)\left(rw + h + \lambda\frac{\left(w - \frac{h}{\Delta}\right)^2}{\frac{h}{\Delta}}\right) + v'(w)\lambda\frac{w - \frac{h}{\Delta}}{\frac{h}{\Delta}} + \frac{\lambda\Delta}{h}\left(v\left(\frac{h}{\Delta}\right) - v(w)\right)$$

If $w^* \neq \frac{(1+\bar{f})h}{\Delta}$ then $v''(w^*) = 0$. Then at $w = w^*$

$$\gamma(v'(w)+1) - \frac{\lambda\Delta}{h} = \lambda\left[v'(w)\frac{w-\frac{h}{\Delta}}{\frac{h}{\Delta}} + \frac{\Delta}{h}\left(v\left(\frac{h}{\Delta}\right) - v(w)\right)\right]$$

$$\left(w - \frac{h}{\Delta}\right)\cdot\left[\gamma(v'(w)+1) - \frac{\lambda\Delta}{h}\right] = \lambda\left(w - \frac{h}{\Delta}\right)\cdot\left[v'(w)\frac{w-\frac{h}{\Delta}}{\frac{h}{\Delta}} + \frac{\Delta}{h}\left(v\left(\frac{h}{\Delta}\right) - v(w)\right)\right]$$

Then substituting this into the principal's HJB at $w = w^*$

$$(r+\gamma)v(w) = \mu + \lambda(f(w)-1) + \gamma(R-w) + v'(w)(rw+h) + (w - h/\Delta)\left(\gamma(v'(w)+1) - \frac{\lambda}{h/\Delta}\right),$$

$$(r+\gamma)v(w) = \mu + \lambda(f(w)-1) + \gamma(R-w) + v'(w)(rw+h) + (w-h/\Delta)\gamma(v'(w)+1) - \lambda f(w),$$

$$(r+\gamma)v(w) = \mu - \lambda + \gamma(R-w) + v'(w)(rw+h) + (w-h/\Delta)\gamma(v'(w)+1),$$

$$(r+\gamma)v(w) = \mu - \lambda + \gamma(R-w) + v'(w)(rw+h+\gamma(w-h/\Delta)) + \gamma(w-h/\Delta),$$

$$(r+\gamma)v(w) = \mu - \lambda + \gamma(R-h/\Delta) + v'(w)(rw+h+\gamma(w-h/\Delta)).$$

Because $v(w)$ is strictly concave in $\left[\frac{h}{\Delta}, w^*\right)$ it follows that

$$v\left(\frac{h}{\Delta}\right) + v'(w)\left(w - \frac{h}{\Delta}\right) < v(w).$$

Thus

$$(r+\gamma)\left(v\left(\frac{h}{\Delta}\right) + v'(w)\left(w-\frac{h}{\Delta}\right)\right) < \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'(w)\left(rw + h + \gamma\left(w - \frac{h}{\Delta}\right)\right).$$

Simplifying this expression

$$(r+\gamma)v\left(\frac{h}{\Delta}\right) < \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'(w)\left(rw + h + \gamma\left(w-\frac{h}{\Delta}\right) - (r+\gamma)\left(w - \frac{h}{\Delta}\right)\right)$$

$$(r+\gamma)v\left(\frac{h}{\Delta}\right) < \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'(w)\left(r\frac{h}{\Delta} + h\right)$$

However it contradicts the fact that function $v(w)$ satisfies (6) at $w = \frac{h}{\Delta}$. □

Proof of Lemma 4 (optimality of no termination)

Proof. To provide a simple sufficient parametric condition for high effort to be optimal I assume that μ is sufficiently large that the principal would rather employ the agent with no effort, than terminate him:

$$\mu - (\lambda + \Delta)(1 - \bar{f}) - c\bar{f} \geq rR.$$

where c is the cost of monitoring that needs to be positive if $\bar{f} = 1$.

Then if $w < \frac{h}{\Delta}$ the agent cannot be incentivized to exert effort. Absent public randomization, principal's value function evolves according to the differential equation

$$(r + \gamma)v(w) = \mu - (\lambda + \Delta)(1 - \bar{f}) - c\bar{f} + \gamma(R - w) + v'(w)(rw + h).$$

Taking the derivative with respect to w:

$$(r + \gamma)v'(w) = -\gamma + v'(w)r + v''(w)(rw + h)$$

$$v''(w) = \frac{\gamma(v'(w) + 1)}{rw + h}$$

Since $v'(w) < 1$ under the optimal contract, it follows that $v''(w)$ is positive. Thus, in the region $w < \underline{w}$ the optimal contract requires public randomization (otherwise $v(w)$ is convex which then requires public randomization again). This implies that for $w < \underline{w}$ the principal's value function is given by

$$v(w) = v(0) \cdot \frac{\frac{h}{\Delta} - w}{\frac{h}{\Delta}} + v\left(\frac{h}{\Delta}\right) \cdot \frac{w}{\frac{h}{\Delta}}$$

A sufficient condition for the principal to not wish to start with $w = 0$ is

$$-(\lambda + \Delta)(1 - \bar{f}) - c\bar{f} \leq -\lambda - (r + \gamma)\frac{h}{\Delta}.$$

The above condition is satisfied if Δ is sufficiently large and $\bar{f} < 1$.

In order for the monitoring frequency $f(w) \leq \frac{w - \frac{h}{\Delta}}{\frac{h}{\Delta}}$ we need to show that the principal does not wish for the w to end up in $[0, \frac{h}{\Delta}]$ where randomization between high and low effort would have been optimal.

As in the proof of Proposition 2 define

$$g(w) = wv'(w) - v(w).$$

In order for Proposition 2 to hold it would be sufficient that

$$1 + g(w) - \lim_{\varepsilon \downarrow 0} g\left(\frac{h}{\Delta} + \varepsilon\right) \geq 0$$
$$1 + g(w) - \lim_{\varepsilon \downarrow 0} g\left(\frac{h}{\Delta} - \varepsilon\right) \leq 0$$

As I showed earlier, the first inequality holds if the players are sufficiently impatient. Need to check the second inequality. Note that $g\left(\frac{h}{\Delta}-\right) = -v(0)$. Thus need to check

$$1 + g(w) + v(0) \leq 0.$$

Since $g(w)$ is decreasing, thus if the above holds at $w = \frac{h}{\Delta}$, it holds for all $w \in \left[\frac{h}{\Delta}, \frac{h}{\Delta(1-\bar{f})}\right]$. Thus the sufficient condition is

$$1 + \frac{\mu - (\lambda + \Delta)(1 - \bar{f}) - c\bar{f} + \gamma R}{r + \gamma} + v'\left(\frac{h}{\Delta}\right)\frac{h}{\Delta} - v\left(\frac{h}{\Delta}\right) \leq 0.$$

This implies that is sufficient to show that

$$v\left(\frac{h}{\Delta}\right) \geq v'\left(\frac{h}{\Delta}\right) \cdot \frac{h}{\Delta} + \frac{r + \gamma + \mu - (\lambda + \Delta)(1 - \bar{f}) - c\bar{f} + \gamma R}{r + \gamma}. \tag{23}$$

Because for $w \geq \frac{h}{\Delta}$ principal's value function $v(\cdot)$ satisfies (16)

$$(r + \gamma)v\left(\frac{h}{\Delta}\right) = \mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'\left(\frac{h}{\Delta}\right)\left(r\frac{h}{\Delta} + h\right)$$

Substituting this into (23) the sufficient condition becomes

$$\frac{\mu - \lambda + \gamma\left(R - \frac{h}{\Delta}\right) + v'\left(\frac{h}{\Delta}\right)\left(r\frac{h}{\Delta} + h\right)}{r + \gamma} \geq v'\left(\frac{h}{\Delta}\right) \cdot \frac{h}{\Delta} + \frac{r + \gamma + \mu - (\lambda + \Delta)(1 - \bar{f}) - c\bar{f} + \gamma R}{r + \gamma}$$
$$\frac{-\gamma\frac{h}{\Delta} + v'\left(\frac{h}{\Delta}\right)\left(-\gamma\frac{h}{\Delta} + h\right)}{r + \gamma} \geq \frac{r + \gamma + \lambda\bar{f} - \Delta(1 - \bar{f})}{r + \gamma}$$

This implies

$$v'(h/\Delta) \cdot \frac{h - \gamma\frac{h}{\Delta}}{r + \gamma} \geq \frac{r + \gamma + \lambda\bar{f} - \Delta(1 - \bar{f}) + \gamma\frac{h}{\Delta}}{r + \gamma},$$
$$v'(h/\Delta)\left(h - \gamma\frac{h}{\Delta}\right) \geq r + \gamma + \lambda\bar{f} - \Delta(1 - \bar{f}) + \gamma\frac{h}{\Delta}.$$

If $\Delta > \gamma$, then $h - \gamma\frac{h}{\Delta} \geq 0$. The lower bound for $v'(h/\Delta)$ is -1. Thus a sufficient condition for the above to be satisfied is

$$\frac{\gamma h}{\Delta} - h \geq r + \gamma + \lambda\bar{f} - \Delta(1-\bar{f}) + \gamma\frac{h}{\Delta}.$$
$$0 \geq r + \gamma + \lambda\bar{f} - \Delta(1-\bar{f}) + h.$$

which is satified if Δ is sufficiently large. A large Δ does not contradict the parametric assumptions of Proposition 2 and can be satisfied for reasonable parameters. □

Proof of Proposition 4 (optimal renegotiation)

Proof. Define by $b(w)$ the value function of the principal under the optimal renegotiation-proof contract. The social surplus is given by $b(w) + w$ and must be concave in w. Otherwise the principal can use public randomization to generate a strict improvement.

Lemma 12. *Under any optimal renegotiation-proof contract $\mathcal{C}^R$ it must be the case that*

$$h \geq \Delta \cdot \Big(f_t \cdot \beta_t + (1-f_t) \cdot w_t\Big). \tag{24}$$

Proof. Suppose there exists an optimal, incentive compatible, renegotiation-proof contract $\mathcal{C}^R = (C^R, F^R)$ for every initial continuation value w for the agent. For each w define the set of effort levels $\mathcal{A}(w)$ such that

$$\mathcal{A}(w) = \left\{\hat{a} = \{\hat{a}_t\}_t \geq 0 : \mathrm{E}_{\hat{a}}\left[e^{-r\tau}C_\tau - \int_0^\tau e^{-rt}\hat{a}_t h\, dt\right] = \mathrm{E}_{\bar{a}}\left[e^{-r\tau}C_\tau - \int_0^\tau e^{-rt} h\, dt\right]\right\}.$$

Then define

$$A(w) = \inf_{\hat{a}\in\mathcal{A}(w)} \mathrm{E}\left[\int_0^\tau e^{-rt}\hat{a}_t h\, dt\right] \tag{25}$$

Function $A(w)$ is the minimum expected amount of effort the agent needs to exert to be indifferent between his deviation and high effort in the incentive compatible contract $\mathcal{C}^R$. Note that it is without loss of generality to think for $A(\cdot)$ being indexed by agent's continuation utilities. It would be equivalent to index it by the associated public history of the contract. The only relevant property is that it is non-negative.

Define $q(w, \pi)$ to be the continuation value of the agent if the probability that the expected compensation C_τ is $\pi \leq 1$. If the agent exerts high effort then, along the equilibrium path, $\pi = 1$.

However if $\pi < 1$, then the agent might wish to deviate to an alternative effort level:

$$q(w,\pi) = \sup_{\hat{a}} \mathrm{E}_{\hat{a}} \left[\pi \cdot e^{-r\tau} C_\tau - \int_0^\tau e^{-rt} \hat{a}_t h \, dt \right].$$

According to the envelope theorem:

$$\frac{\partial q}{\partial \pi}(w,\pi) = \mathrm{E}_{\hat{a}} \left[e^{-r\tau} C_\tau \right] = w + A(w)$$

where $A(w)$ is defined in (25). This is necessary since, otherwise, by following effort $A(w)$ the agent could achieve a higher expected payout than $q(w,\pi)$ in some neighborhood $\pi \in (1-\varepsilon, 1)$, which would contradict the definition of $q(w,\pi)$. Following Proposition 1 of Sannikov (2014), the necessary local incentive compatibility condition is given by:

$$h \leq \Delta \Big(f(w)\beta(w) + (1 - f(w)) \cdot \frac{\partial q}{\partial \pi}(w,\pi) \Big). \tag{26}$$

This is based on Lemma (14) which proves that the agent is only compensated in the high state in a renegotiation-proof contract in which X^A is not privately observed by the agent.

In a renegotiation-proof contract the principal cannot commit to delayed punishments if such incentives are not dynamically credible. Every period the principal can offer a new "forward-looking" contract which does not condition on information prior to that period. This implies that, when choosing (f_t, β_t), in period t, the principal does not take into account the effect this had on incentive compatibility in periods $s < t$. Note that this issue does not arise in contracts in which the agent directly observes performance since the sensitivity of agent's compensation to effort at time s is resolved at time s, and has no implication on incentives at time t.

Using the above argument I show below that (26) is binding under $\mathcal{C}^R$. From the contrary, suppose (26) is strict:

$$\frac{h}{\Delta} < f(w)\beta(w) + (1 - f(w))(w + A(w)).$$

By arguments similar to the proof of Proposition 3 the value function of the principal under the optimal renegotiation-proof contract must satisfy

$$(r+\gamma)b(w) = \max_{\beta, f} \Big[\mu + \lambda(f-1) - cf + \gamma(R - w) + b'(w)(rw + h + \lambda f \beta) + \lambda f (b(w-\beta) - b(w)) \Big].$$

For a given level of monitoring f the first order condition with respect to β at $h = h/\Delta$

$$\lambda f b'(w) - \lambda f(b'(w-\beta)) = 0.$$

This implies that $b'(w-\beta) = b'(w)$. Then taking derivatives of the left and right hand sides using the envelope theorem:

$$(r+\gamma)b'(w) = -\gamma + b''(w)(rw + h + \lambda f\beta) + b'(w)r + \lambda f(b'(w-\beta) - b'(w)) \tag{27}$$

$$\gamma(b'(w)+1) = b''(w)(rw + h + \lambda f\beta) \tag{28}$$

Note that $b''(w) \leq 0$, while $b'(w) \geq -1$. This implies that if the local incentive compatibility constraint is not binding, then $b'(w) = -1$. However this implies that $w \geq \bar{w} = +\infty$ for $\bar{f} = 1$ which is a contradiction. Thus, the local incentive compatibility condition must be binding under the optimal renegotiation-proof contract. Thus

$$h = \Delta \cdot \Big(f_t \cdot \beta_t + (1-f_t)\cdot(w_t + A(w_t))\Big)$$

This implies that

$$h \geq \Delta \cdot \Big(f_t \cdot \beta_t + (1-f_t)\cdot w_t\Big)$$

which is strict whenever $A(w_t) > 0$. □

Lemma 13. *Contract $\mathcal{C}^R$ satisfying* (24) *is incentive compatible if and only if* (24) *holds with equality with probability* 1*.*

Proof. Consider an agent's relaxed best response to a contract problem in which agent's effort cost if he deviates is given by

$$h_t(a) = \pi_t h < h$$

where

$$\pi_t = e^{-\int_0^t \Delta(1-f_s)(1-a_s)\,ds}.$$

Function π_t is a decreasing function in time and $\dot{\pi}_t \leq 0$ if and only if $(1-f_t)(1-a_t) > 0$. In such a relaxed problem the agent's effort cost permanently decreases after a deviation to low effort. This makes deviations more profitable. The payoff of the agent under such an effort cost out of equilibrium is given by:

$$\max_a \mathrm{E}\left[e^{-r\tau}\mathbb{1}\{Z_\tau = 0\}w_\tau - \int_0^\tau h_t(a_t)\,dt\right] \geq \max_a \mathrm{E}\left[e^{-r\tau}\mathbb{1}\{Z_\tau = 0\}w_\tau - \int_0^\tau ha_t\,dt\right]$$

The agent's continuation value along the equilibrium path is w_t. The agent's continuation value if he deviates to alternative effort profiles is then given by $\hat{q}(w, \pi) = \pi \cdot w$. The incentives to exert effort are thus independent of the history. Necessary and sufficient incentive compatibility conditions are given by

$$\frac{\pi_t h}{\Delta} \leq \pi_t f_t \beta_t + \pi_t(1 - f_t)w_t,$$
$$\frac{h}{\Delta} \leq f_t\beta_t + (1 - f_t)w_t.$$

This implies that if (24) holds, then one of the agent's best responses is to not exert effort. In the latter case however the effort costs are 0 since he does not exert any effort and, the fact that the problem was relaxed, does not matter. Contradiction. □

This sequence of lemmas imply that the optimal renegotiation-proof contract sets

$$\frac{h}{\Delta} = f_t \cdot \beta_t + (1 - f_t) \cdot w_t.$$

□

Lemma 14. *Suppose the agent does not observe X^A. For any incentive compatible contract (C, F) there exists a weakly more profitable incentive compatible contract $(\hat{C}, F)$ such that $\hat{C} = 0$ if $Z_\tau = 0$.*

Proof. For a given effort process $a = \{a_t\}_{t\geq 0}$ the cumulative intensity of arrival of processes Y and Z along the path of high effort are given by

$$\Lambda_t^Y = \int_0^t f_s \lambda\,ds,$$
$$\Lambda_t^Z = \int_0^t (1 - f_s)\lambda\,ds.$$

For an alternative effort $\hat{a} = \{\hat{a}_t\}_{t \geq 0}$ we have

$$\hat{\Lambda}^Y_t = \int_0^t f_s \lambda(\hat{a}_s)\, ds,$$
$$\hat{\Lambda}^Z_t = \int_0^t (1 - f_s)\lambda(\hat{a}_s)\, ds.$$

Define two random processes (ξ^I_t, ξ^D_t):

$$\xi^Y_t = \left(\frac{\hat{\Lambda}^Y_t}{\Lambda^Y_t}\right)^{X^Y_t} \cdot e^{\Lambda^Y_t - \hat{\Lambda}^Y_t},$$
$$\xi^Z_t = \left(\frac{\hat{\Lambda}^Z_t}{\Lambda^Z_t}\right)^{X^Z_t} \cdot e^{\Lambda^Z_t - \hat{\Lambda}^Z_t}.$$

It is a standard result that process ξ^Y_t is the Radon-Nikodym derivative between probability measures X^Y_t and $\hat{X}^Y_t$. Since $a_t = 1$ and $\hat{a}_t \leq 1$ we have

$$\xi^Y_t \geq e^{\Lambda^Y_t - \hat{\Lambda}^Y_t}$$
$$\xi^Z_t \geq e^{\Lambda^Z_t - \hat{\Lambda}^Z_t}$$

The probability that there were no unseen negative shocks are

$$\hat{p}_t = \mathrm{P}_{\hat{a}}(Z_t = 0) = \mathrm{E}_{\hat{a}}\left[\mathbb{1}\{Z_t = 0\}\right] = \mathrm{E}_a\left[\mathbb{1}\{Z_t = 0\} \cdot \xi^Z_t\right] = e^{\Lambda^Z_t - \hat{\Lambda}^Z_t} \cdot \mathrm{E}_a\left[\mathbb{1}\{X^Z_t = 0\}\right] = e^{\Lambda^Z_t - \hat{\Lambda}^Z_t} \cdot p_t,$$

where $p_t = \mathrm{P}_{\bar{a}}(Z_t = 0)$. Thus

$$\frac{\hat{p}_t}{p_t} = e^{\Lambda^Z_t - \hat{\Lambda}^Z_t}.$$

Now I show that under the old contract the agent's payoff under the alternative effort profile $\hat{a}$ delivers the agent a weakly higher value than the new contract. Thus if the original contract was incentive compatible, the new contract is also incentive compatible.

$$\begin{aligned}
\hat{w}_t &= \mathrm{E}_{\hat{a},t}\left[e^{-r(\tau-t)}C_\tau\right] \\
&= \mathrm{E}_{\bar{a},t}\left[e^{-r(\tau-t)}C_\tau \cdot \xi^Y_\tau \cdot \xi^Z_\tau\right] \\
&\geq \mathrm{E}_{\bar{a},t}\left[e^{-r(\tau-t)}C_\tau \cdot \xi^Y_\tau \cdot e^{\Lambda^Z_\tau - \hat{\Lambda}^Z_\tau}\right] \\
&= \mathrm{E}_{\bar{a},t}\left[e^{-r(\tau-t)}C_\tau \cdot \xi^Y_\tau \cdot \frac{\hat{p}_\tau}{p_\tau}\right] \\
&= \mathrm{E}_{\hat{a},t}\left[e^{-r(\tau-t)}w_\tau \cdot \hat{p}_\tau\right] \\
&= \mathrm{E}_{\hat{a},t}\left[e^{-r(\tau-t)}C_\tau\right]
\end{aligned}$$

Thus the agent gets compensated less under the new contract if he deviates. Thus if a deviation is profitable under the new contract, then the original contract was not incentive compatible. □

References

G. Baker, R. Gibbons, and K. J. Murphy. Subjective performance measures in optimal incentive contracts. *The Quarterly Journal of Economics*, 109(4):1125–1156, 1994.

G. P. Baker. Incentive contracts and performance measurement. *Journal of Political Economy*, pages 598–614, 1992.

R. Benabou and J. Tirole. Self-confidence and personal motivation. *The Quarterly Journal of Economics*, 117(3):871–915, 2002.

R. Benabou and J. Tirole. Intrinsic and extrinsic motivation. *The Review of Economic Studies*, 70 (3):489–520, 2003.

B. Biais, T. Mariotti, J.-C. Rochet, and S. Villeneuve. Large risks, limited liability, and dynamic moral hazard. *Econometrica*, 78(1):73–118, 2010a.

B. Biais, T. Mariotti, J.-C. Rochet, and S. Villeneuve. Large risks, limited liability, and dynamic moral hazard. *Econometrica*, 78(1):73–118, 2010b.

P. Brémaud. Point processes and queues: martingale dynamics. 1981.

M. Chen, P. Sun, and Y. Xiao. Optimal monitoring schedule in dynamic contracts. 2017.

P. DeMarzo and Y. Sannikov. Optimal security design and dynamic capital structure in a continuoustime agency model. *The Journal of Finance*, 61(6):2681–2724, 2006.

P. M. DeMarzo and M. J. Fishman. Optimal Long-Term Financial Contracting. *Review of Financial Studies*, 20(6):2079–2128, July 2007.

W. Fuchs. Contracting with repeated moral hazard and private evaluations. *American Economic Review*, 2007.

M. Harris and A. Raviv. Some results on incentive contracts with applications to education and employment, health insurance, and law enforcement. *American Economic Review*, 68(1):20–30, 1978.

F. Hoffmann, R. Inderst, and M. Opp. Only time will tell: A theory of deferred compensation. 2017.

B. Holmstrom. Moral hazard and observability. *The Bell Journal of Economics*, pages 74–91, 1979.

A. S. Hornstein and M. Zhao. Corporate capital budgeting decisions and information sharing. *Journal of Economics and Management Strategy*, 20(4):1135–1170, 2011.

C. Laux. Limited-liability and incentive contracting with multiple projects. *RAND Journal of Economics*, pages 514–526, 2001.

A. Lizzeri, M. A. Meyer, and N. Persico. The incentive effects of interim performance evaluations. 2002.

W. B. MacLeod. Optimal contracting with subjective evaluation. *American Economic Review*, 93 (1):216–240, 2003.

G. Manso. Motivating innovation. *The Journal of Finance*, 66(5):1823–1860, 2011.

K. R. Murphy and J. Cleveland. *Understanding performance appraisal: Social, organizational, and goal-based perspectives*. Sage, 1995.

D. Orlov, A. Skrzypacz, and P. Zryumov. Persuading the regulator to wait. 2018.

T. Piskorski and M. M. Westerfield. Optimal dynamic contracts with moral hazard and costly monitoring. *Journal of Economic Theory*, 166:242–281, 2016.

R. Rajan, H. Servaes, and L. Zingales. The cost of diversity: The diversification discount and inefficient investment. *The Journal of Finance*, 55(1):35–80, 2000.

K. Ray. The retention effect of withholding performance information. *Accounting Review*, 82(2): 389–425, 2007.

Y. Sannikov. A continuoustime version of the principal–agent problem. *Review of Economic Studies*, 75(3):957–984, 2008.

Y. Sannikov. Moral hazard and long-run incentives. *Unpublished working paper, Princeton University*, 2014.

D. Scharfstein and J. Stein. The dark side of internal capital markets: Divisional rentseeking and inefficient investment. *The Journal of Finance*, 55(6):2537–2564, Nov. 2000.

A. Smolin. Dynamic evaluation design. 2017.

F. Varas. Contracting Timely Delivery with Hard to Verify Quality. 2013.

J. Zhu. Optimal Contracts with Shirking. *Review of Economic Studies*, 80(2):812–839, Apr. 2013.